Optimal Filtering for Patterned Displays

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***Abstract*—Displays with repeating patterns of colored subpixels gain spatial resolution by setting individual subpixels rather than by setting entire pixels. This paper describes optimal filtering that produces subpixel values from a high-resolution input image. The optimal filtering is based on an error metric inspired by psychophysical experiments. Minimizing the error metric yields a linear system of equations, which can be expressed as a set of filters. These filters provide the same quality of font display as standard anti-aliasing at a point size 25% smaller. This optimization forms the filter design framework for Microsoft's ClearType.**

***Index Terms*—Anti-aliasing, ClearType, fonts, image processing, liquid crystal displays, optimal filtering, patterned displays.**

I. Introduction

FOR PATTERNED displays such as LCD's, a pixel is a concept, not a physical device. A patterned display consists of a repeating pattern of singly-colored subpixels, which can be grouped for convenience into full-colored pixels. There is an opportunity to increase the effective resolution of a patterned display by treating the subpixels separately. The potential luminance resolution is the subpixel resolution. However, treating each subpixel purely as a luminance source, while ignoring color, causes large amounts of color error.

The tradeoff between luminance resolution and color error can be balanced by the use of a perceptual error metric, which estimates the perceived visual effect of luminance and color errors. This paper discusses optimal filtering to produce subpixel values that minimize a perceptual error metric. The optimal filtering provides a visually pleasing balance between increased resolution and color fidelity.

In previous work, researchers have suggested minimizing a perceptual error metric in order to construct halftoning algorithms [1]–[3]. Halftoning is a nonlinear process that requires repeated evaluation of an error metric in an error diffusion loop. In contrast, this paper uses direct linear optimal filtering, which permits high-speed font rasterization.

II. A Perceptual Error Metric

The optimal filtering takes full-color image samples γ_{kd} as input, where k indexes the (perhaps multidimensional) spatial samples, and d indexes the color space of the image. For convenience, the γ_{kd} are assumed to be spatially sampled at the same positions as the subpixels.

The optimal filtering produces singly-colored image samples α_k as output, which control the luminance of each subpixel in the display. The optimal filtering chooses the α_k to be "close" to the $\gamma_{kd},$ as measured by a perceptual error metric.

The error between γ_{kd} and α_k is measured in a perceptually relevant color space. There is evidence that the human visual system separates image data into a brightness channel and two opponent color channels: red minus green and blue minus yellow [4]. The error in the opponent color space is

$$E_{ck} = M_{ck}\alpha_k - \sum_{d=0}^{2} C_{cd}\gamma_{kd} \tag{1}$$

where C_{cd} and M_{ck} are matrices that transform α_k and γ_{kd} into an opponent color space. The matrix M_{ck} encodes the spatial pattern of subpixel color.

The error E_{ck} is then transformed into frequency space. The quadratic norm of the error is measured independently at each frequency. Each frequency norm is then multiplied by a perceptually-motivated weight. Let the transformed frequency representation of the error be

$$\hat{E}_{cn} = \sum_{k=0}^{N-1} E_{ck}\phi_{kn} \tag{2}$$

where

- n indexes the (perhaps multidimensional) frequency space;
- N number of spatial samples;
- ϕ_{kn} basis functions of the frequency transformation.

The optimal filter minimizes

$$\mathcal{E} = \sum_{c=0}^{2}\sum_{n=0}^{N-1} W_{cn}\hat{E}_{cn}\hat{E}^*_{cn} \tag{3}$$

where the superscript $*$ indicates the complex conjugate. The weights W_{cn} can be determined from experiments that measure the sensitivity of the visual system to gratings of different color and frequency. The weight W_{cn} is a squared-amplitude weight.

Applying calculus to (1)–(3) yields the optimum of (3)

$$\sum_{c,\,n,\,j} W_{cn}\left(\alpha_j M_{cj} - \sum_{d} C_{cd}\gamma_{jd}\right) M_{ck}P_{kjn} = 0 \tag{4}$$

where $P_{kjn} = \phi^*_{jn}\phi_{kn} + \phi^*_{kn}\phi_{jn}$. Equation (4) is in the form of a linear system

$$\sum_{j=0}^{N-1} A_{kj}\alpha_j = r_k \tag{5}$$

where $A_{kj} = \sum_{c,\,n} W_{cn}M_{cj}M_{ck}P_{kjn}$ is a symmetric matrix and $r_k = \sum_{c,\,n,\,j,\,d} W_{cn}M_{ck}P_{kjn}C_{cd}\gamma_{jd}$. In general, the optimal α_k for a patterned display can be computed via the solution of the linear system (5).

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III. Optimal Filtering for Striped Displays

Consider the common LCD display that consists of individually addressable subpixels arranged in vertical stripes of the same color. The repeating pattern of colors is RGB. Optimal filtering is applied only in the horizontal direction perpendicular to the striping, for speed of implementation. The error is decomposed into its Fourier representation. Under these assumptions, (4) becomes

$$2\sum_{c,n,j} W_{cn}\left(\alpha_j M_{cj} - \sum_d C_{cd}\gamma_{jd}\right) \cdot M_{ck}\cos\left(\frac{2\pi(k-j)n}{N}\right) = 0 \tag{6}$$

where k is the horizontal position in the scanline, and N is the width of the scanline.

