On Sensor Network Lifetime and Data Distortion

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***Abstract*— Fidelity is one of the key considerations in data collection schemes for sensor networks. A second important consideration is the energy expense of achieving that fidelity. Data from multiple correlated sensors is collected over multi-hop routes and fused to reproduce the phenomenon. However, the same distortion may be achieved using multiple rate allocations among the correlated sensors. These rate allocations would typically have different energy cost in routing depending on the network topology. We consider the interplay between these two considerations of distortion and energy. First, we describe the various factors that affect this trade-off. Second, we discuss bounds on the achievable performance with respect to this trade-off. Specifically, we relate the network lifetime L_t to the distortion D of the delivered data. Finally, we present low-complexity approximations for the efficient computation of the $L_t(D)$ bound.**

I. Introduction

Since their conception, sensor networks[1] are finding applications in a variety of problem domains spanning security, scientific explorations, education, and entertainment. The underlying system in most of these applications essentially consists of a group of sensors collecting data about a phenomenon of interest which is fused to reproduce some desired attribute of the phenomenon. In this work, we consider the performance of the data collection process in terms of the achieved distortion, as related to a key system resource: energy.

In most systems, the achievable distortion in data reconstruction is related to the minimum rate required, in the form of a rate-distortion function, that quantifies the minimum amount of rate required to achieve a certain level of distortion at the point of reconstruction. This trade-off is more critical when the data needs to be transported over a communication network, thus entailing the use of network resources such as bandwidth and access costs. Sensor networks are systems where apart from bandwidth, energy is a major resource bottleneck. It is of interest in such systems, to characterize the distortion performance in terms of energy as well. As an example consider a sensor network of wireless cameras. In many cases e.g. surveillance, it may well be that a high quality image of the scene is not always required and we may be willing to give up fidelity for extending the lifetime of the network. We wish to determine precisely this trade-off between fidelity and lifetime.

The energy consumed depends not just on the data rate but also on the routing scheme used. As an example, consider the network shown in Figure 1. The phenomenon to be sensed is present near node A, and it is to be reproduced at the base station. Some of the key issues to be considered for determining the energy distortion performance are:

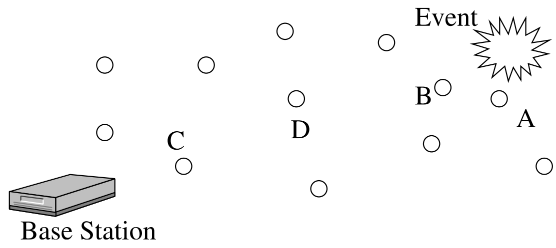

Fig. 1. A sensor network monitoring a phenomenon of interest.

Choice of Sensors: The same level of distortion may be achievable through various choices of sensors acting as data sources. For each choice of sources, the energy cost of data delivery across the network varies. For instance, sensor A may need a lower data rate, since it measures the phenomenon at high SNR but require the use of longer routes, while sensor D may need a much a larger data-rate for the same distortion but send it using a shorter route.

Choice of Routes: The cost may vary even for the same data-rate beginning with the same source sensors depending on the route used. For instance, the cost of route $A \rightarrow B \rightarrow D \rightarrow C \rightarrow Base$ will be different from $A \rightarrow Base$ due to number of nodes involved and the dependence of transmit power upon distance. Data may even be spread across multiple routes to maximally exploit the available batteries.

Protocol Overheads: The data collected is routed over multiple hops. The energy cost of initiating additional data collection at nodes already on an existing multi-hop route, may be lower than at nodes not on the chosen route due to wake-up and initiation overheads. The fidelity advantage from such nodes may however be lower than that from nodes not on the chosen route.

In-network Aggregation: As the data is routed, multiple streams from different sources may be aggregated within the network, reducing the cost of communication, though adding some processing cost, and limiting some choices of routes.

The above choices affect how energy is consumed at different nodes in the network and this expense must be subject to the battery availability at different nodes. The distribution of the sensed phenomenon is not necessarily uniform, and routing choices may be affected by this.

A. Related Work

The problems of minimizing energy cost and maximizing lifetime have been considered before for fixed data rate requirements. In [2], [3], an upper bound on the network lifetime was derived when the data source and data rate are known. A distributed procedure to find such capacity achieving routes was discussed in [4]. Other practical energy aware routing schemes have been explored as well [5], [6], [7]. Further energy considerations have also been explored, such as minimizing the transmission cost [8], [9] and the placement of the nodes for energy efficiency [10].

