Scoop: Decentralized and Opportunistic Multicasting of Information Streams

D. Gunawardena, T. Karagiannis, A. Proutiere, E. Santos-Neto and M. Vojnović

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D. Gunawardena, T. Karagiannis, A. Proutiere and M. Vojnović are with Microsoft Research, Cambridge, UK. {dinang, thomkar, aproutie, milanv}@microsoft.com. E. Santos-Neto is with the University of British Columbia, Canada. Work by E. Santos-Neto was performed in part while an intern with Microsoft Research Cambridge, UK.
Abstract – We consider the problem of delivering information streams to interested mobile users that leverages both access to the infrastructure and device-to-device data transfers. Our goal is to design practical relaying algorithms that aim at optimizing a global system objective that accounts for the user interest in content with respect to the type of the content and its delivery time, and account for resource constraints such as storage and transmission costs.

We first provide evidence that significant performance gains can be achieved in practice by extending the information dissemination from one to two hops and that only diminishing benefits are achieved through paths of larger length. We also show that correlation of delay through paths is typically significant, thus asking for system design that would allow for general user mobility.

We propose a class of relaying strategies (referred to as SCOOP) that aim at optimizing a global system objective, are fully decentralized, require only locally observed states by individual devices, and allow for general user mobility. These properties characterize a practical scheme whose efficiency is demonstrated using real-world mobility traces.

1. INTRODUCTION

The concept of opportunistic communications has emerged as an alternative and augmentation of traditional networks for devices that experience intermittent connectivity. In such networks, besides the regular access to wireless or wired networks, mobile devices may exploit opportunistic device-to-device data transfers to increase network performance and achieve dissemination of information.

While initially targeted for disaster recovery, vehicular or challenged networks that are delay-tolerant (DTNs), opportunistic communications have recently attracted additional interest as a means to reduce the communication cost both for Internet Service Providers (ISPs) and individual users, especially for the case of 3G networks. Applications that constantly push information streams and content to mobile devices (e.g., news broadcasting, Facebook and Twitter feeds, podcasting [15, 12, 10]) are commonplace and their data volumes are projected to significantly increase [19], posing a challenge to the existing infrastructure. Operators, thus, consider opportunistic data transfers to alleviate congestion in their backhaul networks, e.g., in 3G networks [1] in the backhaul, they can leverage the social structure of the network. An additional solution, from the perspective of device-to-device transfers, is the use of opportunistic relaying to prevent extra charges imposed when exceeding monthly data limits or when roaming.

Typically, proposals for routing or, more generally, information dissemination in DTNs, either attempt to keep a single copy of a message to deliver in the network (i.e., forwarding protocols, e.g., [11]), or replicate messages at transfer opportunities to find a path to the destination (i.e., epidemic routing). Proposed solutions attempt to limit the message replication in the network by deploying various heuristics, such as limiting the number of existing replicas (e.g., [24, 5]), inferring the likelihood of the message delivery (e.g., [17, 3]), or leveraging the social structure of the network [8]. To limit message replication, most of the proposed protocols try to infer device mobility and track the expected delays towards various nodes in order to make informed decisions on which messages to relay. This implies that nodes have to estimate delays, and in order to achieve this, simplifying assumptions about user mobility are often admitted, for example, that delays through different paths are independent. However, as our results show, the independence assumption does not hold practice. Given the status quo, an outstanding problem is to devise a practical message relaying algorithm that aims at maximizing a priori defined global system objective for general user mobility.

This work proposes a class of decentralized and opportunistic relaying strategies (referred to as SCOOP) that aim at optimizing a priori defined global system objective. The admitted global system objective captures the value of information streams to users by accounting in a natural way for both users’ preference across various information streams with respect to the content and timeliness of delivery. The optimization problem also accounts for both storage and transmission costs. SCOOP features the following desired properties: (1) it aims at optimizing a well-defined global system objective, (2) it supports multi-point to multi-point communication, i.e., a multicast delivery of information streams, unlike previous proposals of point-to-point (unicast) routing schemes, (3) it is decentralized and requires only local observations, and (4) it allows for general user mobility, and thus does not require any independence assumptions with regard to message forwarding paths and is thus practical.

SCOOP is much simpler than existing state-of-the-art relaying strategies such as RAPID [3]. Indeed, the decision to relay a message from a given information stream, when the relay meets the corresponding source, depends on a single control variable (the probability to relay the message) that is identical for all messages of the stream. This contrasts, for example, with RAPID where this decision is taken on a per-message basis, depending on whether or not the message has been already relayed by other nodes and on delay estimates. This simplicity yields more practical implementations and increased scalability, and yet does not impact the performance as we show in our experimental results.

