This document is the supplemental material to the ECML-PKDD'12 submission.

A Exponential Integral

Suppose that x is a random variable with Gaussian distribution, i.e., $p(x) := \mathcal{N}(x; \mu, \sigma^2)$, we present the derivations of the expectation for the $\exp(x)$ w.r.t. x as follows:

$$
\begin{aligned}
&E_{x\sim p(x)}(\exp(x)) = \int_x \frac{\exp(x)}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \mathrm{d}x \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\mu^2}{2\sigma^2}\right) \int_x \exp\left(-\frac{x^2 - 2x(\mu+\sigma^2)}{2\sigma^2}\right) \mathrm{d}x \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\mu + \frac{\sigma^2}{2}\right) \int_x \exp\left(-\frac{(x-(\mu+\sigma^2))^2}{2\sigma^2}\right) \mathrm{d}x \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\mu + \frac{\sigma^2}{2}\right) \sqrt{2\pi\sigma^2} \\
&= \exp(\mu + \sigma^2/2).
\end{aligned}
$$

B Log Gaussian Integral

Suppose x is a random variable with Gaussian distribution $p(x) : \mathcal{N} \sim (\mu, \sigma^2)$ and $q(x)$ is a Gaussian, $\mathcal{N} \sim (\mu_1, \sigma_1^2)$, let us show how to derive the expectation of $\log q(x)$ w.r.t. x as follows:

$$
\begin{aligned}
&E_{x\sim p(x)}(\log q(x)) = E_{x\sim p(x)}\left(\log\left(\frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right)\right)\right) \\
&= -\frac{1}{2}\log(2\pi\sigma_1^2) - \frac{1}{2\sigma_1^2} E_{x\sim p(x)}(x-\mu_1)^2 \\
&= -\frac{1}{2}\log(2\pi\sigma_1^2) - \frac{1}{2\sigma_1^2}\left(E_{x\sim p(x)}(x^2) - 2\mu_1\mu + \mu_1^2\right) \\
&= -\frac{1}{2}\log(2\pi\sigma_1^2) - \frac{1}{2\sigma_1^2}\left(\sigma^2 + \mu^2 - 2\mu_1\mu + \mu_1^2\right).
\end{aligned}
$$

C Solution to (19)

First note that (19) is equivalent to

$$
\frac{1}{2(1+\sigma^2\exp(\kappa))} - \exp(\kappa) = \frac{\kappa-\mu}{\sigma^2} - s_i,
$$

By setting $z = \exp(\kappa)$ and $\frac{\kappa-\mu}{\sigma^2} - s_i = A$, we can convert (19) into a quadratic equation with respect to z

$$2\sigma^2 z^2 + 2(A\sigma^2 + 1)z + 2A - 1 = 0,$$

with its positive root given by

$$z = \frac{-(A\sigma^2 + 1) + \sqrt{(A\sigma^2 - 1)^2 + 2\sigma^2}}{2\sigma^2}. \tag{25}$$

Note that we choose the positive root because $\exp(\cdot)$ cannot be negative. Given $\frac{\kappa-\mu}{\sigma^2} - s_i = A$ and (25), we have

$$z = \frac{\mu + s_i\sigma^2 - 1 - \kappa + \sqrt{(\kappa - \mu - s_i\sigma^2 - 1)^2 + 2\sigma^2}}{2\sigma^2}. \tag{26}$$

We plug $\exp(\kappa) = z$ into (26), and get

$$\kappa = \log\left(\frac{\mu + s_i\sigma^2 - 1 - \kappa + \sqrt{(\kappa - \mu - s_i\sigma^2 - 1)^2 + 2\sigma^2}}{2\sigma^2}\right). \tag{27}$$