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Dashboard Mechanisms for Online Markets

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A Grand Challenge: understand and guide computation in the wild

- computational primitive: local/individual/strategic optimization.
- objective: good global outcomes
- a key application area: “online markets”
  - uber, airbnb, twitter, wikipedia, tinder, mechanical turk, ...

Observation: most environments do not allow “truthful mechanisms”; need theory for non-truthful mechanism design.
Assumptions for Online Markets

Basic Assumptions: cf. online advertising.

- longlived agents with persistent values: \( v = (v_1, \ldots, v_n) \).
- good algorithm \( x(v) = (x_1(v), \ldots, x_n(v)) \) exists for selecting allocation from agent values; \( x \) is monotone.
- environment is stochastic (\( x(\cdot) \) is continuous and differentiable.)
- required standard non-truthful payment rule, e.g., winner-pays-bid.

Basic Question: is there a winner-pays-bid mechanism \( \hat{x} \) that implements algorithm \( x \) in equilibrium* for all \( v \).
(for what \( x \) does such a mechanism exist?)
Context and Results

Context: “price of anarchy” for winner-pays-bid auctions:

- substitutes: constant [e.g. Lucier, Borodin ’10; Syrgkanis, Tardos ’13]
- complements: linear [e.g. Lucier, Borodin ’10; Dütting, Kesselheim ’15]

Results:

1. general frameworks for winner-pays-bid mechanisms, c.f., VCG.

2. Nash implementation:
   - multi-unit: can implement $x \in$ proportional weights.
   - substitutes: open question
   - complements: cannot implement proportional weights.

3. persuadable agents: can implement any monotone continuous $x$. 
Basic Question: is there a winner-pays-bid mechanism $\tilde{x}$ that implements algorithm $x$ in Nash equilibrium for all $v$. (for what $x$ does such a mechanism exist)

Definitions:

- agents have values: $v = (v_1, \ldots, v_n)$
- winner-pays-bid mechanism $\tilde{x}$ on bids $b = (b_1, \ldots, b_n)$.
  - allocation to $i$: $\tilde{x}_i(b)$
  - payment from $i$: $b_i \tilde{x}_i(b)$
  - utility of $i$: $(v_i - b_i) \tilde{x}_i(b)$
- bid function: $\beta$ gives Nash equilibrium for all $v$:
  $$\beta_i(v_i) \in \arg\max_b (v_i - b) \tilde{x}_i(b, \beta_{-i}(v_{-i}))$$
- $\tilde{x}$ implements $x$ if $x(v) = \tilde{x}(\beta(v))$.

Example Algorithm: proportional weights: map values to weights, selects winner w.p. proportional to weight; e.g., exponential weights.
Main Idea

Main Idea: combine inference with implementation.

Example: two-agent proportional-bid mechanism

- allocation rule: $\tilde{x}_1(b) = b_1 / (b_1 + b_2)$.
- For equilibrium bids $b$, agent’s first-order condition identifies value:
  - Agent 1 with value: $v_1$ chooses $b_1 = \arg\max_b (v_1 - b) \tilde{x}_1(b, b_2)$
  - equilibrium bid satisfies first-order condition: $v_1 = b_1 + \tilde{x}_1(b_1) / \tilde{x}_1'(b_1) = 2b_1 + b_1^2 / b_2$.
  - E.g., for $b_1 = 1, b_2 = 2$ obtain: $v_1 = 5/2$.

Consequence: in hindsight mechanism know agents’ values exactly!

Goal: use this idea to implement any algorithmic outcome $x$. 
Thm: [Myerson '81] a mechanism \( \tilde{x} \) and bid function \( \beta \) is in BNE iff 
\[
x(v) = \tilde{x}(\beta(v))
\]
satisfies:

1. **monotonicity (M):** \( x_i(v) \) is monotone in \( v_i \).
2. **payment identity (PI):** 
   \[
   p_i(v) = v_i x_i(v) - \int_0^{v_i} x_i(z, v_{-i}) \, dz + p_i(0, v_{-i})
   \]
   and usually \( p_i(0, v_{-i}) = 0 \).
Main Idea: combine inference with implementation.