The key assumption that underlies the design of the relaying strategies proposed in this paper is the restriction to forward messages along paths of length at most two hops (referred to as two-hop relaying), i.e., messages are transferred to a user either through a direct contact with a source or through another user acting as a relay. While this restriction may degrade the efficiency of the information dissemination compared with a relaying strategy that would allow for longer-length paths, we show that in practice, forwarding...
along paths of length at most two already provides nearly optimal performance. Our data analysis further shows that relaying paths in mobile networks are typically positively correlated, and thus any independence assumptions are not valid in practice. Interestingly, positive correlations persist across a wide range of communication delays. In summary, our contributions are:

- Through the analysis of several real-world traces, we show that two-hops are enough for opportunistic relaying of information and that relaying paths are positively correlated (§2). This provides a justification to restrict the design to two-hop relaying. The observed positive correlations suggest that deriving information dissemination schemes using an underlying user mobility model under which delays through distinct paths are statistically independent would not be justified. Such positive correlations are indeed allowed by our framework which admits a general user mobility described by a stationary ergodic process, and thus allowing for user mobility to be statistically non-identical across users and in general statistically dependent across individual users and time.
- We formulate a natural global system objective and devise a decentralized relaying strategy that aims at optimizing this global system objective (§3). This is unlike to proposals that deploy various heuristic relaying strategies that are not necessarily optimal with respect to an underlying global system objective because they are not designed for the given objective or rely on some simplifying assumptions on user mobility that may not hold in practice. Our decentralized relaying strategy is derived as a sub-gradient scheme for an a priori defined global system objective combining the techniques from the Smoothed Perturbation Analysis (SPA) [e.g. [7]] and stochastic approximation (e.g. [14]).
- We describe a baseline implementation SCOOP and demonstrate the performance and practicality of the proposed framework through simulations using real-world mobility traces (§4). Specifically, our results show that, overall, SCOOP achieves comparable, and sometimes higher, delivery rates to that of an omniscient RAPID-like scheme.

To the best of our knowledge, our work would be first to present a class of relaying strategies for multicast content delivery in mobile ad-hoc networks that optimize a priori global system objective and allow for general user mobility. Specifically, in contrast to previous work, our framework alleviates making any specific assumptions on user mobility that may not hold in practice, such as statistical independence of inter-contact times between mobile devices, homogeneity of distributions of inter-contact times between distinct pairs of mobile devices, and assuming specific parametric families for the distributions of inter-contact times.

## 2. MULTI-HOP RELAYING

In this section, we examine the benefits of multi-hop relaying strategies through a set of real-world datasets. Table 1 summarizes the different datasets used in this study. Overall, the traces have widely different properties in terms of their duration, the environment, and the type of contacts studied, such as contacts of human mobility, e.g., Bluetooth contacts of human carried devices (Infocom trace) [23], device presence in WiFi hotspots (UCSD trace) [18], contacts from moving vehicles, e.g., opportunistic data transfers across the DieselNet buses [2], and GPS inferred contacts (SF Taxis trace) [21]. This section addresses the following questions:

Q1: Do a few number of hops suffice for content relaying or are long paths required to achieve acceptable performance?

Q2: What are the properties of the discovered multi-hop paths, and in particular, are the paths independent?

In the remainder of this section we provide support for the following two claims: (1) two-hop relaying brings most of the benefits when considering multi-hop relaying and (2) dependence across two-hops paths is significant, and thus independence assumptions do not appear to be valid.

### 2.1 Two Hops are Enough!

We examine the benefits of exploiting multiple hops in opportunistic information dissemination by tracking a message dissemination through contacts for the four traces in Table 1. In particular, we are interested in the message dissemination time defined as the time it takes for a message originated from a node to reach all nodes connected to the source through a path of length limited to some fixed number of hops. To measure this dissemination time, we randomly choose a source node, and observe how information originated from this node spreads through the network allowing for \( k \)-hop paths only, with \( k \) varying from 1 (i.e., direct contacts only) to “infinite” (i.e., the total number of devices in the trace).

Fig. 1 shows the results of this analysis for all the traces studied. In all cases, it is evident that, for all practical purposes, using just two hops yields nearly the same performance as using “any-hop” paths to disseminate information. The information relayed using direct contacts (i.e., one-hop relaying) only reaches a fraction of the population for the DieselNet and UCSD traces, equal to roughly 60%; in general, exploiting direct contacts only results in significant delays compared to multi-hop forwarding. Going beyond two hops brings marginal delay benefits, observation which holds irrespective of the type and properties of the trace, and irrespective of the source node chosen. Specifically, we find

<table>
<thead>
<tr>
<th>Name</th>
<th>Technology</th>
<th>Duration</th>
<th>Devices</th>
<th>Contacts</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCSD</td>
<td>WiFi</td>
<td>77 days</td>
<td>275</td>
<td>116,383</td>
<td>2002</td>
</tr>
<tr>
<td>Infocom</td>
<td>Bluetooth</td>
<td>3 days</td>
<td>197</td>
<td>42,569</td>
<td>2005</td>
</tr>
<tr>
<td>DieselNet</td>
<td>WiFi</td>
<td>20 days</td>
<td>74</td>
<td>5,260</td>
<td>2007</td>
</tr>
<tr>
<td>SF Taxis</td>
<td>GPS</td>
<td>24 days</td>
<td>535</td>
<td>18,3M</td>
<td>2008</td>
</tr>
</tbody>
</table>
that the improvement of two-hop paths compared to one-hop paths is typically at least one order of magnitude with regard to the dissemination time. These benefits significantly diminish going beyond two-hop paths (see Table 2).

