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Dynamic Matching in School Choice
Efficient Seat Reassignment after Late Cancellations

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School Choice

Goal: Let students choose where they go to school
School Choice: Theory
[Balinski & Sönmez '99, Abdulkadiroglu & Sönmez '03]

Students
Have preferences (rank-ordered lists)

Schools
give priority to students (e.g. neighborhood, sibling)

Goal: Let students choose their school while respecting priorities
School Choice: Theory Informs Practice
(e.g. NYC, Boston, Denver, Washington D.C., Seattle, New Orleans)

December → March → September

Deferred Acceptance
Round 1 Assignment
School Starts

NYC [Abdulkadiroglu, Pathak & Roth '05]; Boston [Abdulkaridoglu et al. '05]
NYC School Choice: Practice

~80,000 seats assigned*

Deferred Acceptance
Round 1 Assignment

School Starts

Round 1 Outcome: Most efficient assignment that respects priorities

*NYC Public High Schools, 2004-2011 average
NYC School Choice: Practice

December

March

~80,000 seats assigned*

Deferred Acceptance

Round 1 Assignment

September

School Starts

Round 1 Outcome: Most efficient assignment that respects priorities

*NYC Public High Schools, 2004-2011 average
NYC School Choice: Practice

December  
March  
September

~8,000 (10%) seats vacated

Deferred Acceptance  
Round 1 Assignment  
School Starts

School Start Outcome: Inefficient assignment subject to respecting priorities

*NYC Public High Schools, 2004-2011 average
Our Contribution

December

March

September

~8,000 (10%) seats vacated

Deferred Acceptance

Round 1 Assignment

Reassignment

School Starts

School Start Outcome: Efficient assignment subject to respecting priorities

*NYC Public High Schools, 2004-2011 average
School Choice: Practice

~8,000 (10%) seats vacated

Waitlist System
(e.g. NYC K, Boston, Denver, Washington D.C., Seattle)

Students waitlisted at all schools they prefer to their Round 1 assignment

*NYC Public High Schools, 2004-2011 average
School Choice: Practice

- December
- March
- September

~8,000 (10%)* seats vacated

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Waitlist:

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School Choice: Practice

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School Choice: Practice

December | March | September

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(e.g. NYC K, Boston, Denver, Washington D.C., Seattle)

Students waitlisted at all schools they prefer to their Round 1 assignment

Waitlist: [Diagram of people and buildings]

*NYC Public High Schools, 2004-2011 average
School Choice: Practice

December

March

September

~8,000 (10%)* seats vacated

Poor operations:
Congestion,
High cost of reassignment

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School Choice: Practice

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*NYC Public High Schools, 2004-2011 average
School Choice: Practice

December  March  September

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Congestion, High cost of reassignment

Waitlist System
(e.g. NYC K, Boston, Denver, Washington D.C., Seattle)

Students waitlisted at all schools they prefer to their Round 1 assignment

Waitlist:

*NYC Public High Schools, 2004-2011 average
School Choice: Practice

- Poor operations: Congestion, High cost of reassignment
- Poor allocative efficiency: No time to reach efficient assignment

Waitlist System
(e.g. NYC K, Boston, Denver, Washington D.C., Seattle)
Students waitlisted at all schools they prefer to their Round 1 assignment

~8,000 (10%) seats vacated

*NYC Public High Schools, 2004-2011 average"
Reassignment Problem

~8,000 (10%) seats vacated

Goal: Design a seat reassignment mechanism with...

- **Smooth operations:**
  No congestion, minimal reassignment

- **High allocative efficiency:**
  Students assigned to schools they most want while respecting priorities

- **Good incentives:**
  Students want to enter in both rounds and report their preferences truthfully

Same Round 1
Our Contribution

We propose a seat reassignment mechanism with...

