Nested data parallelism in Haskell

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Road map

Multicore

Parallel programming essential

Task parallelism

- Explicit threads
- Synchronise via locks, messages, or STM

Modest parallelism
Hard to program

Data parallelism

Operate simultaneously on bulk data

Massive parallelism
Easy to program

- Single flow of control
- Implicit synchronisation

Haskell has **three** forms of concurrency

- Explicit threads
  - Non-deterministic by design
  - Monadic: **forkIO** and **STM**
- Semi-implicit
  - Deterministic
  - Pure: **par** and **seq**
- Data parallel
  - Deterministic
  - Pure: parallel arrays
  - Shared memory initially; distributed memory eventually; possibly even GPUs

```
main :: IO ()
  = do { ch <- newChan
    ; forkIO (ioManager ch)
    ; forkIO  (worker 1 ch)
    ... etc ... }
```

```
f :: Int -> Int
f x = a `par` b `seq` a + b
    where
       a = f (x-1)
       b = f (x-2)
```

Data parallelism

The key to using multicores

Flat data parallel

Apply sequential operation to bulk data

- The brand leader
- Limited applicability (dense matrix, map/reduce)
- Well developed
- Limited new opportunities

Nested data parallel

Apply parallel operation to bulk data

- Developed in 90's
- Much wider applicability (sparse matrix, graph algorithms, games etc)
- Practically un-developed
- **Huge opportunity**

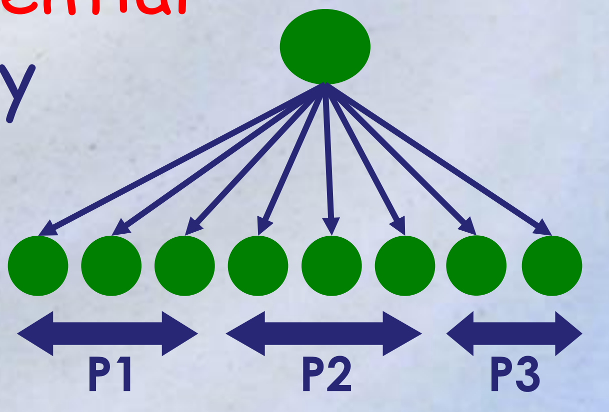
Flat data parallel

e.g. Fortran(s), *C
MPI, map/reduce

- The brand leader: widely used, well understood, well supported

```
foreach i in 1..N {
    ...do something to A[i]...
}
```

- BUT: "something" is sequential
- Single point of concurrency
- Easy to implement: use "chunking"
- Good cost model



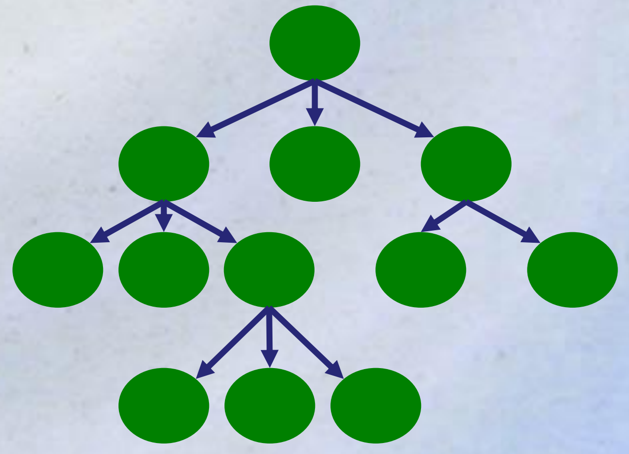
1,000,000's of (small) work items

Nested data parallel

- Main idea: **allow "something" to be parallel**

```
foreach i in 1..N {
    ...do something to A[i]...
}
```

- Now the parallelism structure is recursive, and un-balanced
- Still good cost model
- Hard to implement!



