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Designing simple, efficient, composable mechanisms

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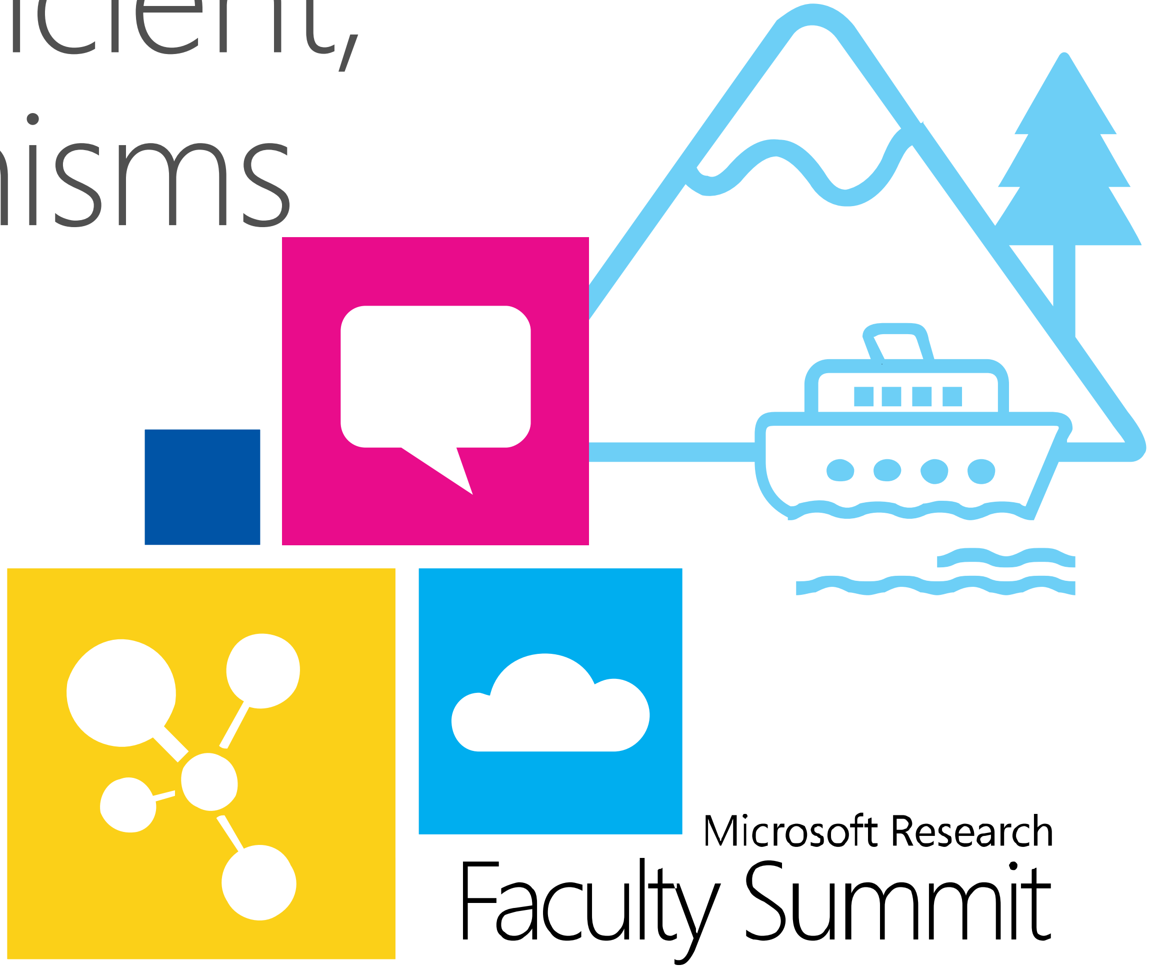


Goods

Effort

Information

Ad Impressions

The classical simple auction

Basic auction: single item Vickrey auction

$2 $5 $7 $3 $4

Pays $5

Player utility $v_i - p_i$ — item value –price paid

Vickrey auction (second price)
- truthful
- efficient
- simple

Goals for mechanism design

Goals:

Simple to understand rules

Simple to participate (truthful)