- Smooth operations:
  No congestion, minimal reassignment

- High allocative efficiency:
  Students assigned to schools they most want while respecting priorities

- Good incentives:
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December, March, September

~8,000 (10%) seats vacated
Our Contribution

1. Reassignment mechanisms with desirable properties
   Permuted Lottery Deferred Acceptance (PLDA) mechanisms
   Exploit indifference classes in school priorities
   Characterize in terms of incentive and efficiency properties

2. Optimizing over PLDA mechanisms
   High efficiency: Sufficient condition for equivalent assignments
   Smooth operations: Reverse lottery minimizes reassignment

3. Simulations using NYC Public High School Data
   Corroborates theoretical findings
Possible Solutions

Assign Seats Later
- Students know outside options
- All school systems want to be last
- Public schools have no commitment devices

Oversubscribe Schools
- Currently used by many principals
- Unknown demand (difficult to predict)
- Costly to accommodate additional students

Coordinate Reassignment
- Maximize welfare and minimize reassignment
- Provide correct incentives for students
- Small change to existing system
Possible Solutions

Assign Seats Later
- Students know outside options
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Coordinate Reassignment
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Related Literature

Mechanism Design
- Mechanisms with desirable properties (e.g. strategyproofness, fairness, efficiency)
  [Compte & Jehiel ‘08, Kesten ‘10, Morrill ‘15, Roth & Sotomayor ‘90]
- Axiomatic characterizations of mechanisms
  [Abdulkadiroglu et al. ‘10, Che & Tercieux ‘15, Morrill ‘13]

Operational Issues
- Policy levers, e.g. priority tie-breaking
  [Abdulkadiroglu et al. ‘09, Arnosti ‘15, Ashlagi et al. ‘15]
- Optimization in school choice
  School choice as assortment planning [Shi ‘17],
  Optimal allocation without money [Ashlagi & Shi ‘15],
  Optimization with TTC [Leshno & L. ‘17]

Reassignment
- Dynamic stable matching
  [Compte & Jehiel ‘08, Combe et al. ‘16, Kesten & Dur ‘17, Westkamp ‘12]
  [Bando ‘12, Kennes ‘14, Pereyra ‘13]
  [Damiano & Lam ‘05, Doval ‘16, Kadam & Kotowski ‘14, Kurino ‘09]

Continuum Model
- Fluid models
  [Adlakha & Johari ‘13, Balseiro et al. ‘15, Talluri & van Ryzin ‘06, Whitt ‘02]
- Queueing Policies
  [Lee & Srinivasan ‘89, Whitt ‘02]
Outline

1. A Class of Reassignment Mechanisms

2. Optimizing over PLDA mechanisms

3. Simulations using NYC Public High School Data
1. A Class of Reassignment Mechanisms

The Permutated Lottery Deferred Acceptance (PLDA) mechanisms
Deferred Acceptance

[ Gale & Shapley '62 ]

Input:

- **Students**
  - have strict ordinal preferences $\succ$ over schools

- **Schools**
  - have strict priorities $\succ^s$ over students

Output: Efficient allocation that respects priorities
Deferred Acceptance

[[Gale & Shapley '62]]

**Input:**

- **Students**
  - have strict ordinal preferences $\succ$ over schools

- **Schools**
  - have strict priorities $\succ^S$ over students

**Deferred Acceptance** [[Gale & Shapley '62, Azevedo & Leshno '16]]

- Each school has an admissions cutoff
- Students above the admissions cutoff for a school have the school in their **virtual assortment**
- Students choose favorite school in their virtual assortment - clears the market

**Output:** Efficient allocation that respects priorities
Deferred Acceptance

Input:

Students

have strict ordinal preferences

\( \succeq \)

over schools

Schools

have strict priorities \( \succ \) over students

Deferred Acceptance [Gale & Shapley ’62, Azevedo & Leshno ’16]

Each school has an admissions cutoff

Students above the admissions cutoff for a school have the school in their virtual assortment

Students choose favorite school in their virtual assortment - clears the market

Output: Efficient allocation that respects priorities
Deferred Acceptance with Single Tie-Breaking

**Input:**

- **Students:** have strict ordinal preferences \(\succ\) over schools
- **Schools:** turn weak priority groups into strict priorities

**Single tie-breaking:** Break ties within priority groups at each school by giving each student a random lottery number \(L\), single lottery \(L\) used at all schools

**Output:** Efficient allocation that respects priorities
Deferred Acceptance with Single Tie-Breaking

Input:

Students
have strict ordinal preferences
\( \succ \) over schools

Schools
turn weak priority groups into strict priorities

Single tie-breaking: Break ties within priority groups at each school by giving each student a random lottery number \( L \), single lottery \( L \) used at all schools

Output: Efficient allocation that respects priorities
School Choice in NYC

- December
- March: ~8,000 (10%) seats vacated
- September

Round 1
Deferred Acceptance
Single Tie-Breaking

Current "Round 2"
Waitlist System
Deferred Acceptance with Single Tie-Breaking

Input:

- **Students**: have strict ordinal preferences \( \succ \) over schools
- **Schools**: turn weak priority groups into strict priorities

Single tie-breaking: Break ties within priority groups at each school by giving each student a random lottery number \( L \), single lottery \( L \) used at all schools

Output: Efficient allocation that respects priorities
School Choice in NYC

- Round 1: Deferred Acceptance, Single Tie-Breaking
- Current "Round 2": Waitlist System

- ~8,000 (10%) seats vacated

Timeline:
- December
- March
- September
School Choice in NYC

- December
  - Deferred Acceptance
  - Single Tie-Breaking

- March
  - \(~8,000 \text{(10\%)}\) seats vacated

- September
  - Current "Round 2"
  - Waitlist System
Our Proposal: Permutated Lottery DA

- **Round 1**
  - Deferred Acceptance
  - Single Tie-Breaking

- **Proposed Round 2**
  - Permutated Waitlist System

- ~8,000 (10%) seats vacated
Our Proposal: Permutated Lottery DA

Round 1
Deferred Acceptance
Single Tie-Breaking

Proposed Round 2
Deferred Acceptance
with Single Tie-Breaking
with first round guarantee
via permuted Lottery P(L)
Permutated Lottery Deferred Acceptance (PLDA)

Proposed Round 2
Deferred Acceptance with Single Tie-Breaking
with first round guarantee
via permuted Lottery P(L)

Round 1 lottery order
Round 2 "forward lottery"
Round 2 "independent lottery"
Permutated Lottery Deferred Acceptance (PLDA)

Proposed Round 2

Deferred Acceptance with Single Tie-Breaking
with first round guarantee
via permuted Lottery P(L)
Main Results

Minimizing Congestion

PLDA based on reversing the lottery minimizes #reassignments

Round 1 lottery order

Round 2 "reverse lottery"
Main Results

Minimizing Congestion

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Why PLDA Mechanisms?

PLDA mechanisms are the unique mechanisms that satisfy desirable incentive and efficiency properties
Main Results

Minimizing Congestion

PLDA based on reversing the lottery minimizes #reassignments

Why PLDA Mechanisms?

PLDA mechanisms are the unique mechanisms that satisfy desirable incentive and efficiency properties

Minimizing Reassignment vs. Maximizing Efficiency

All PLDA mechanisms give the same final assignment under natural conditions on aggregate demand.
Model
Model

Schools $S = s_1, s_2, \ldots, s_n$, outside option $s_{n+1} \notin S$

Capacities $q_i$

Student types $\theta = \left(\succ^\theta, \sim^\theta, p^\theta\right) \in \Theta$

first round preferences $\succ^\theta$, second round preferences $\sim^\theta$

priority groups $p^\theta$ at each school

Students are given by $\lambda = (\theta^\lambda, l^\lambda) \in \Lambda = \Theta \times [0, 1]$

first round lottery number $l^\lambda \in [0, 1]$
**Model Assumptions**

**Consistent Preferences:** Students' second-round preferences obtained from first-round preferences via truncation

![Diagram showing first-round preferences and second-round preferences]

**Large Market:** Continuum model, probability measure $\eta$ over student types $\Lambda = \Theta \times [0, 1]$
Model Timeline

Outside option revealed

$(\succ, p) \quad \text{Round 1} \quad \succ \quad \text{Round 1: } \mu \text{ via DA} \quad \succ \quad \text{Round 2} \quad \succ \quad \text{Round 2: } \hat{\mu} \text{ via M}$
Model Assumptions

**Consistent Preferences:** Students' second-round preferences obtained from first-round preferences via truncation

First-round preferences  
Second-round preferences

**Large Market:** Continuum model, probability measure $\eta$ over student types $\Lambda = \Theta \times [0, 1]$ lottery
Model Timeline

Outside option revealed

$(\succ, p)$ Round 1  Round 1: $
\succ
$
report $\succ^r$
$\mu$ via DA

Round 2  Round 2:
report $\succ^r$
$\hat{\mu}$ via M
Model Timeline

Outside option revealed

\((\succ, p)\) Round 1 Round 1: report \(\succ^r\) \(\mu\) via DA

Same Round 1: Deferred Acceptance with Single Tie-breaking

\(\prec\) Round 2 Round 2: report \(\prec^r\) \(\hat{\mu}\) via M
Model Timeline

Outside option revealed

$(\succ, p)$ Round 1 Round 1: $
succ$
report $
r^*$
$
\mu$
via DA

Round 2 Round 2:
report $
succ^r$
$\hat{\mu}$
via M

Proposed Round 2:
Deferred Acceptance with permuted
Single Tie-breaking
Why PLDA Mechanisms?
Why PLDA Mechanisms?