Still 1,000,000's of (small) work items

Nested DP is great for **programmers**

- Fundamentally more modular
- Opens up a much wider range of applications:
  - Sparse arrays, variable grid adaptive methods (e.g. Barnes-Hut)
  - Divide and conquer algorithms (e.g. sort)
  - Graph algorithms (e.g. shortest path, spanning trees)
  - Physics engines for games, computational graphics (e.g. Delauny triangulation)
  - Machine learning, optimisation, constraint solving

Nested DP is tough for **compilers**

- ...because the concurrency tree is both irregular and fine-grained
- But it can be done! NESL (Blelloch 1995) is an existence proof
- Key idea: "flattening" transformation:

Nested data parallel program
(the one we want to write)

Compiler

Flat data parallel program
(the one we want to run)

Array comprehensions

[:Float:] is the type of parallel arrays of Float

```
vecMul :: [:Float:] -> [:Float:] -> Float
vecMul v1 v2 = sumP [: f1*f2 | f1 <- v1 | f2 <- v2 :]
```

sumP :: [:Float:] -> Float

An array comprehension: "the array of all f1*f2 where f1 is drawn from v1 and f2 from v2"

Operations over parallel array are computed in parallel; that is the only way the programmer says "do parallel stuff"

NB: no locks!

Sparse vector multiplication

A sparse vector is represented as a vector of (index,value) pairs

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul sv v = sumP [: f*(v!i) | (i,f) <- sv :]
```

v!i gets the i'th element of v

Parallelism is proportional to length of sparse vector

Sparse matrix multiplication

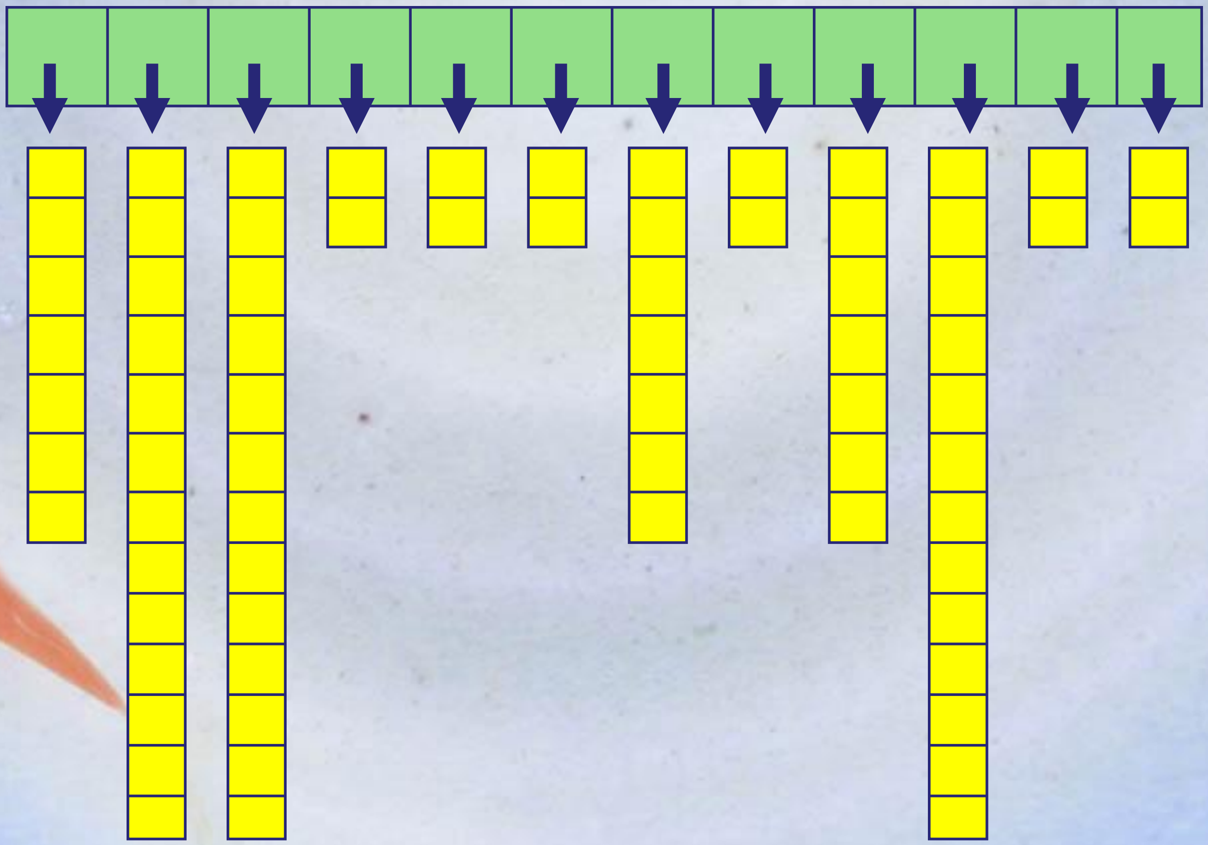
A sparse matrix is a vector of sparse vectors