Simple to run

Good outcome

- efficient
- high revenue

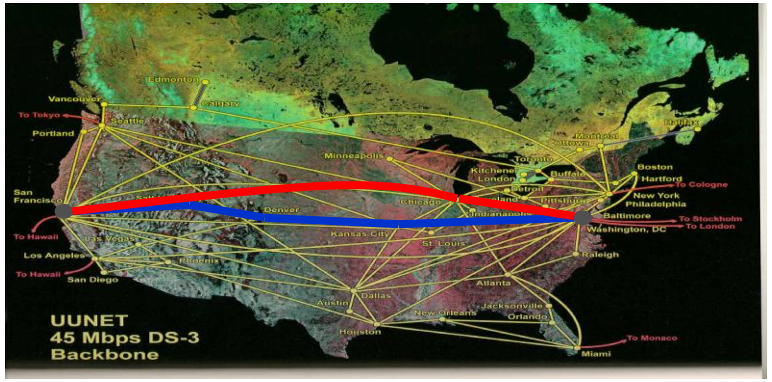
Classical mechanism design: truthful

Extension of Vickrey auction VCG (truthful and efficient),

but not simple

Simple vs optimal

Simple mechanism can lead to good outcome.
Optimal outcome is not practical.



Traffic subject to congestion delays

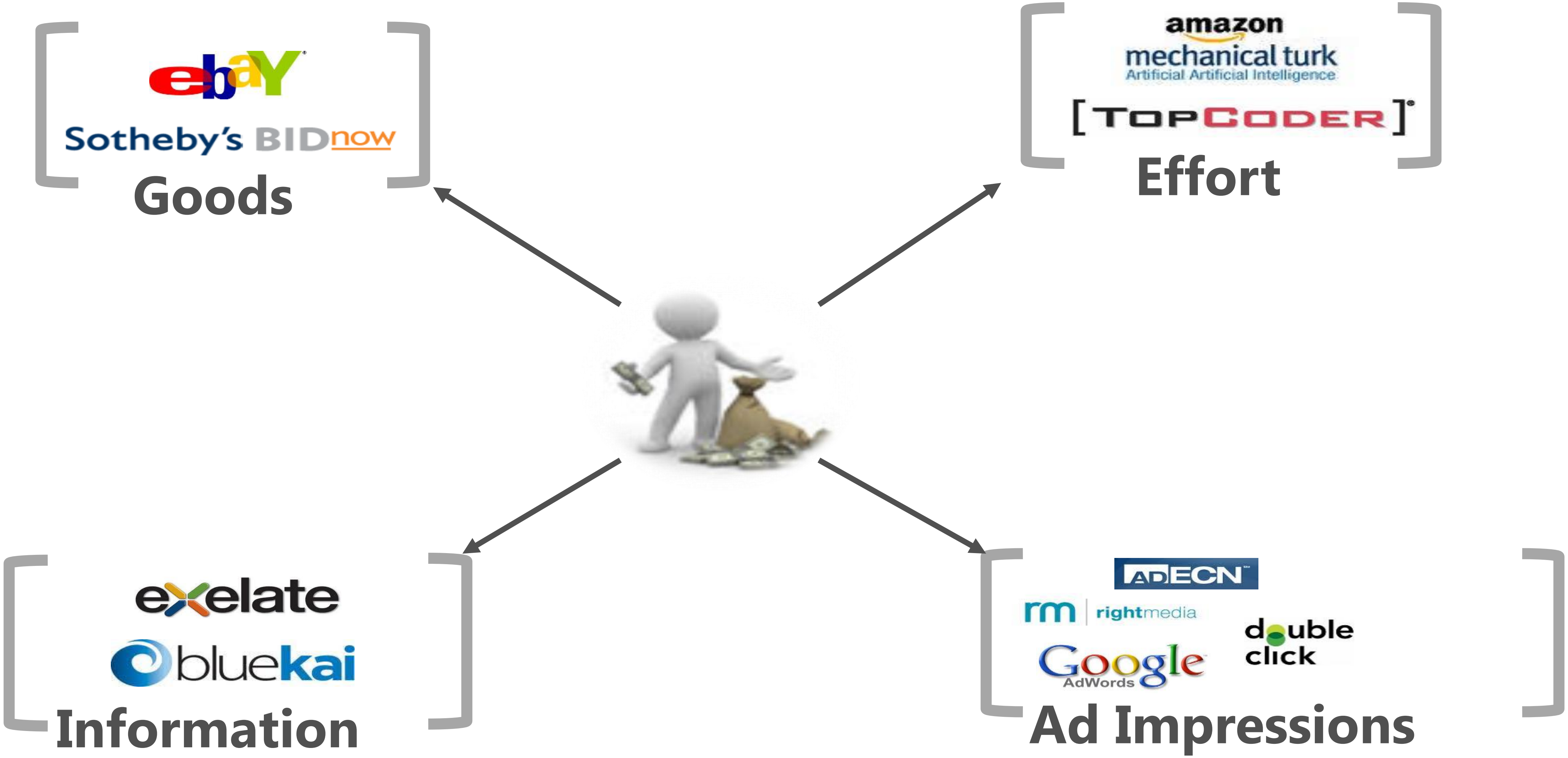
Congestion game =cost (delay) depends on congestion on edges