### Table 2: Median delivery delay vs. number of hops.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCSD</td>
<td>25 days</td>
<td>2.5 days</td>
<td>1 day</td>
<td>1 day</td>
</tr>
<tr>
<td>Infocom</td>
<td>6 hr</td>
<td>6 hr</td>
<td>6 hr</td>
<td>6 hr</td>
</tr>
<tr>
<td>DieselNet</td>
<td>8 days</td>
<td>40 min</td>
<td>40 min</td>
<td>40 min</td>
</tr>
<tr>
<td>SF Taxis</td>
<td>4 hr</td>
<td>15 min</td>
<td>7 min</td>
<td>3 min</td>
</tr>
</tbody>
</table>

From the system design perspective, it is important that two-hop relaying schemes can achieve delays close to the “optimal”, as restricting to two-hop relaying schemes significantly simplifies the design of relaying strategies.

#### 2.2 Paths are Positively Correlated

Typically, opportunistic relaying algorithms operate by replicating messages at device contacts based on pre-defined heuristic rules [24, 5], and/or attempt to optimize a utility function using some simplifying assumptions about user mobility or global information state [3, 13]. In most cases, content relaying ignores any possible relationships among the various relay nodes and similarly analytical tractability favors the assumption of statistically independent relaying paths. However, in practice, one would expect that correlations among relaying nodes do exist, and that such correlations might result in sub-optimal forwarding and duplication of the content of interest. For example, devices carried by friends or co-workers might exhibit similar daily patterns with regard to their contacts with other devices. Thus, content duplication in such cases, where delay patterns between two devices are highly correlated, would bear little or no benefit in practice.

Having established that two-hop paths are sufficient, we now concentrate on analyzing the independence hypothesis by studying possible correlations across two-hop relaying paths. To this end, for a source device \( s \), and a destination device \( d \), we examine the time it takes (i.e., delay) for a message originated at device \( s \) to reach device \( d \) through a relay device \( r \), for all possible \((s, r, d)\) paths. We estimate the path delay by sampling at regular intervals throughout the trace, thus creating a delay time series per \((s, r, d)\) path. For example, one could sample once per day at 10 am, where the delay would specify the time passed since \( d \) last received content from \( s \) through \( r \), assuming that \( s \) always has new content to offer. Two paths for the same \((s, d)\) pair exhibit correlations (i.e., most points in the CDF would specify the time passed since \( d \) last received content from \( s \) through \( r \), assuming that \( s \) always has new content to offer. Two paths for the same \((s, d)\) pair exhibit correlations (i.e., most points in the CDF highlight that in the vast majority of cases, most paths per \((s, d)\) pair exhibit correlations (i.e., most points in the CDF are far away from 0). Table 3 displays the median correlation coefficients. Overall, positive correlations are prominent, while uncorrelated pairs seem limited. This implies that carefully selecting relays is crucial to optimize content distribution, as disseminating content through positively correlated paths might lead to sub-optimal performance.

The observed correlation is present irrespective of the dissemination delay. Fig. 3 examines the median value of the correlation coefficient across relay paths conditioned on their respective delay values. Specifically, for two relay paths.

#### Table 3: Correlation of two-hop paths.

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCSD</td>
<td>0.98</td>
</tr>
<tr>
<td>Infocom</td>
<td>0.5</td>
</tr>
<tr>
<td>DieselNet</td>
<td>0.2</td>
</tr>
<tr>
<td>SF Taxis</td>
<td>0.75</td>
</tr>
</tbody>
</table>
(s, r1, d) and (s, r2, d), the figure examines their correlation coefficient value versus the maximum of the respective mean path delays, and aggregates the correlation values by plotting the median per delay value. Maximum over the delay of the two paths was chosen to ensure that the delay of the two paths examined is bounded by the value on the x-axis. As previously observed, positive correlations are prominent irrespective of the mean path delays in all traces. Further, no clear trend is observed, with the median correlation coefficient value remaining roughly invariant to the mean path delay values.

Finally, we remark that not all paths per (s, d) pair exhibit correlations. We have examined the fraction of paths per (s, d) pair for which the correlation coefficient is within some interval (−δ, δ) for small values of δ > 0. Depending on the value of δ (e.g., from 0.01 to 0.09), the median fraction of uncorrelated paths varies between 1% to 5%. This amounts to roughly 1,500 to 7,000 paths per (s, d) for the SF-Taxis trace, or 10 to 300 paths for the Infocom trace. This indicates existence of paths with statistically independent delays that can be leveraged for the content dissemination task.

3. RELAYING ALGORITHMS

This section introduces a natural global objective and describes a distributed relaying scheme that aims at optimizing this objective. The global objective captures both user preferences over information channels and their timeliness of delivery. The proposed scheme does not rely on any specific assumptions about user mobility, and opportunistically and optimally exploit mobility so as to deliver content to users.