```
smMul :: [:[:(Int,Float):]:] -> [:Float:] -> Float
smMul sm v = sumP [: svMul sv v | sv <- sm :]
```

Nested data parallelism here!
We are calling a parallel operation, svMul, on every element of a parallel array, sm

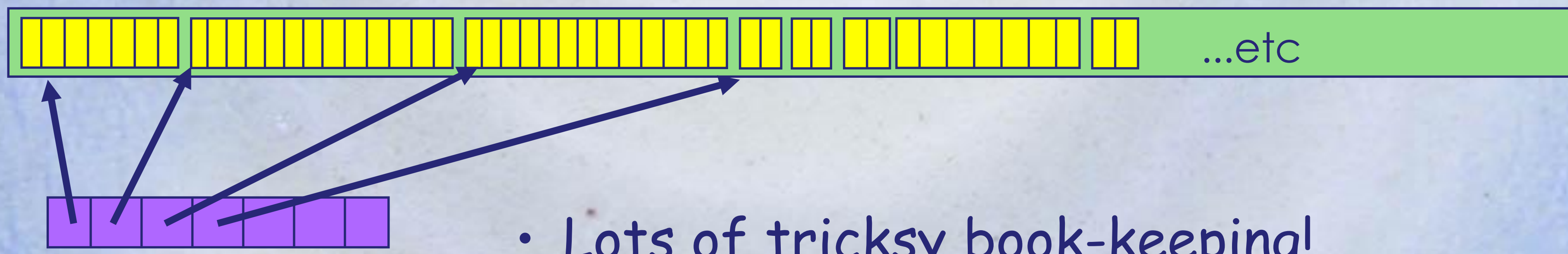
Hard to implement well

- Evenly chunking at top level might be **ill-balanced**
- Top level along might **not be very parallel**



The flattening transformation

- Concatenate sub-arrays into one big, flat array
- Operate in parallel on the big array
- Segment vector keeps track of where the sub-arrays are



- Lots of tricksy book-keeping!
- Possible to do by hand (and done in practice), but very hard to get right
- Blelloch showed it could be done systematically

Parallel search

```
type Doc = [: String :] -- Sequence of words
type DocBase  = [: Document :]

search :: DocBase -> String -> [: (Doc,[:Int:]):]
```

Find all Docs that mention the string, along with the places where it is mentioned
(e.g. word 45 and 99)

Parallel search

```
type Doc = [: String :]
type DocBase  = [: Document :]

search :: DocBase -> String -> [: (Doc,[:Int:]):]

wordOccs :: Doc -> String -> [: Int :]
```

Find all the places where a string is mentioned in a document (e.g. word 45 and 99)

Parallel search

```
type Doc = [: String :]
type DocBase  = [: Document :]

search :: DocBase -> String -> [: (Doc,[:Int:]):]
search ds s = [: (d,is) | d <- ds
                        , let is = wordOccs d s
                        , not (nullP is) :]

wordOccs :: Doc -> String -> [: Int :]
```

```
nullP :: [:a:] -> Bool
```

Parallel search

```
type Doc = [: String :]
type DocBase  = [: Document :]

search :: DocBase -> String -> [: (Doc,[:Int:]):]

wordOccs :: Doc -> String -> [: Int :]
wordOccs d s = [: i | (i,s2) <- zipP positions d
                    , s == s2 :]
    where
      positions :: [: Int :]
      positions = [: 1..lengthP d :]
```