Simple vs optimal

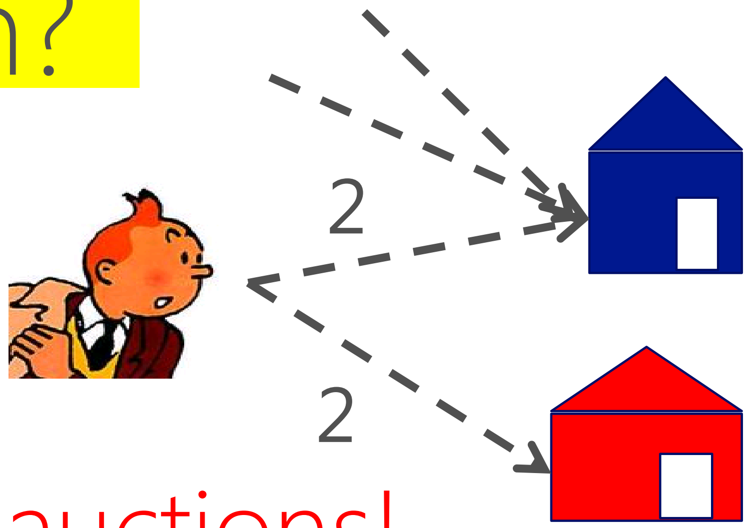
Simple mechanism can lead to good outcome.
Optimal outcome is not practical.

Also true in many other applications:
- routing, bandwidth sharing, load balancing,
- and also Internet auctions

Multiple opportunities: composition



Truthful auctions and composition?



Second price auction

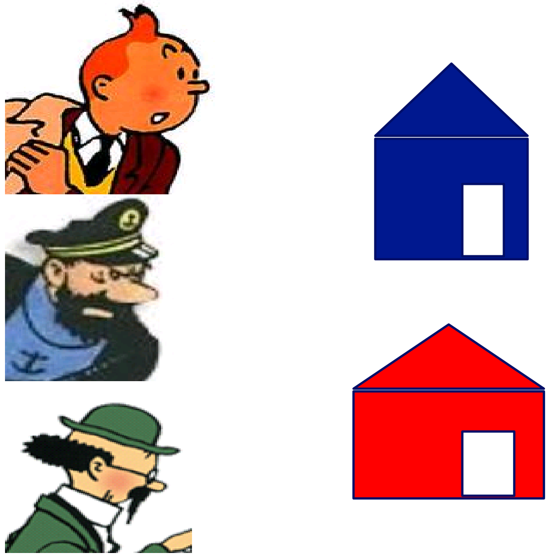
truthful and simple

But players may participate in multiple auctions!

Two simultaneous second price auctions? No!

How about sequentially? No!

Auctions as Games



Simultaneous second price?

Christodoulou, Kovacs, Schapira ICALP'08
Bhawalkar, Roughgarden SODA'10

AdAuctions (GSP)

Paes-Leme, T FOCS'10, Lucier, Paes-Leme + CKKK EC'11

First price?

Hassidim, Kaplan, Mansour, Nisan EC'11

Sequential auction?

Paes Leme, Syrgkanis, T SODA'12, EC'12

Question: how good outcome to expect?

some are composition of simple auctions

Our Framework

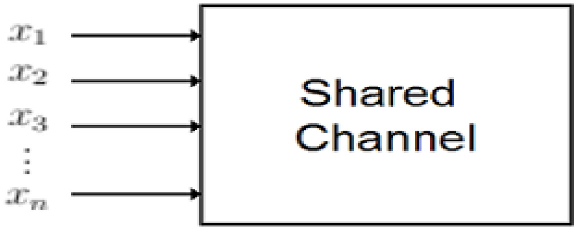
Possible outcomes : X

User i has value $v_i(x)$ for each outcome $x \in X$

quasi-linear utility:

outcome x and price p has utility $v_i(x)-p$ for user i.