3.1 Objectives

3.1.1 Channels

We consider a system that consists of a set of information channels I, assumed to be finite, and a set of users U. Each user is interested in the content of some of these channels. Channel i publishes messages at instances of a stationary and ergodic process at rate 0 < λi < ∞ of messages per second. Users may receive messages directly from a source of the corresponding channel or through another user acting as a relay. For each channel i, we assume that a message published at time t may be of interest to a user if it reaches this user no later than t + ti, where ti is a deadline associated with messages of channel i.

3.1.2 User Mobility

We assume that user mobility is a stationary and ergodic process, thus allowing for general user mobility. For example, individual user movements are allowed to be statistically non identical and correlations are allowed across time and across individual user movements. User mobility is naturally assumed to be independent of message generation processes at the sources. As a consequence, we may define the stationary one-hop and two-hop delays: Di.u is the time it takes for
a channel-$i$ message to reach user $u$ without the help of any relay, and $D_{t,r,u}$ is the time it takes for a channel-$i$ message to reach user $u$ through relay $r$. Note that it might well be that the two-hop delays of channel-$i$ messages to user $u$ through different relays are correlated.

3.1.3 Probabilistic Relaying Strategies

We consider randomized relaying strategies that are specified by $x \in [0,1]^{[I] \times [U]}$ where $x_{i,r}$ represents the probability that user $r$ relays a message of channel $i$. User $r$ provides, for relaying purposes, a finite storage of size $B_r$ messages. Denote by $s$ and $t$ two consecutive contact times between a relay $r$ and a source of channel $i$. At time $t$, relay $r$ considers all messages published by channel $i$ in the time interval $[\max(s, t-t_i), t]$ in decreasing order of message age, and downloads each such message with probability $x_{i,r}$, where draws for different messages are independent. Note that no transmission constraints are considered here, as relays are interested in providing, a finite storage of size $B_r$ messages.

We may however extend our algorithm and analysis to model transmission constraints at individual user devices, which are implicit in the definition of the delivery probability $p_{i,u}(x)$. We will provide an explicit characterization of the delivery probability, $p_{i,u}(x)$, later in this section. Notice that in the above optimization problem, there is no cost for relays to download messages from sources or to transmit these messages to interested users. In a more realistic setting, relays may wish to limit the number of transmissions, for example to save battery power. Assume that the cost for relay $r$ to transmit and receive messages to be relayed at average rate $a_r$ is captured through a cost function $C_r(a_r)$, assumed to be increasing, continuously differentiable, and convex. This is accommodated by replacing the objective function in (1) by:

$$\sum_{i \in I, u \in U} V_{i,u}(p_{i,u}(x)) = \sum_{r \in R} C_r(a_r(x))$$

where $a_r(x)$ represents the average transmission and reception rate of relay $r$ under strategy $x$. The analysis and the relaying strategies proposed here to solve SYSTEM can be extended to include transmission costs.

3.2 Sub-gradient Algorithms

In this section we focus on describing relaying strategies that aim at solving SYSTEM, introduced in (1). Our strategies are based on sub-gradient method that amounts to updating the relay probabilities as follows, for every channel $i$ and relay $r$,

$$\frac{d}{dt} x_{i,r} = \sum_{j \in I, u \in U} V'_{j,u}(p_{j,u}(x)) \frac{\partial}{\partial x_{i,r}} p_{j,u}(x).$$

(3)

Under this dynamics, the objective function in SYSTEM increases over time and converges to a maximum value. Due to space limitations, this paper skips the presentation of structural properties of the optimization problem (1), but note that we were able to establish uniqueness of optima under some simplifying assumptions.

The difficulty of this approach lies in evaluating the gradient in (3), i.e. for every channel $j$ and user $u$, we need to evaluate $\partial p_{j,u}(x)/\partial x_{i,r}$, for every channel $i$ and relay $r$. To address this challenge, we combine techniques from Smoothed Perturbation Analysis (SPA) (e.g., [7]), and stochastic approximation (e.g., [14]). In what follows, in order to sim-

\[2\] We provide details in Appendix.
plify the presentation, we use linear utility functions so that \( V_{i,u}(p_a(x)) = w_{i,u} p_a(x) \), for some positive constant \( w_{i,u} \), but note that the analysis readily extends to more general classes of utility functions.

### 3.2.1 Smoothed Perturbation Analysis

We show how to evaluate the gradient of the function \( p_{j,u}(x) \), for every channel \( j \) and relay \( r \), using smoothed perturbation techniques [7]. This yields an explicit characterization of the gradient in terms of expectations of some random variables whose realizations can be locally observed by users and estimated by an online procedure that we describe in §3.3.

The age \( \tilde{A}_{j,u} \) of a message of channel \( j \) when received by user \( u \), if received at all, exceeds the deadline \( t_j \) for user \( u \), if the message could not have been received within deadline by user \( u \) through neither a direct contact with a source of the message nor via any relay. We characterize this event in the following.