Our goal is to explore the trade-off between required distortion, D, (rather than a known data rate) and the achievable lifetime L_t for the system.

B. Key Contributions

We ask a new question: what is the lifetime of a sensor network attempting to reproduce a given phenomenon at a required distortion, under given sensor noise behavior, energy availability, communication energy model and routing options. The problem of determining lifetime is different from minimizing the transmission cost, as the use of multiple higher cost routes may be required to maximally exploit the energy resources. Further, our problem is different from determining the lifetime for a given data rate requirement, since we target the distortion performance. We show how to formulate this new problem in terms of known rate-distortion relationships, and also discuss a computationally tractable heuristic to solve it. To the best of our knowledge it is the first attempt to capture the lifetime-distortion relationship in a joint framework.

We model the problem and describe the relevant parameters involved in the next section. Section III shows how the energy-distortion trade-off may be determined for this system model. We then show a more computationally tractable method to determine the trade-off in section IV, and provide an illustrative example. Extensions to the problem and on-going work are discussed in section V which also concludes the paper.

II. System Overview and Problem Description

We consider the following system to model data collection in a sensor network. A network of N nodes is deployed to monitor a region of interest (Figure 2).

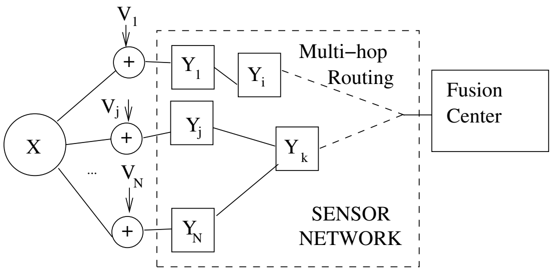

Fig. 2. System model: a subset of the sensor nodes communicate their observations to a fusion center using multi-hop routes across the network.

A. Observation Model

Suppose a phenomenon or target $\{X(t)\}_{t=1}^{\infty}$ is present at some fixed location within the region covered by the sensor network[1]. $X(t)$ is assumed to be a temporally memoryless, zero mean Gaussian random variable with variance σ_X^2.

The network has N sensors. The reading at sensor i at time t is denoted $Y_i(t)$ for $i \in \{1, ..., N\}$ and is related to X as:

$$Y_i(t) = X(t) + V_i(t) \tag{1}$$

where $V_i(t)$ is a memoryless zero mean Gaussian random variable with variance σ_i^2, that models sensor noise. Also, $\{V_i(t), V_j(t)\}$ are independent for all $i \neq j$.

The SNR at a sensor depends on its distance from the phenomenon, and we model the σ_i^2 as proportional to the square of this distance. The locations of all the nodes are known and stay fixed.

The user of the network is interested in an estimate $\hat{X}(t)$ of $X(t)$, derived from the observations $\{Y_i(t)\}_{i=1}^{N}$, with a specification on maximum distortion D. The usual mean squared error distortion measure is used:

$$D^{(k)} = \frac{1}{k}\sum_{t=1}^{k} E\left((X(t) - \hat{X}(t))^2\right) \tag{2}$$

measured over a block length k in time.

We assume that the optimal fusion algorithm to achieve the minimum possible distortion in estimation is used for reconstruction, i.e., the correlation among observations is exploited in estimation.

In our model, a subset of the observations $\{Y_i(t)\}_{i=1}^{N}$, is communicated to a common fusion center where these observations are processed to generate the estimate of the phenomenon $\hat{X}(t)$. While it is conceivable that smaller subsets of the observations may be partially aggregated as they traverse the network, and then these aggregates may be processed in turn to generate the final estimate, we consider the above model for analytical tractability.

B. Energy Model

The observations are communicated over multiple hops to the fusion center. Each transmission and reception consumes energy proportional to the number of bits transferred and the distance between the transmitter and the receiver. This is modeled as in [11]. The energy consumption per bit at the transmitter, $P_{tx}(i,j)$ i when communicating with a node j, is given as:

$$P_{tx}(i,j) \quad = \quad \alpha_1 + \alpha_2 d(i,j)^2 \tag{3}$$

where $d(i,j)$ is the distance between the transmitter and the receiver, and α_1, α_2 are radio dependent constants. The first term models a constant consumption in the radio electronics and the second term models the distance dependent transmission cost. Suppose the reception energy is P_{rx} per bit, and the

[1] An alternative formulation where the phenomenon is a field spread across a subset of the region covered by the sensor network is also of interest. However, we do not consider that model here.

energy cost of sampling the transducers is P_{sense} per bit. The energy availability at each node is denoted $\{E_i\}_{i=1}^N$.