Combinatorial Auctions

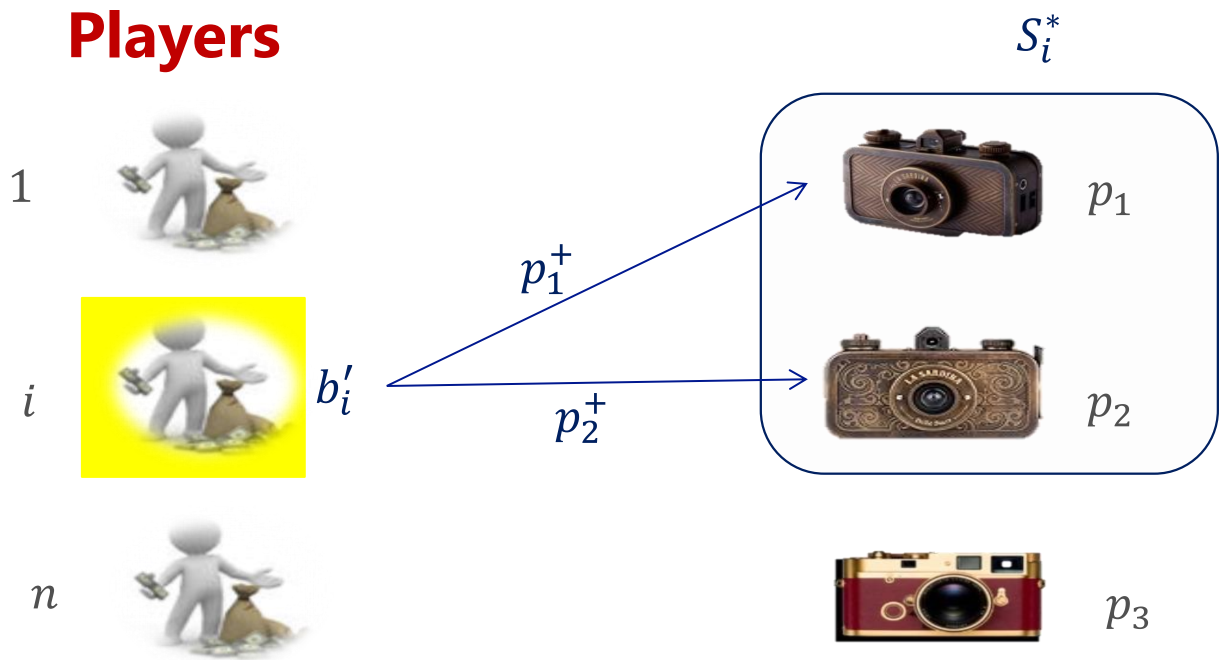


Bandwidth Sharing

Public projects

Simultaneous first-price auctions

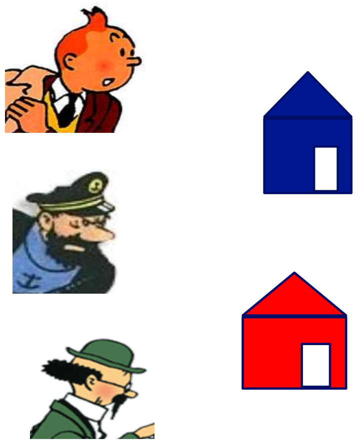
Theorem [Bikhchandani'96] Any pure Nash equilibrium of a simultaneous first price auction in the full information setting has optimal welfare $OPT = \sum_i v_i(S_i^*)$.



Pure Nash sets prices

Market clearing prices
$\Rightarrow$ socially optimal allocation

What makes a mechanism good?



Desired properties of robust mechanism

Quality of outcomes in auctions

√ 1^{st} price and Pure Nash

Which auctions have high quality outcomes?

What if stable solution is not found?

Can we have guarantees outside of Nash equilibrium?

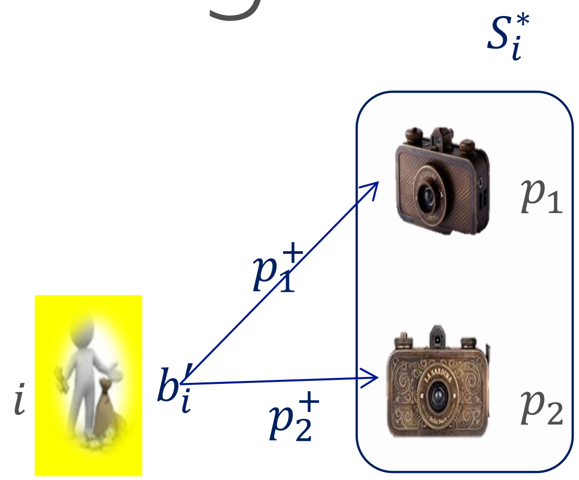
What if other player's values are not known

Can we have guarantees in Bayesian settings?

Each player plays in many games

Still guarantee high quality outcome?