```
zipP    :: [:a:] -> [:b:] -> [:(a,b):]
lengthP :: [:a:] -> Int
```

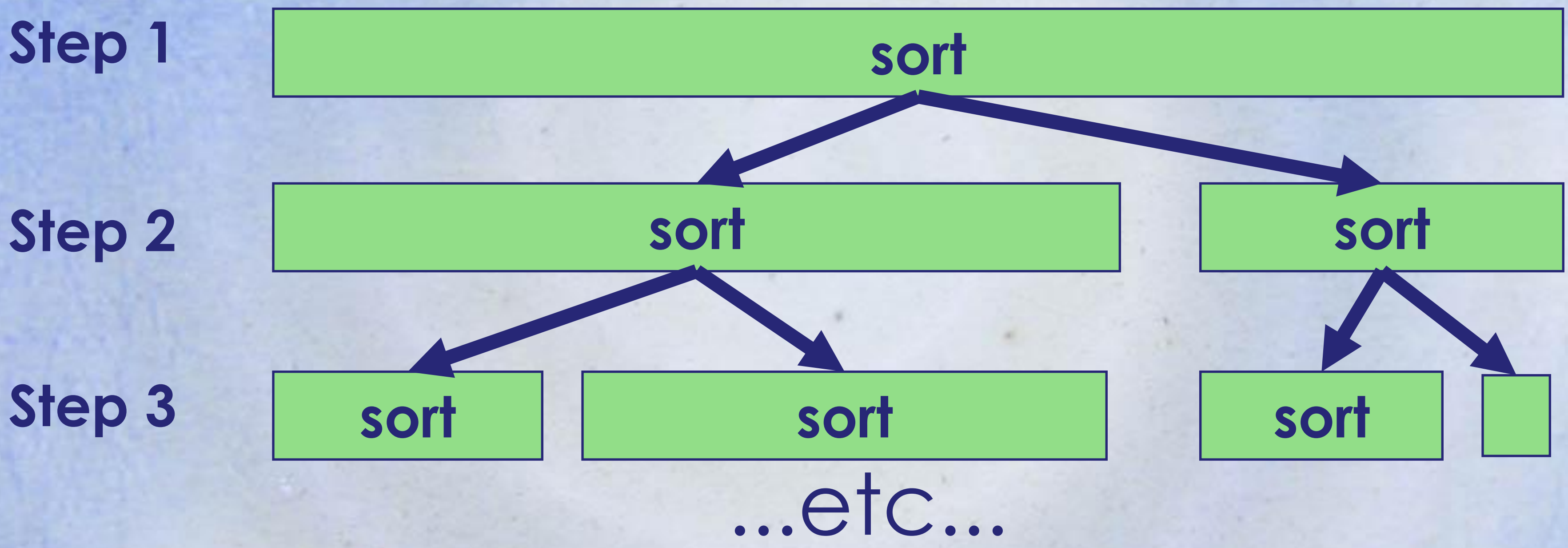
Data-parallel quicksort

```
sort :: [:Float:] -> [:Float:]
sort a = if (length a <= 1) then a
         else sa!0 +++ eq +++ sa!1
   where
      m = a!0
      lt = [: f | f<-a, f<m :]
      eq = [: f | f<-a, f==m :]
      gr = [: f | f<-a, f>m :]
      sa = [: sort a | a <- [:lt,gr:] :]
```

Parallel filters

2-way nested data parallelism here!

How it works



- All sub-sorts at the same level are done in parallel
- Segment vectors track which chunk belongs to which sub problem
- Instant insanity when done by hand

Fusion

- Flattening is not enough