- PLDA mechanisms inherit the **nice properties** of DA.

**Theorem.** PLDA mechanisms:
- Respect guarantees and priorities
- Are constrained Pareto efficient
- Are two-round strategy-proof
Why PLDA Mechanisms?

☐ PLDA mechanisms inherit the nice properties of DA.

**Theorem.** PLDA mechanisms are the **only** mechanisms that:

- Respect guarantees and priorities
- Are constrained Pareto efficient
- Are two-round strategy-proof
- Are anonymous and averaging

and whose first round is DA with single tie-breaking, when every school has a single priority group.
Why PLDA Mechanisms?

- PLDA mechanisms inherit the **nice properties** of DA.

**Theorem.** PLDA mechanisms:
- Respect guarantees and priorities
- Are constrained Pareto efficient
- Are two-round strategy-proof

- PLDAs are the **only** mechanisms with these properties.
Why PLDA Mechanisms?

- PLDA mechanisms inherit the **nice properties** of DA.

**Theorem.** PLDA mechanisms:
- Respect guarantees and priorities
- Are constrained Pareto efficient
- Are two-round strategy-proof

- PLDAs are the **only** mechanisms with these properties.

- PLDA mechanisms are **simple to explain and implement.**

  - Run Deferred Acceptance again! (Centralized PLDA)
  - Use waitlist system again! (Decentralized PLDA)
2. Optimizing over PLDA Mechanisms
Optimal PLDA Allocations

Theorem: If the order condition holds, then all PLDA mechanisms produce equivalent allocations.

Theorem: The reverse lottery (RLDA) allocation minimizes the measure of reassigned students among equivalent allocations.
Optimal PLDA Allocations

Theorem: If the order condition holds, then all PLDA mechanisms produce type equivalent allocations.

Theorem: The reverse lottery (RLDA) allocation minimizes the measure of reassigned students among type equivalent allocations.

Definition:
Two allocations are type equivalent if they provide each student with the same distribution for their final assignment.
Optimal PLDA Allocations

Theorem: If the order condition holds, then all PLDA mechanisms produce type equivalent allocations.

Theorem: The reverse lottery (RLDA) allocation minimizes the measure of reassigned students among type equivalent allocations.

Definition:
Two allocations are type equivalent if they provide each student with the same distribution for their final assignment. Equivalently, for every student type they assign the same measure of that type to each school:

\[ \eta \left( \{ \lambda : \theta^\lambda = \theta, \hat{\mu}(\lambda) = s_i \} \right) = \eta \left( \{ \lambda : \theta^\lambda = \theta, \hat{\mu}'(\lambda) = s_i \} \right) \quad \forall \theta \in \Theta, s_i \in S. \]
Reverse Lottery Minimizes Reassignment

Theorem: If the order condition holds, then all PLDA mechanisms produce type equivalent allocations.

**Theorem:** The reverse lottery (RLDA) allocation minimizes the measure of reassigned students among type equivalent allocations.

**Proof idea:** Reversing the lottery shortcuts the improvement chains.

*E.g.*

Forward lottery:

\[ s_4 > s_3 > s_2 > s_1 \]

Reverse lottery:

\[ s_1 > s_2 > s_3 > s_4 \]
Reverse Lottery Minimizes Reassignment

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Equivalence of PLDA Allocations

**Theorem:** If the order condition holds, then all PLDA mechanisms produce type equivalent allocations.
Equivalence of PLDA Allocations

Theorem: If the order condition holds, then all PLDA mechanisms produce type equivalent allocations.

Definition:
The order condition holds if schools have the same order of popularity in both rounds.
Equivalence of PLDA Allocations

Theorem: If the **order condition** holds, then all PLDA mechanisms produce type equivalent allocations.

**Definition:**
The **order condition** holds if schools have the same order of popularity in both rounds.

E.g. Common Ranking
Equivalence of PLDA Allocations

**Theorem:** If the *order condition* holds, then all PLDA mechanisms produce type equivalent allocations.

**Definition:**
The *order condition* holds if schools have the same order of popularity in both rounds.

E.g. Common Ranking
E.g. Uniform Departures
Equivalence of PLDA Allocations

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E.g. Common Ranking
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Equivalence of PLDA Allocations

Theorem: If the order condition holds, then all PLDA mechanisms produce type equivalent allocations.