Smooth = approximately market clearing
(can happen robustly)



Recall: Market clearing prices optimality proof:

Player i can claim her optimal set S_i^* to get value

$$utility_i \geq v_i(S_i^*) - \sum_{\{j \in S_i^*\}} p_j$$

Approximately market clearing: Player i has a bid b'_i, such that if current bids are b_{-i} and item prices are p_j we get

$$utility_i(b_i', b_{-i}) \geq \lambda\, v_i(S_i^*) - \mu \sum_{\{j \in S_i^*\}} p_j$$

b_i', should not depend on b_{-i}

Price of Anarchy

Theorem (Syrkganis-T'13) Auction game (λ,μ)-smooth game, then the price of anarchy is at most λ/max(1, μ) $\leq \lambda/\mu$.

Robust: also true for

- for mixed equilibria and learning outcomes
- for Bayesian game, if player types are independent
- preserved in composition (under no complements)

Global Efficiency Theorem (Syrgkanis-T'13) A market composed of simultaneous (λ, μ)-smooth mechanisms achieves at least $\frac{\lambda}{\max\{1,\mu\}}$ of optimal welfare at no-regret learning outcomes even under incomplete information, when players have complement free valuations across mechanisms.

Example 1: First price auction for a single item

User of value v_i bid $b'_i = \frac{1}{2} v_i$, utility

Claim: $utility_i(b'_i, b_{-i}) \geq \frac{1}{2} v_i - 1 p_i$

Proof

- Either wins and has utility $v_i - p_i = \frac{1}{2} v_i$
- Or looses and hence price was $p_i \geq \frac{1}{2} v_i$

Other examples of smooth auction games

First price auction (1-1/e,1) smooth

See also Hassidim et al EC'12, Syrkhanis'12

All pay auction (½,1)-smooth

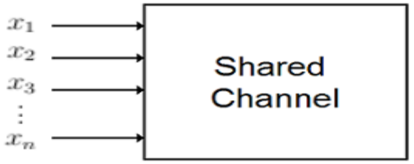
First position auction (GFP) is (½,1)-smooth

Other applications include:

- public goods
- bandwidth allocation (Johari-Tsitsiklis),
- etc

Combinatorial Auctions

Public Projects



Bandwidth Allocation

Learning outcome

b_1^1 b_2^1 … b_n^1

b_1^2 b_2^2 … b_n^2

b_1^3 b_2^3 … b_n^3

b_1^t b_2^t … b_n^t

time

Maybe here they don't know how to bid, who are the other players, …

Run Auction on ($b_1^1, b_2^1, \ldots, b_n^1$)

By here they have a better idea…

Run Auction on ($b_1^t, b_2^t, \ldots, b_n^t$)

Vanishingly small regret for any fixed strat x:

$$\sum_t \text{utility}_i(b_i^t, b_{-i}^t) \geq \sum_t \text{utility}_i(x, b_{-i}^t) - o(T)$$

Bayesian extension theorem

Theorem (Syrkganis-T'13) If an auction game is (λ,μ)-smooth, then Bayesian Price of anarchy is at most λ/max(1, μ), assuming player types are independent

Special strategy b' may depend on opponent (not on their strategy), so not useful….

Proof idea: consider random draw w, and take (λ,μ)-smooth deviation for valuations $(\mathrm{v_i}, \mathrm{w}_{-i})$. Take expectation.

Simultaneous composition

See next

Theorem(Syrkganis-T'13) simultaneous mechanisms M_j each (λ,μ)-smooth and players have no complements across mechanisms, then composition is also (λ,μ)-smooth

Corollary: Simultaneous first price auction has price of anarchy of e/(e-1) if player values have no complements

- Simultaneous all-pay auction: price anarchy 2
- Mix of first price and all pay, PoA at most 2

No complements across mechanisms

Marginal value for any allocation from some mechanism can only decrease, as he gets non-empty allocations from more mechanisms

No assumption about allocation structure and valuation within mechanism

We use fractionally subadditive across mechanisms

$\supseteq$ submodular

Extensions

Sequential composition

Smooth mechanisms compose sequentially when valuations are generalized unit-demand

Second-price and no-overbidding

Provide a generalization of the no-overbidding assumption

Give extended smoothness framework that can capture second price type of auctions under no-overbidding assumptions

Hard budget constraints on payments

Same efficiency guarantees with respect to new welfare benchmark: Optimal welfare achievable after capping a players valuation by his budget

Simple, composable, efficient mechanisms

Smooth mechanisms: natural generalization of market clearing prices

Many simple mechanisms are smooth

Smooth mechanisms compose well (assuming no complements across mechanisms)

Good outcome quality (Nash, Bayesian Nash, learning outcomes, budget constraints)

Designing simple and smooth mechanisms

Thanks!