```
vecMul :: [:Float:] -> [:Float:] -> Float
vecMul v1 v2 = sumP [: f1*f2 | f1 <- v1 | f2 <- v2 :]
```

- Do not
  1. Generate [: f1*f2 | f1 <- v1 | f2 <- v2 :] (big intermediate vector)
  2. Add up the elements of this vector
- Instead: multiply and add in the same loop
- That is, **fuse** the multiply loop with the add loop
- Very general, aggressive fusion is required

Purity pays off

- Two key transformations:
  - Flattening
  - Fusion
- Both depend utterly on purely-functional semantics:
  - no assignments
  - every operation is a pure function

The data-parallel languages of the future will be functional languages

What we are doing about it

NESL

a mega-breakthrough but:

- specialised, prototype
- first order
- few data types
- no fusion
- interpreted

Substantial improvement in

- **Expressiveness**
- **Performance**

Haskell

- broad-spectrum, widely used
- higher order
- very rich data types
- aggressive fusion
- compiled

- **Shared memory initially**
- **Distributed memory eventually**
- **GPUs anyone?**

Not a special purpose data-parallel compiler! Most support is either useful for other things, or is in the form of library code.

Main contribution: an optimising data-parallel compiler implemented by modest enhancements to a full-scale functional language implementation

Four key pieces of technology

1. Flattening
   - specific to parallel arrays
2. Non-parametric data representations
   - A generically useful new feature in GHC
3. Chunking
   - Divide up the work evenly between processors
4. Aggressive fusion
   - Uses "rewrite rules", an old feature of GHC

Not a special purpose data-parallel compiler! Most support is either useful for other things, or is in the form of library code.

Step 0: desugaring

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul sv v = sumP [: f*(v!i) | (i,f) <- sv :]
```

```
sumP :: Num a => [:a:] -> a
mapP :: (a -> b) -> [:a:] -> [:b:]
```

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul sv v = sumP (mapP (\(i,f) -> f * (v!i)) sv)
```

Step 1: Vectorisation

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul sv v = sumP (mapP (\(i,f) -> f * (v!i)) sv)
```

```
sumP      :: Num a => [:a:] -> a
*^        :: Num a => [:a:] -> [:a:] -> [:a:]
fst^      :: [:(a,b):] -> [:a:]
bpermuteP :: [:a:] -> [:Int:] -> [:a:]
```

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul sv v = sumP (snd^ sv  *^  bpermuteP v (fst^ sv))
```

Scalar operation * replaced by vector operation *^

Vectorisation: the basic idea

```
mapP f v
```

➡

```
f^ v
```

```
f  :: T1 -> T2
f^ :: [:T1:] -> [:T2:]   -- f^ = mapP f
```

- For every function f, generate its **lifted version**, namely f^
- Result: a functional program, operating over flat arrays, with a fixed set of primitive operations *^, sumP, fst^, etc.
- Lots of intermediate arrays!

Vectorisation: the basic idea

```
f  :: Int -> Int
f x = x+1

f^ :: [:Int:] -> [:Int:]
f^ x = x +^ (replicateP (lengthP x) 1)
```

This	Transforms to this
Locals, x	x
Globals, g	g^
Constants, k	replicateP (lengthP x) k

```
replicateP :: Int -> a -> [:a:]
lengthP :: [:a:] -> Int
```

Vectorisation: the key insight

```
f  :: [:Int:] -> [:Int:]
f a = mapP g a = g^ a

f^ :: [:[:Int:]:] -> [:[:Int:]:]
f^ a = g^^ a        --???
```

Yet another version of g???

Vectorisation: the key insight

```
f  :: [:Int:] -> [:Int:]
f a = mapP g a = g^ a

f^ :: [:[:Int:]:] -> [:[:Int:]:]
f^ a = segmentP a (g^ (concatP a))
```

First concatenate,
then map,
then re-split