We first need to introduce some notation for a message of channel \( j \). Without loss of generality, we assume that this message was generated at time equal to 0. Let \( \tilde{A}_{j,r,u} \) denote the age of the message of channel \( j \) at earliest time instant at which it could have been received by user \( u \) through relay \( r \) (if it was downloaded by relay \( r \)). Let \( N_{j,r,u} \) denote the number of messages admitted by relay \( r \) in the time interval \( (D_{j,r}, D_{j,r,u}] \), where \( D_{j,r} \) is the one-hop delay from \( j \) to relay \( r \) and \( D_{j,r,u} \) is the two-hop delay from \( j \) to \( u \) through relay \( r \). Notice that each message admitted in the latter time interval moves the message of channel \( j \) towards the head of the queue. The age \( \tilde{A}_{j,r,u} \) is less than \( t_j \) if and only if both of the following two conditions hold true: (1) there exists a path to user \( u \) through relay \( r \) within deadline \( t_j \), i.e. \( D_{j,r,u} \leq t_j \) and \( 2 \) the message is not evicted by the buffer policy at relay \( r \), i.e. \( N_{j,r,u} < B_r \). Therefore, we have

\[
\{ \tilde{A}_{j,r,u} \leq t_j \} = \{ D_{j,r,u} \leq t_j \} \cap \{ N_{j,r,u} < B_r \}.
\]

Note that \( \tilde{A}_{j,r,u} \) is defined for each message of channel \( j \) and may have a finite value even if the message was not downloaded by relay \( r \). To account for this, we define

\[
\tilde{A}_{j,r,u} = \begin{cases} 
\tilde{A}_{j,r,u} & \text{if } R_{j,r} = 1 \\
\infty & \text{otherwise}
\end{cases}
\]

where \( R_{j,r} \) is a binary indicator that takes value 1 if the message was admitted by relay \( r \) and value 0, otherwise.

Now, observe that \( A_{j,u} > t_j \) holds if and only if (1) the message could not have been delivered through a direct contact of user \( u \) with a source, i.e. \( D_{j,u} > t_j \), and (2) there exists no path to deliver the message through a relay within the deadline, i.e. \( A_{j,r,u} > t_j \), for every relay \( r \). In other words, we have

\[
\{ A_{j,u} > t_j \} = \{ D_{j,u} > t_j \} \cap \{ A_{j,r,u} > t_j \}.
\]

In order to present the main result of this section, we need to introduce some new notation. Let \( N'_{j,r,u} \) be the number of channel-\( i \) messages downloaded by relay \( r \) in the time interval \( (D_{j,r}, D_{j,r,u}] \) and let \( K'_{j,r,u} \) be the number of channel-\( i \) messages that are observed by relay \( r \) in \( (D_{j,r}, D_{j,r,u}] \) but not downloaded. Denote by \( A'_{j,r,u} \) the age of a message of channel \( j \) when arriving at user \( u \), assuming that relay \( r \) is not used to disseminate the message. Notice that \( A'_{j,r,u} > t_j \) holds if and only if \( D_{j,u} > t_j \) and \( A_{j,r,u} > t_j \), for every relay \( r' \neq r \).

Finally, let us define the following indicator, for a message of channel \( j \), relay \( r \), and user \( u \),

\[
I_{j,r,u} = \mathbb{I}_{A'_{j,r,u} > t_j} \mathbb{I}_{D_{j,r,u} \leq t_j}.
\]

We can now state the main result of this section that characterizes the gradient of the function \( p_{j,u}(x) \), for every channel \( j \) and relay \( r \). This is a key result that will enable us to devise optimal relaying strategies. The proof of the theorem is presented in §3.4.

**Theorem 3.1.** For every channel \( j \in I \) and user \( u \in \mathcal{U} \), the gradient of the function \( p_{j,u}(x) \) is given by, for every channel \( i \) and relay \( r \),

\[
\frac{\partial}{\partial x_{i,r}} p_{j,u}(x) = \mathbb{E}_x \left[ \mathbb{I}_{A'_{j,r,u} > t_j} \mathbb{I}_{D_{j,r,u} \leq t_j} \right] \mathbb{I}_{i = i} - \mathbb{E}_x [I_{j,r,u} R_{r,i}(N'_{j,r,u} \mathbb{I}_{N_{j,r,u} = B_r + K'_{j,r,u}} + K'_{j,r,u} \mathbb{I}_{N_{j,r,u} = B_r - 1})].
\]

(4)

The component of the gradient, \( \partial p_{j,u}(x)/\partial x_{i,r} \), consists of a positive and a negative element that admit the following intuitive interpretations. First, the positive element is zero for every \( i \neq j \); for \( i = j \), it corresponds to the probability that the message of channel \( i \) could have been delivered through relay \( r \) and not through any other path. Second, the negative element can be interpreted as a negative externality term that captures the effect of increasing the relaying probability \( x_{i,r} \) on the probability of delivery of channel-\( j \) messages. This term measures the number of channel-\( i \) messages downloaded by relay \( r \) during the time the channel-\( j \) message, which was dropped by \( r \) just before meeting user \( u \), was in the buffer of relay \( r \); and the number of channel-\( i \) messages that were rejected by relay \( r \) during the time the channel-\( j \) message was in the buffer and it was at the head of the queue (next to be evicted) when relay \( r \) meets user \( u \).