Definition:
The order condition holds if schools have the same order of popularity in both rounds under RLDA:

\[ C_{\pi,i} > C_{\pi,j} \Rightarrow \hat{C}_{\pi,i}^R \geq \hat{C}_{\pi,j}^R \text{ for all } \pi, i, j. \]
Equivalence of PLDA Allocations

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"Round 2 RLDA cutoffs are in the same order as Round 1 cutoffs"
Equivalence of PLDA Allocations

**Theorem:** If the *order condition* holds, then all PLDA mechanisms produce type equivalent allocations.

**Definition:**
The *order condition* holds if schools have the same order of popularity in both rounds under RLDA.

**Remark:** When the order condition does not hold, PLDA mechanisms do not always produce type equivalent allocations.
Equivalence of PLDA Allocations

Proof Intuition:
When the order condition holds, all student types have the same distribution of virtual assortments in the second round

Virtual assortment:
Set of schools for which a student is above the admissions cutoff
Equivalence of PLDA Allocations

Proof Intuition:
When the order condition holds, all student types have the same distribution of virtual assortments in the second round.

One priority class:
First round virtual assortments: \{1,2,...,n\}, \{2,...,n\}, ..., \{n\}

Order condition:
RLDA second round virtual assortments: \{1,2,...,n\}, \{2,...,n\}, ..., \{n\}
RLDA virtual assortments: \{1,2,...,n\}, \{2,...,n\}, ..., \{n\}
Equivalence of PLDA Allocations

Proof Intuition:
When the order condition holds, all student types have the same distribution of **virtual assortments** in the second round.

One priority class:
First round virtual assortments: \{1,2,\ldots,n\}, \{2,\ldots,n\}, \ldots, \{n\}

Order condition:
RLDA second round virtual assortments: \{1,2,\ldots,n\}, \{2,\ldots,n\}, \ldots, \{n\}
RLDA virtual assortments: \{1,2,\ldots,n\}, \{2,\ldots,n\}, \ldots, \{n\}

**RLDA cutoffs are maximally misaligned**
Any PLDA virtual assortments: \{1,2,\ldots,n\}, \{2,\ldots,n\}, \ldots, \{n\}
Equivalence of PLDA Allocations

Proof Intuition:
When the order condition holds, all student types have the same distribution of virtual assortments in the second round.

One priority class:
1. Guess PLDA cutoffs that match RLDA virtual assortments.
2. PLDA cutoffs in the same order as first round cutoffs, as the RLDA cutoffs are maximally misaligned.
3. Compare with RLDA virtual assortments to show that guessed cutoffs are market-clearing.
Equivalence of PLDA Allocations

**Proof Intuition:**
When the order condition holds, all student types have the same distribution of *virtual assortments* in the second round

**One priority class:**
1. Guess PLDA cutoffs that match RLDA virtual assortments
2. PLDA cutoffs in the same order as first round cutoffs, as the **RLDA cutoffs are maximally misaligned**
3. Compare with RLDA virtual assortments to show that guessed cutoffs are market-clearing

**General priorities:**
Reduce to the case with one priority class
3. Simulations on NYC School Data
Simulations: Data

NYC High School Admissions Data*

- Student preferences and school priorities
  (2004-2005 main round of admissions)
- Enrollment in 2005-2006 school year

Simulations

- Student first-round preferences, school priorities and capacities
directly from data
- Student second-round preferences from enrollment data:
  assume all students either leave or retain the same preferences

*2004-2005 NYC High School Admission, N=81,884, n=653
Simulations: Reassignment

Family of PLDA mechanisms with parameter $\alpha$

$$P(l) = \alpha l + l'$$

$l, l' \sim \mathcal{N}(0, 1)$ independent

2004-2005 NYC High School Admission, $N=81,884$, $n=653$
Simulations: Allocative Welfare

Table 1: Simulation Results, 2004-2005 NYC High School admissions

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$k = 1$ (%)</th>
<th>$k \leq 2$ (%)</th>
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<tbody>
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<td>50.27</td>
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<td>1.00</td>
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<td>0.00</td>
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<td>RLDA: $-\infty$</td>
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2004-2005 NYC High School Admission, N=81,884, n=653
## Simulations: Allocative Welfare

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**Remark:** Table gives allocative efficiency of final assignment. In practice, decentralized FLDA waitlists take months to clear and may never reach the final assignment.