```
concatP  :: [:[:a:]:] -> [:a:]
segmentP :: [:[:a:]:] -> [:b:] -> [:[:b:]:]
```

Shape

Flat data

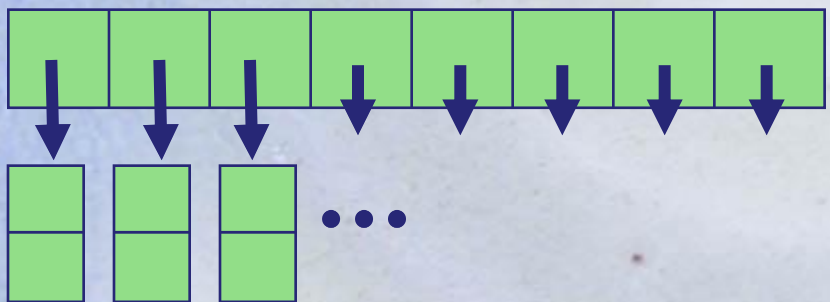
Nested data

Payoff: f and f^ are enough. No f^^

Step 2: Representing arrays

`[:Double:]` Arrays of pointers to boxed numbers are Much Too Slow

`[:(a,b):]` Arrays of pointers to pairs are Much Too Slow



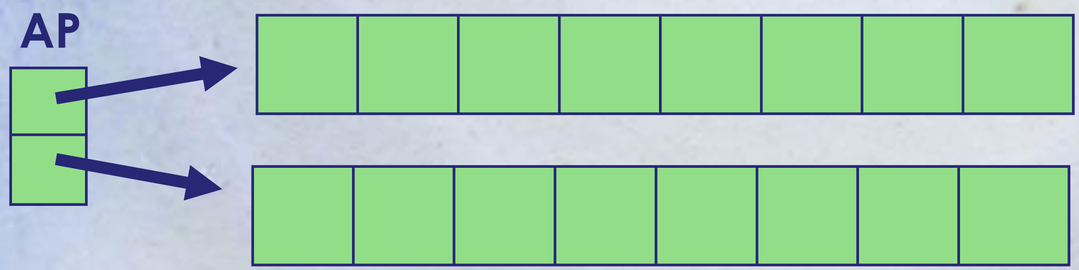
Idea!
Representation of an array depends on the element type

Step 2: Representing arrays

[POPL05], [ICFP05], [TLDI07]

```
data family [:a:]

data instance [:Double:] = AD ByteArray
data instance [:(a,b):]  = AP [:a:] [:b:]
```



- Now *^ is a fast loop
- And fst^ is constant time!

```
fst^ :: [:(a,b):] -> [:a:]
fst^ (AP as bs) = as
```

Step 2: Nested arrays

Shape

Flat data

```
data instance [:[:a:]:] = AN [:Int:] [:a:]

concatP  :: [:[:a:]:] -> [:a:]
concatP (AN shape data) = data

segmentP :: [:[:a:]:] -> [:b:] -> [:[:b:]:]
segmentP (AN shape _) data = AN shape data
```

Surprise: concatP, segmentP are constant time!

Higher order complications

```
f   :: T1 -> T2 -> T3

f1^ :: [:T1:] -> [:T2:] -> [:T3:] -- f1^ = zipWithP f
f2^ :: [:T1:] -> [:(T2 -> T3):]   -- f2^ = mapP f
```

- f1^ is good for [: f a b | a <- as | b <- bs :]
- But the type transformation is not uniform
- And sooner or later we want higher-order functions anyway
- f2^ forces us to find a representation for [:(T2->T3):]. Closure conversion [PAPP06]

Step 3: chunking

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul (AP is fs) v = sumP (fs  *^  bpermuteP v is)
```

- Program consists of
  - Flat arrays
  - Primitive operations over them (*^, sumP etc)
- Can directly execute this (NESL).
  - Hand-code assembler for primitive ops
  - All the time is spent here anyway
- But:
  - intermediate arrays, and hence memory traffic
  - each intermediate array is a synchronisation point
- Idea: chunking and fusion

Step 3: Chunking

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul (AP is fs) v = sumP (fs  *^  bpermuteP v is)
```

1. **Chunking**: Divide is,fs into chunks, one chunk per processor
2. **Fusion**: Execute sumP (fs *^ bpermute v is) in a tight, sequential loop on each processor
3. **Combining**: Add up the results of each chunk

Step 2 alone is not good for a parallel machine!

Expressing chunking

```
sumP :: [:Float:] -> Float
sumP xs = sumD (mapD sumS (splitD xs))
```

```
splitD    :: [:a:] -> Dist [:a:]
mapD      :: (a->b) -> Dist a -> Dist b
sumD      :: Dist Float -> Float

sumS      :: [:Float:] -> Float       -- Sequential!
```

- **sumS** is a tight sequential loop
- **mapD** is the true source of parallelism:
  - it starts a "gang",
  - runs it,
  - waits for all gang members to finish

Expressing chunking

```
*^ :: [:Float:] -> [:Float:] -> [:Float:]
*^ xs ys = joinD (mapD mulS
              (zipD (splitD xs) (splitD ys))
```