We further assume that the distortion required is well within the data capacity of the network and only energy is the key consideration; hence we do not explicitly model the bandwidth constraints in this work.

III. Characterizing the Energy Distortion Relationship

The performance criterion of interest is the network lifetime, L_t. It is defined as the time duration for which the network can sustain the data flows required to reproduce the phenomenon at or below a specified distortion D.

Using the above energy cost and observation models, the optimization problem for finding the maximum lifetime L_t for given distortion D, can be stated as in Figure 3; the constraints are explained below. Here we assume that the initial battery available at each node is known, and the desired distortion is fixed at D. The solution to the optimization problem yields not only the achievable lifetime bound, but also the routing schedule and sensor selections.

A. Distortion Constraints

The sensors which generate data can be any subset of $\{Y_i(t)\}_{i=1}^N$. The constraint on the subset is that the rate of transmission after encoding of the data collected should be sufficient to estimate $X(t)$ at required distortion. We show how to write this constraint in terms of a known rate distortion relationship.

For a given set of sensors, $\{Y_i\}_{i=1}^N$, the required total rate R_Σ required to achieve a desired distortion is known from the $R_\Sigma(D)$ bound for the Gaussian CEO problem [12].The feasible rate allocation vectors $\mathcal{R}_N = \{R_1, ..., R_N\}$ which can help operate at that sum rate are described in [13], [14], [15], [16]. We first use these results to capture the relationship between the sensor rates and distortion. Following [14], and modifying the notation for our system model we can characterize this set to consist of all vectors $\{R_1, ..., R_N\}$ that satisfy equations (7) and (8) in Figure 3. Here $\{r_i\}_{i=1}^N$ are auxiliary variables, and are among the unknowns for the optimization.

Note that this only defines the region in which the rate vectors may be selected in order to satisfy the distortion constraints. As mentioned before the cost to the system is not the rate itself but the energy required to support this rate over the multi-hop network. Thus, the constraints on the rate vectors cannot be used by themselves to select appropriate sensors, and the energy cost of each such vector must be considered.

B. Energy Constraints

Assume that the fusion center is node $N + 1$, and is not energy constrained. The cost of each route from the selected sensors, and the load distribution of the data among the available routes will determine the routing energy cost. However, since the number of routes can be exponential in the number of nodes, it has been found to be more tractable to take an equivalent view of the routes in terms of data flows across each link in the network [2], [3], [4]. The combination of all the routes can be mapped to data flows across each link by taking a sum of the data delivered for each route that uses that link. Suppose the total data flow from a node i to a node j is denoted f_{ij}. Then the total cost of these flows determines the total routing cost.

This cost can be expressed at each node in terms of the transmission cost of the entire flow exiting the node, reception cost of the flow entering a node, and the sensing cost for the amount of flow generated. The total energy cost over the entire lifetime of the network, can be obtained by multiplying by L_t and this total cost should be lower than the battery reserve, E_i, for each node. This is precisely stated in equation (6). Additionally, conservation of flows immediately leads to constraint (5).

The key difference from [2], [3], [4] is that in constraints (5) and (6), rather than considering known data rates generated at a fixed set of sensor nodes, we optimize over the specific sensor rates required to satisfy the distortion specification. This modification also implies that the technique used to linearize the program in [2], [3] is no longer applicable, and we shall offer alternatives in the next section.

The remaining constraints, equation (4), simply state that the flows, lifetime, and sensor rates cannot be negative. The auxiliary variables used in the distortion constraints are also known to be non-negative [14].

IV. Computationally Tractable Solution

The optimization problem as stated above is useful for capturing a multitude of energy and distortion issues in a joint framework. However, the problem is non-linear, due to the nature of the distortion constraints. It is well known that optimization tools can handle linear programming much more efficiently than non-linear programs. We propose a heuristic to reduce the complexity of optimization: we linearize the non-linear constraints and also reduce the number of these constraints.