The gradient in (4) can be estimated in an online fashion by relays using only locally observable information, as we describe in the next section.

### 3.2.2 Stochastic Approximation

We identify an online algorithm for updating the relaying probabilities by relays based on locally observed information and show convergence to the sub-gradient dynamics introduced in (3).

We consider updating of the relaying probabilities by a relay \( r \) and introduce the following variables per message \( m \) of channel \( c \) that are locally observable by relay \( r \). Let us introduce \( Y_{i,r}(m) \) defined as follows

\[
Y_{i,r}(m) = \sum_{u \in \mathcal{U}} (\alpha_{i,s,u}(m) - \beta_{i,s,u}(m))
\]

3Hereinafter, for a relation \( A \), \( \mathbb{I}_A \) is equal to 1 if \( A \) is true, and 0, otherwise.
where

\[ \alpha_{r,t,u}(m) = \left( w_{r,u} \mathbb{1}_{\mathcal{A}_{r,u}(m) \geq t_u} \mathbb{1}_{\mathcal{A}_{r,u}(m) \leq t_u} \right) \mathbb{1}_{c=i} \]

and

\[ \beta_{r,t,u}(m) = w_{r,u} \mathbb{1}_{\mathcal{A}_{r,u}(m) > t_u} \mathbb{1}_{\mathcal{D}_{r,t,x}(m) \leq t_c} \mathcal{R}_c(m) \times \]

\[ \times \left[ K_{r,t,u}(m) \mathbb{1}_{\mathcal{N}_{r,t,u}(m) = B_i} + K'_{r,t,u}(m) \mathbb{1}_{\mathcal{N}_{r,t,u}(m) = B_i - 1} \right]. \]

For an interpretation of the expected values of \( \alpha_{r,t,u}(m) \) and \( \beta_{r,t,u}(m) \) we refer to the discussion following Theorem 3.1.

Remark that relay \( r \) can observe \( Y_{r,m}(m) \) when it receives feedback from users for message \( m \). Each user \( u \) interested in messages \( m \) must inform relay \( r \) whether other relays were able to successfully deliver message \( m \) to user \( u \). This can be achieved by letting each user \( u \) interested in message \( m \) keep a record whether message \( m \) could have been received within deadline through a unique path, if it could have been received at all. Then, when for the first time \( T_{r,u}(m) \) after the deadline of message \( m \) expires, relay \( r \) and user \( u \) meet, user \( u \) sends the required information to \( r \) which allows to compute the part of \( Y_{r,m}(m) \) corresponding to user \( u \). We assume that relay \( r \) updates the relaying probability \( x_{r,j} \) at instances \( T_{r,u}(m) \), for each message \( m \) and interested user \( u \) by an online update rule that we describe in the following.

We denote with \( S_r(n) \) the \( n \)-th feedback from a user to relay \( r \) (notice that \( S_r(n), n \geq 0 \) is a superposition of the instances \( (T_{r,u}(m), \forall m, \forall u) \)). Denote by \( c(n) \) the channel of the corresponding message, and by \( u(n) \) the user from which relay \( r \) receives feedback. We update the relaying probabilities \( \{x_{r,j}, i \in I \} \) using the following stochastic approximation algorithm per each new feedback received, for \( 0 < \epsilon < 1 \),

\[ x_{r,j}(n+1) = x_{r,j}(n) + \epsilon \sum_{j \in \mathcal{U}} \frac{\lambda'_j}{\lambda'_1} \left( \alpha_{r,t,u}(n) - \beta_{r,t,u}(n) \right) \]

where \( \lambda'_j \) is the publishing rate of messages of channel \( j \) as observed by relay \( r \). Notice that \( \lambda'_j \) is equal to the publishing rate of channel \( j \) messages if the relay meets a source of channel \( j \) at a positive rate. Remark that the update rule (5) conveniently aggregates feedback from different users in an online fashion.

We show that the update rule (5) approximates the subgradient algorithm specified in (3). Let \( \bar{x}(t) \) be a continuous-time process, defined for channel \( i \) and relay \( r \) as follows:

\[ \bar{x}_{r,i}(t) = x_{r,i}(n), \text{ for } t \in [\varepsilon S_r(n), \varepsilon S_r(n+1)]. \]

We next present a convergence result whose proof is provided in § 3.4.2.

**Theorem 3.2.** For the stochastic approximation algorithm (5), \( \bar{x}(t) = (\bar{x}_{r,i}(t), i \in I, r \in \mathcal{U}) \) uniformly converges over compact time intervals, for asymptotically small parameter \( \varepsilon > 0 \), to the solution of the following system of ordinary differential equations, for every channel \( i \) and relay \( r \),

\[ \frac{d}{dt} \bar{x}_{r,i}(t) = \frac{1}{\tau_r} \sum_{j \in I, u \in \mathcal{U}} w_{j,u} \frac{\partial}{\partial x_{r,j}} p_{j,u}(\bar{x}(t)) \]

where \( \tau_r := 1/(\sum_{j \in I} \lambda'_j) \).