```
splitD    :: [:a:] -> Dist [:a:]
joinD     :: Dist [:a:] -> [:a:]
mapD      :: (a->b) -> Dist a -> Dist b
zipD      :: Dist a -> Dist b -> Dist (a,b)

mulS :: ([:Float:],[: Float :]) -> [:Float:]
```

- Again, mulS is a tight, sequential loop

Step 4: Fusion

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul (AP is fs) v = sumP (fs  *^  bpermuteP v is)
 = sumD . mapD sumS . splitD . joinD . mapD mulS $
      zipD (splitD fs) (splitD (bpermuteP v is))
```

- Aha! Now use rewrite rules:

```
{-# RULE
    splitD (joinD x)   = x
    mapD f (mapD g x)  = mapD (f.g) x #-}
```

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul (AP is fs) v = sumP (fs  *^  bpermuteP v is)
 = sumD . mapD (sumS . mulS) $
      zipD (splitD fs) (splitD (bpermuteP v is))
```

Step 4: Sequential fusion

```
svMul :: [:(Int,Float):] -> [:Float:] -> Float
svMul (AP is fs) v = sumP (fs  *^  bpermuteP v is)
 = sumD . mapD (sumS . mulS) $
       zipD (splitD fs) (splitD (bpermuteP v is))
```

- Now we have a sequential fusion problem.
- Problem:
  - lots and lots of functions over arrays
  - we can't have fusion rules for every pair
- New idea: stream fusion

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Four key pieces of technology

1. **Flattening**
   - specific to parallel arrays
2. **Non-parametric data representations**
   - A generically useful new feature in GHC
3. **Chunking**
   - Divide up the work evenly between processors
4. **Aggressive fusion**
   - Uses "rewrite rules", an old feature of GHC

Not a special purpose data-parallel compiler! Most support is either useful for other things, or is in the form of library code.

And it goes fast too…

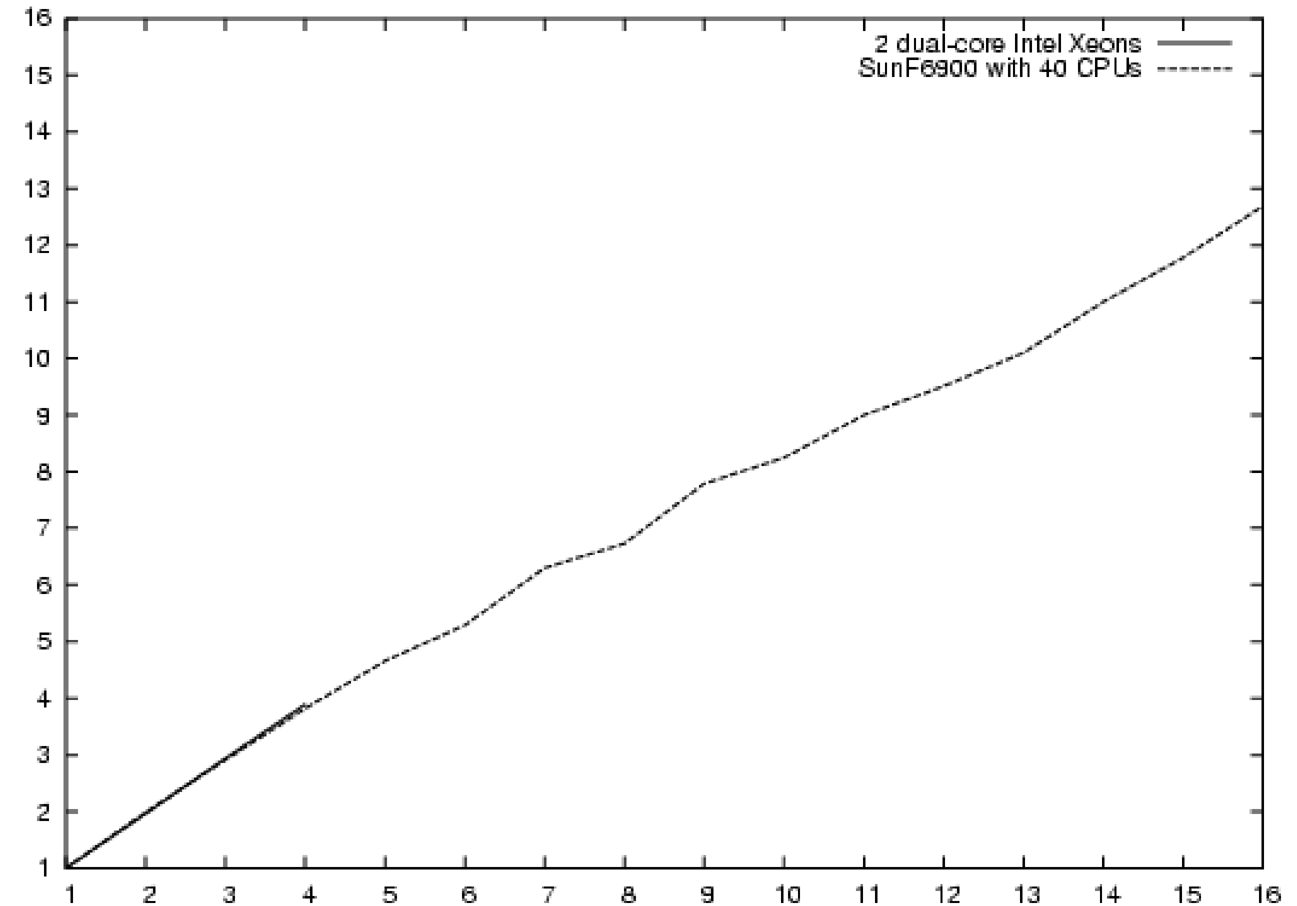

Figure 2. Speedup of smvm (x-axis is number of PEs)

1-processor version goes only 30% slower than C

Perf win with 2 processors

Summary

- **Data parallelism** is the only way to harness 100's of cores
- **Nested DP** is great for programmers: far, far more flexible than flat DP
- Nested DP is tough to implement, but we (think we) know how to do it
- **Huge opportunity**: almost no one else is dong this stuff!
- Functional programming is a massive win in this space: Haskell prototype in 2008
- **WANTED**: friendly guinea pigs

http://haskell.org/haskellwiki/GHC/Data_Parallel_Haskell

Extra slides

Stream fusion for lists