Consider first the rate allocation constraints (7) and (8). Observe that in (7), the quantity $(1 - 2^{-2r_k})$ is positive. If we replace this quantity with zero, the magnitude of the right hand side will increase, since this quantity occurs behind a negative sign and the log function is monotonic. Thus, the inequality in (7) is strengthened with this replacement, and any rate vector which satisfies the new inequality, is a feasible rate vector for achieving the required distortion. With this replacement, (7) becomes linear:

$$\sum_{k \in A} R_k \geq \sum_{k \in A} r_k + \frac{1}{2}\log_2 \frac{1}{D} - \frac{1}{2}\log_2\left(\frac{1}{\sigma_X^2}\right) \tag{9}$$

The inequality (8) is also non linear. We consider a variable substitution to consider a strengthened and linearized version for this inequality. Consider an auxiliary variable r' which is chosen as the smallest among all r_k:

$$r_k \geq r' \quad \forall k \in \{1, ..., N\} \tag{10}$$

$$\textbf{max } L_t$$

Subject to: $f_{ij} \geq 0, \quad R_i \geq 0, \quad r_i \geq 0, \forall i,j \in \{1,...,N\}$ (4)

Flow Conservation:

$$\sum_{d\in\{1,N+1\},d\neq i} f_{id} - \sum_{s\in\{1,N+1\},s\neq i} f_{si} = R_i, \quad i \in \{1,...,N\} \tag{5}$$

Energy Constraints:

$$L_t \left[\sum_{d\in\{1,N+1\},d\neq i} P_{tx}(i,d) f_{id} + \sum_{s\in\{1,N+1\},s\neq i} P_{rx}(s,i) f_{si} + P_{sense} R_i \right] \leq E_i, \quad i \in \{1,...,N\} \tag{6}$$

Distortion Constraints:

$$\sum_{k\in A} R_k \geq \sum_{k\in A} r_k + \frac{1}{2}\log_2 \frac{1}{D} - \frac{1}{2}\log_2 \left[\frac{1}{\sigma_X^2} + \sum_{k\in A^c} \frac{1-2^{-2r_k}}{\sigma_k^2} \right] \forall \text{ non-empty } A \subseteq \{1,...,N\} \tag{7}$$

$$\frac{1}{\sigma_X^2} + \sum_{k=1}^{N} \frac{1-2^{-2r_k}}{\sigma_k^2} \geq \frac{1}{D} \tag{8}$$

Fig. 3. Optimization problem for determining the energy distortion trade-off for a given network.

We now rewrite (8) with this replacement for all k:

$$\frac{1}{\sigma_X^2} + (1 - 2^{(-2r')}) \sum_{k=1}^{N} \frac{1}{\sigma_k^2} \geq \frac{1}{D} \tag{11}$$

Re-arranging, one obtains:

$$r' \geq \frac{1}{2}\log_2\left(\frac{1}{T}\right) \quad \text{where} \quad T = \left[1 - \frac{\frac{1}{D} - \frac{1}{\sigma_X^2}}{\sum_{i=1}^{N}\frac{1}{\sigma_i^2}} \right] \tag{12}$$

which is linear. Since (6) is still non-linear, a further variable replacement is necessary to linearize the program, which can be realized by considering:

$$f'_{ij} = f_{ij}L_t, \quad R'_i = R_i L_t, \quad r'_i = r_i L_t, \quad r'' = r' L_t \tag{13}$$

in all the constraints. With the above variable replacements and the modified constraints (9), (10) and (12), the optimization problem can be expressed as a linear program. The new linear program is clearly sub-optimal as we have reduced the search space in the process of linearization.

On the other hand, we could get a lower bound on the lifetime-distortion relationship by relaxing the inequalities instead of strengthening them. Inequality (7) can be weakened if we approximate $2^{-2r_k} = 0$, obtaining:

$$\sum_{k\in A} R_k \geq \sum_{k\in A} r_k + \frac{1}{2}\log_2 \frac{1}{D} - \frac{1}{2}\log_2 \left[\frac{1}{\sigma_X^2} + \sum_{k\in A^c} \frac{1}{\sigma_k^2} \right] \tag{14}$$

Similarly, in (8), if we replace all r_k by the greatest among them, the inequality is weakened, yielding a new inequality as $r_i \leq r' \quad \forall k \in \{1,...,N\}$. Solving this relaxed but linear version of the optimization problem will lead to possibly infeasible solution. However, we are guaranteed that the exact lifetime distortion bound lies between the solution to this relaxed linear program and the strengthened linear program.

Further computational complexity arises from the fact that the number of constraints in (7) depends on the number of possible subsets of $\{1,...,N\}$, which is $2^N - 1$. For large N, this may be intractable. One approximation here is to consider only sensors with SNR above a particular threshold, i.e., within a certain distance from the phenomenon as the source sensors. The remaining sensor nodes act as relays. With this approximation, the number of constraints is polynomial in N. The optimization problem then yields a rate allocation among these selected sensors only.