### 3.3 A Baseline Implementation

For concreteness, we describe an implementation of the stochastic approximation algorithm (5). We describe the state kept by individual users and the actions performed at user encounters.

#### 3.3.1 Relay \( r \)

Relay \( r \) maintains a buffer of messages observed from sources which includes real messages whose payload was downloaded and also virtual messages that are messages observed by the relay whose payload was not downloaded. Note that at any time, there are at most \( B_r \) real messages in the buffer, where \( B_r \) is a configuration parameter, while virtual messages do not consume the buffer of relay \( r \) and some control information is maintained for these messages in order to compute \( \alpha_{r,t,u}(m) \) and \( \beta_{r,t,u}(m) \) for each message \( m \) of channel \( i \) and each interested user \( u \). Relay \( r \) further maintains a reference to the last message dropped from the buffer. Refer to Figure 4 for an illustration of a relay’s buffer structure. We now describe the procedures run by relay \( r \) when meeting a source and a user, respectively.

**Relay \( r \) meets a channel-\( i \) source.** Relay \( r \) first updates its estimate, \( \lambda'_i \), the publish rate of fresh channel-\( i \) messages as observed by relay \( r \). This is done by using a recursive estimator such as exponential weighting smoothing that is commonplace in the design of networking systems. It then downloads each message with probability \( x_{r,i} \) in decreasing order of age, and during this procedure, updates the reference to the last dropped message.

**Relay \( r \) meets user \( u \).** Relay \( r \) first transmits all messages from its buffer to user \( u \) which are of interest to this user (user is subscribed to this channel and the age of a message is smaller than the deadline). The relay maintains two records per message \( m \), \( \mathcal{D}_r[m][i][u] \) and \( \mathcal{D}_r[m][i][u] \), where \( m \) is identifier of a message, \( i \) is identifier of a channel, and \( u \) is identifier of a user, which we describe in the follow-
ing. Notice that these records are created only if a message is in either state head-of-the-queue or last-dropped at an encounter with a user u and the user expressed interest for message m. At such an event, if m is at the head-of-the-queue, for each channel i, dec_h[m][i][u] is created and set to the difference of the number channel-i messages in the buffer (real and virtual) and the number of real messages in its buffer (notice that this difference corresponds to the parameter $K'_c,i,r,a(m)$) where c is the channel of message m). On the other hand, if message m is the last dropped message, for each channel i, dec_ld[m][i][u] is created and set to the number of real channel-i messages in its buffer (notice that this corresponds to $N'_{c,i,r,u}(m)$). The records dec_h[m][i][u] and dec_ld[m][i][u] are kept by relay r until feedback from user u for message m is received and at that time are used to adjust the relaying probabilities for relay r, which we describe in more detail shortly.

Finally, relay r receives feedback from user u and updates its relaying probabilities. Specifically, for a message m of channel c, user u sends a ternary feedback $(f_1(m), f_2(m), f_3(m))$ where $f_1(m)$, $f_2(m)$, and $f_3(m)$ are binary values that are used to adjust the relaying probabilities as follows, for a fixed configuration parameter $\varepsilon > 0$,

$$x_{i,r} \leftarrow x_{i,r} + \frac{\varepsilon}{K_r} \left[f_1(m) \mathbb{I}_{x_{i-1}} - f_2(m) \left(f_1(m) \text{dec}_h[m][i][u] + (1 - f_3(m)) \text{dec}_ld[m][i][u]\right)\right].$$

Notice that $f_1(m)$ signals whether an increment of the relaying probability $x_{i,r}$ should be made, $f_2(m)$ signals whether a decrement of the relaying probabilities of relay r should be made, and $f_3(m)$ signals whether the decrement is because m was either in the head-of-the-queue or the last-dropped state.

Garbage collection. For each message m observed by relay r, relay r maintains a list of receivers that need to provide feedback for this message. These are receivers that observed message m for the first time from the buffer of relay r or in the last-drop state at relay r. The state maintained for message m is deleted by relay r when feedback is received from all receivers that needed to provide feedback.

3.3.2 Receiver u

For each message m of interest for receiver u, the latter maintains a list of relays, inc_list[m], which at the time when the feedback collection is completed, contains identities of relays through which m was observed within deadline and the payload of this message could not have been downloaded from neither a source nor another relay, and which thus should receive a positive feedback. Similarly, user u maintains a list of relay identities, dec_list[m], for which message m was observed in either the head-of-the-queue or the last-dropped state, and which thus should receive a negative feedback.

Receiver u meets relay r. For each observed message m, receiver u maintains a boolean variable seen_real[m], which will be used to distinguish the case where user u could have downloaded the payload of message m from more than 1 user (either a source or a relay), or otherwise.

We first describe the updates of dec_list[m]. If the variable seen_real[m] is equal to 0 (i.e. message m has not been downloaded earlier), then, if message m is either head-of-the-queue or last-dropped at relay r, then r is appended to dec_list[m]. Otherwise, if seen_real[m] is equal to 1, then any entries from dec_list[m] are deleted (because there existed a path to deliver message m to receiver u).