```
map f (filter p (map g xs))
```

- Problem:
  - lots and lots of functions over lists
  - and they are **recursive** functions
- New idea: make map, filter etc non-recursive, by defining them to work over **streams**

Stream fusion for lists

```
data Stream a where
    S :: (s -> Step s a) -> s -> Stream a

data Step s a = Done | Yield a (Stream s a)

toStream :: [a] -> Stream a
toStream as = S step as
  where
    step [] = Done
    step (a:as) = Yield a as

fromStream :: Stream a -> [a]
fromStream (S step s) = loop s
  where
    loop s = case step s of
               Yield a s'  -> a : loop s'
               Done        -> []
```

Non-recursive!

Recursive

Stream fusion for lists

```
mapStream :: (a->b) -> Stream a -> Stream b
mapStream f (S step s) = S step' s
  where
    step' s = case step s of
                Done        -> Done
                Yield a s' -> Yield (f a) s'

map :: (a->b) -> [a] -> [b]
map f xs = fromStream (mapStream f (toStream xs))
```

Non-recursive!

Stream fusion for lists

```
map f (map g xs)

= fromStream (mapStream f (toStream
    (fromStream (mapStream g (toStream xs))))

=                 -- Apply (toStream (fromStream xs) = xs)
  fromStream (mapStream f (mapStream g (toStream xs)))

=                 -- Inline mapStream, toStream
  fromStream (Stream step xs)
   where
     step [] = Done
     step (x:xs) = Yield (f (g x)) xs
```

Stream fusion for lists

```
fromStream (Stream step xs)
     where
       step [] = Done
       step (x:xs) = Yield (f (g x)) xs

=                    -- Inline fromStream
  loop xs
    where
        loop [] = []
        loop (x:xs) = f (g x) : loop xs
```

- Key idea: mapStream, filterStream etc are all non-recursive, and can be inlined
- Works for arrays; change only fromStream, toStream