Both the upper and lower bounds are plotted for a randomly generated network topology in Figure 4, with ten sensors and the number of source sensors constrained to be the three closest ones to the phenomenon. The energy parameters used are from a hardware described in [11]: $\alpha_1 = 45 \times 10^{-9}, \alpha_2 = 10 \times 10^{-12}, \beta = 135 \times 10^{-9}$ and $\alpha_3 = 50 \times 10^{-9}$. The battery is chosen to be $E_i = 180nJ, \forall i$, as in [2]; larger battery will yield larger lifetime. Suppose the phenomenon is present close to sensor 1, $\sigma_X^2 = 10$, and noise variances are proportional to the distance:

$$\sigma_1^2 = 0.01, \quad \sigma_i^2 = \sigma_1^2 d^2(1,i), i \in \{2,...,N\}$$

The distortion is converted to dB: $D(dB) = 10\log_{10}(D/\sigma_X^2)$. The bound is also explored for multiple random topologies, generated in a $100m \times 100m$ area with 10 sensors each. We evaluate the relaxed linear program for all these networks, when the first 3 sensors are chosen to be the sources. The results using the strengthened version are qualitatively similar. The lifetime-distortion relationship averaged over ten random instances of such a network is shown in Figure 5, along with the standard deviation across the random topologies.

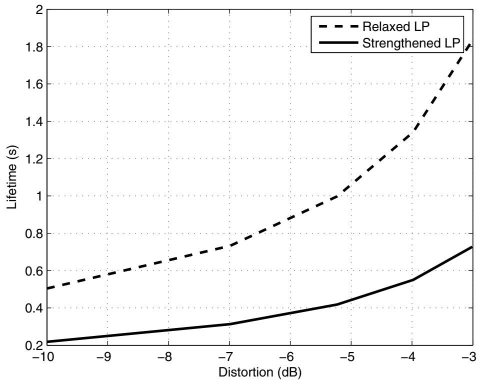

Fig. 4. Solutions to linearized versions of the optimization problem for a random network topology. The distortion shown is normalized with respect to σ_X^2.

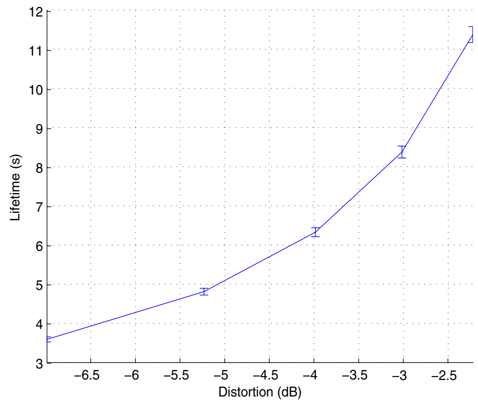

Fig. 5. Lifetime vs Distortion plot. The optimization solution obtained using the tractable method, averaged over 10 random topologies. The standard deviation across random topologies is shown by the error bars. The distortion is normalized with respect to σ_X^2.

The above heuristic is only a starting point, and can be used to get order of magnitude estimates on the achievable lifetime for required distortion, with very low computational complexity. However, determining a solution close to the optimal is also of interest. As part of ongoing work, we are exploring methods to use the two solutions obtained from the above linearized versions of the problem, to initiate a gradient descent or other search strategy which leads to a closer estimate of the $L_t(D)$ relationship.

V. Conclusions and Future Work

We presented an optimization framework to derive a relationship between the lifetime of a network and achieved distortion performance, rather than assuming a known data traffic schedule. We stated this new problem in terms of known rate distortion bounds and discussed a computationally tractable heuristic to evaluate this bound. This is a first step toward developing a framework for energy and distortion performance for multi-hop sensor networks. We are considering other computationally tractable methods to obtain closer estimates of the $L_t(D)$ relationship than the initial heuristic. In addition, we are exploring better methods to use the non-linear constraints by restricting data generation to a small subset of nodes. Further, the system model presented in section II does not account for all the issues mentioned in section I. In particular, we assumed that data is not aggregated in-network as it propagates toward the fusion center. Also, protocol overheads of selecting multiple sensors were not explicitly accounted for. Future work would involve incorporating these factors. A distributed sensor selection and routing scheme to achieve the lifetime bound is also desirable.

VI. Acknowledgments

This material is based upon work supported by the National Science Foundation (NSF) under Grant Nos. ANI-00331481 and 0306408 and the Center for Embedded and Networked Sensing, UCLA.

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