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2004-2005 NYC High School Admission, $N=81,884$, $n=653$
Simulations Match Theory

- PLDA mechanisms give similar allocative welfare

- RLDA minimizes reassignment

This occurs even though the order condition does not hold.
Reassignment in School Choice

Goal: Reassignment Mechanism M with

- Smooth operations:
  Minimal reassignment

- High allocative efficiency:
  Students assigned to schools they want

- Good incentives:
  Students report preferences truthfully
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Goal: Reassignment Mechanism M with

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- Good incentives:
  Students report preferences truthfully

Same Round 1

Reverse the Lottery!
Reassignment in School Choice

Goal: Reassignment Mechanism $M$ with

- Smooth operations: Minimal reassignment
- High allocative efficiency: Students assigned to schools they want

Good incentives: Incentive compatible, respects guarantees and priorities, constrained Pareto efficient

Characterization Theorem: $M$ is a PLDA mechanism.
Reassignment in School Choice

- **December**
- **March**
- **September**

**Goal:** PLDA Mechanism M with

- **Smooth operations:**
  Minimal reassignment

- **High allocative efficiency:**
  Students assigned to schools they want

- **Good incentives:**
  Incentive compatible, respects guarantees and priorities, constrained Pareto efficient

**Properties of Round 1**

**Same Round 1**

**Key Idea:** Correlating the lotteries lets us control reallocation
Reassignment in School Choice

Goal: PLDA Mechanism M with

- Smooth operations:
  Minimal reassignment

- High allocative efficiency:
  Students assigned to schools they want

- Good incentives:
  Incentive compatible, respects guarantees and priorities, constrained Pareto efficient

Type Equivalence: All PLDAs have same allocative efficiency*

*when the order condition holds
Reassignment in School Choice

- **Goal:** PLDA Mechanism $M$ with
  - **Smooth operations:** Minimal reassignment
  - **High allocative efficiency:** Students assigned to schools they want
  - **Good incentives:** Incentive compatible, respects guarantees and priorities, constrained Pareto efficient

Reverse Lottery Minimizes Reassignment*

*when the order condition holds*
Reassignment in School Choice

**Goal:** PLDA Mechanism $M$ with

- **Smooth operations:** Minimal reassignment
- **High allocative efficiency:** Students assigned to schools they want
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NYC High School Admissions Data: Simulations Match Theory
Reassignment in School Choice

Proposal to NYC and BPS:

- Continue using current decentralized waitlists
- Reverse the lottery before providing schools with waitlists
  - Maintain allocative welfare
  - Significantly reduce congestion
  - 'More fair'
Reassignment in School Choice

Implementation in NYC and BPS systems
Data and field experiment for NYC Kindergarten Gifted and Talented school assignment

Extensions
"Inertia bonus" for staying at the Round 1 assigned school
Incorporating student arrivals
Reassignment in School Choice

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My Research Agenda

Goal: Smoothly operating marketplace

Can we improve operations in an existing marketplace?

Can we design assignment algorithms for new marketplaces?
My Research Agenda

Goal: Smoothly operating marketplace

Can we improve operations in an existing marketplace?
- Reassignment in school choice: Virtual assortments [this talk]
- TTC in school choice: Cutoff structure of TTC [Leshno & Lo '17]
- Costly information acquisition [Imgorlica, Leshno, Lo & Lucier '17]

Can we design assignment algorithms for new marketplaces?
- Reassignment: Permutated Lottery Deferred Acceptance [this talk]
- Price-Discovery with Hidden Gross Substitutes [Lo, Sidford & Weyl]
- Costly information acquisition [Imgorlica, Leshno, Lo & Lucier '17]
Managing Marketplace Information

Question:
Can matching platforms improve welfare by facilitating learning?

E.g. Stable Matching:
Learning about preferences is often costly.

How can matching platforms reduce unnecessary costly learning?

Matching Markets with Costly Information Acquisition
[Nicole Immorlica, Jacob Leshno, L., Brendan Lucier]
My Research Agenda

- Donations
- Subscriptions
- Ridesharing
- Housing
- School Choice
- Healthcare

Goal:
Smoothly Operating Marketplace
My Research Agenda

Operations

Goal:
Smoothly Operating Marketplace

Donations

Ridesharing

Economics
Game Theory

School Choice

Computer Science

Subscriptions

Housing

Ny state of Health

Healthcare
Thank You!