The opponent color space used in the optimization is defined by combining the XYZ coordinates for standard NTSC phosphors with the estimated opponent color coordinates for the human visual system [5]. Other phosphors can also be used. The matrix C_{cd} is thus

$$C_{cd} = \begin{pmatrix} 0.47 & 0.51 & 0.02 \\ -0.63 & 0.30 & 0.08 \\ -0.16 & -0.34 & 0.50 \end{pmatrix} \tag{7}$$

and $M_{ck} = 3C_{c,\,k \bmod 3}$.

The visual system acts as a low-pass spatial filter. The following low-pass weighting model is based on psychophysical experiments [4], [5]

$$\begin{aligned} W_0(f_n) &= \min\{1,\,(8/f_n)^8\}, \\ W_1(f_n) &= \min\{1,\,(5/f_n)^4,\,W_0(f_n)\}, \\ W_2(f_n) &= \min\{1,\,(3/f_n)^4,\,W_0(f_n)\} \end{aligned} \tag{8}$$

where f_n is the frequency in cycles per degree (CPD) of the nth Fourier coefficient. To get best results when the user is examining a screen closely, we assume that the pixel Nyquist (0.5 cycles for every three subpixels or $f_{N/6}$) is 8 CPD. This corresponds to a viewing distance of 9.17 in for a 100 ppi display.

Combining (6)–(8) yields a linear system for deriving display values α_k from input values γ_{kd}. Solving this linear system creates a linear mapping from γ_{kd} to α_k. For each color d, the linear mapping is empirically (to within machine precision) a block Toeplitz matrix, with a block size of three. The results of multiplication by the three block Toeplitz matrices are summed together to yield α_k.

Therefore, the solution to the overall linear system can be expressed as a set of nine filters, one for every combination of input and subpixel color. For every subpixel color, the outputs of the three input color filters are summed together and a correction for the display's nonlinear response is applied. Expressing the solution as nine filters is much more efficient $O(N)$ than solving a decomposed linear system $O(N^2)$.

The coefficients of the nine filters can be derived by repeatedly solving the linear system (5) for a scanline of length 300 with $\gamma_{kd} = \delta_{kl}\delta_{de}$ for some l and e. The responses of three contiguous RGB outputs are collected, as l and e are varied over a spatial range and over the input colors. These responses form the filter coefficients, as shown in Fig. 1. The x-axis of Fig. 1 is measured in subpixel units. The vertical dashed lines show the location of the three subpixels controlled by the nine filters. For example, the three filters R→R, G→R, and B→R apply to the R, G, and B input images, respectively. The outputs of these three filters are then summed together, and the sum is applied to the R subpixel whose location corresponds to the dashed line over the R.

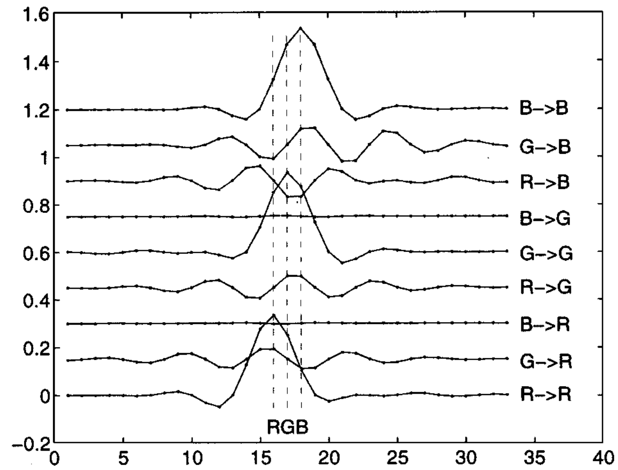

Fig. 1. Nine optimal filters for striped displays. Each filter is displaced upwards to show the shape of all the filters.

There are two differences between these optimal filters and standard antialiasing filters. First, the same-color R→R, G→G, and B→B filters look like normal antialiasing filters. However, each same-color filter is centered underneath the location of the corresponding subpixel instead of at the center of the pixel. Normal antialiasing methods compute the red and blue subpixel values as if they were coincident with the green subpixel and then display the red and blue components shifted 1/3 of a pixel to the left or right. If an object in an image contains more than one primary color and if conventional antialiasing is employed, the shifting of these primaries leads to blurring. By displacing the antialiasing filters, the optimal filters eliminate this blurring at the expense of slight color fringing.

The second difference is that all input colors are coupled to all subpixel colors. The coupling is strongest near the pixel Nyquist frequency, which adds luminance sharpness near edges.

IV. Results and Conclusions

Printing on paper is a subtractive process. Any printed image of an LCD appears too dark. One must view an RGB striped LCD to appreciate the effect of the optimal filtering. Sample digital images of Times Roman fonts filtered with both a 61-tap Lanczos filter and with the optimal filters are available at [6]. In those images, eight-point font glyphs filtered with a Lanczos filter are as crisp as six-point glyphs filtered with optimal filters. The optimal filters allow the use of 25% smaller fonts with no degradation of font quality. These smaller fonts can permit the display of approximately 1.8 times more text on an LCD than previously possible.

The derivation of these optimal filters serves as the filter design framework for Microsoft's ClearType. This derivation can easily be extended into two dimensions to sharpen the display of images of other patterned displays such as the Bayer pattern.

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