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Consequence: two equations for payments: (a) payment-identity and (b) winner-pays bid $p_i(v) = \beta_i(v) x_i(v)$. Solve for $\beta_i(v) = p_i(v) / x_i(v)$. 
Nash implementation

Basic Idea: \( \tilde{x}(b) = x(\beta^{-1}(b)) \)

Issues:
1. bid function \( \beta : \mathbb{R}^n \rightarrow \mathbb{R}^n \) may not be one-to-one.
2. image of \( \beta \) may not be product space. [see paper]

Prop: \( \exists \) winner-pays-bid \( \tilde{x} \) for \( x \iff \beta \) for \( x \) is one-to-one.
Thm: [Myerson '81] a mechanism \( \tilde{x} \) and bid function \( \beta \) is in BNE iff

\[ x(v) = \tilde{x}(\beta(v)) \]

satisfies:

1. **monotonicity (M):** \( x_i(v) \) is monotone in \( v_i \).
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**Consequence:** two equations for payments: (a) payment-identity and (b) winner-pays bid
\[ p_i(v) = \beta_i(v) x_i(v) \]. Solve for \( \beta_i(v) = p_i(v)/x_i(v) \).
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Prop: $\exists$ winner-pays-bid $\tilde{x}$ for $x \leftrightarrow \beta$ for $x$ is one-to-one.

Thm: $\beta$ is one-to-one if Jacobian of $\beta$ is positive definite. [Gale, Nikaido ’65]

Thm: If $x$ is proportional weights for multi-unit environment then
(a) Jacobian of $\beta$ is positive definite and
(b) can compute $\beta^{-1}(b)$ in polynomial time.

Thm: If $x$ is proportional weights for environment w. complements then $\beta$ is not one-to-one.
Challenges for Nash Implementation:

- agents need to find the Nash equilibrium.
- cannot Nash implement general algorithms $x$.

Solution: publish a bidding dashboard, cf. Google, Booking.com, etc.

Definition: persuadable agents will follow dashboard if following dashboard converges to best response.

Definition: a dashboard mechanism is $(\tilde{y}, \tilde{x})$ with:

1. dashboard $\tilde{y}$: estimated bid-allocation rule $\tilde{y}_i : \mathbb{R} \rightarrow [0, 1]$ for $i$.
2. mechanism $\tilde{x}$: bid-allocation rule $\tilde{x} : \mathbb{R}^n \rightarrow [0, 1]^n$. 
Historical-Values Dashboard

**Definition:** the *historical-values dashboard mechanism* is:

1. publish bidding dashboard with bid-allocation rule for historical estimated values.
2. solicit bids: $b$
3. estimate values from bids assuming best response to dashboard: $\tilde{v}$
4. output outcome of algorithm on estimated values: $x(\tilde{v})$
5. charge winners their bids.

**Theorem:** For monotone, continuous $x$, following dashboard converges to best response in two rounds.

**Corollary:** Any monotone, continuous $x$ can be implemented for persuadable agents by a dashboard mechanism.
Connections to Literature

**Implementation Theory:** [survey: Jackson '01]
- characterize $\alpha$ where exist mechanism with unique equilibrium
- any monotone $\alpha$ is implementable in Nash equilibrium.
- sequential mechanisms, cross reporting, not practical.
- **our suggestion:** need to restrict to simple, practical mechanisms.

**Information Design:** [Kamenica, Gentzkow '11; Dughmi, Xu '16]
- agent with prior to make decision.
- informed principal signals agent.
- agent updates prior
- agent acts on posterior
- **our suggestion:** relax assumptions $\Rightarrow$ allows principal to learn
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