We next describe the updates of inc_list[m]. If message m is observed for the first time by receiver u and is in the buffer of relay r, inc_list[m] is initialized to r and seen_real[m] is set to 0, if m is a virtual message, and set to 1, otherwise. Otherwise, if message m was already observed at an earlier instance, then we distinguish two cases. First, if message m is a real message, then any entries from inc_list[m] are removed and r is appended, if seen_real[m] is equal to 0; then, seen_real[m] is set to 1. Second, m is a virtual message, then r is appended to inc_list[m], if seen_real[m] is equal to 0.

Finally, feedback is computed as follows. For each message m such that there exists an entry r in either inc_list[m] or dec_list[m] and the deadline of message m expired, the feedback is set as follows. If r is in inc_list[m], then $f_1(m) = 1$, otherwise, $f_1(m) = 0$. If r is in dec_list[m] then $f_2(m) = 1$, otherwise $f_2(m) = 0$. If receiver u has downloaded message m, then $f_3(m) = 1$, otherwise $f_3(m) = 0$. Notice that conditional on $f_2(m) = 1$, $f_3(m) = 1$ means that message m was in the head-of-the-queue state when r and u were in contact, and otherwise, in the last-dropped state. Feedback $(f_1(m), f_2(m), f_3(m))$ is communicated to relay r.

Receiver u meets source s. If message m is observed from source s within deadline, then any entries are removed from both inc_list[m] and dec_list[m], and seen_real[m] is set to 1.

3.4 Proofs of Theorems 3.1 and 3.2

3.4.1 Proof of Theorem 3.1

First, note that for every relay r \( \in U \), we have

$$1 - p_{j,a}(x) = \mathbb{P}_s[A_{j,a} > t_j]$$

$$= \mathbb{E}_s[1_{X_{j,a} > t_j} (1 - \mathbb{I}_{A_{j,a} > t_j} R_{j,a})]$$

$$= \mathbb{E}_s[A_{j,a} > t_j] - \mathbb{E}_s[I_{j,a}] R_{j,a} \mathbb{I}_{A_{j,a} > B_a}.$$ 

Since the ages of messages through paths other than those traversing relay r do not depend on $x_{j,r}$, in order to compute the partial derivative of $p_{j,a}(x)$ with respect to $x_{j,r}$, it suffices to consider only the second term in the right-hand side of the above equation.

Let $M'_{j,a}$ be the number of channel-i messages that are observed by relay r in the time interval $(D_{j,a}, D_{j,a}]$. Notice
that \( M'_{i,r,a} = N'_{i,r,a} + K'_{i,r,a} \) for every channels \( i \) and \( j \) and relay \( r \). In order to ease the notation, we will use the following shorthand notation \( N ≡ N_{j,r,a}, N' ≡ N'_{j,r,a}, N^{-i} ≡ N - N'_{j,r,a} \), and \( M' ≡ M'_{j,r,a} \).

Notice that the following holds
\[
\mathbb{E}[I_{j,r,a} R_{j,r} \mathbb{I}_{N < B_r}] = \mathbb{E}_x[I_{j,r,a} R_{j,r} \mathbb{P}_x[N < B_r|M', N^{-i}]]
\]
and, then consider
\[
h(x,r) = \mathbb{P}_x[N < B_r|M', N^{-i}]
\]
Since conditional on \( M' \) and \( N^{-i} \), \( N' \) is a binomial random variable with parameters \( M' \) and \( x_i \), we have
\[
h(y) = \sum_{j=0}^{M'} \binom{M'}{j} y^j (1-y)^{M'-j}.
\]
Taking the derivative, we obtain
\[
h'(y) = -M' \binom{M'-1}{B_r-1} y^{B_r-1} (1-y)^{M'-1-B_r} - M' \mathbb{P}[Z = B_r-1] (1-y)^{M'-1}.
\]
where \( Z \) is a binomial random variable \( (M'-1, x_i) \). Now, it is readily showed that for any two binomial random variables \( Z \sim \text{Bin}(m-1, p) \) and \( Y \sim \text{Bin}(m, p) \),
\[
\mathbb{P}[Z = z] = \left(1 - \frac{z}{m}\right) \mathbb{P}[Y = z] + \frac{z+1}{m} \mathbb{P}[Y = z+1].
\]
Therefore, since \( N' \) is a binomial random variable \( (M', x_i) \), conditional on \( M' \), we have
\[
h'(x_i,r) = -\mathbb{E}_x[(M'-N')1_{N-B_r-1} + N'R_I_{N=B_r}]. \quad (6)
\]
We use the latter identity for the following two cases.

**Case 1:** \( j \neq i \). In this case, we have
\[
\frac{\partial}{\partial x_{i,r}} p_{j,r}(x) = \mathbb{E}_x[I_{j,r,a} R_{j,r} h'(x_{i,r})]
\]
\[
= -\mathbb{E}_x[I_{j,r,a} R_{j,r} h'(x_{i,r})](M'-N')1_{N-B_r-1} + N'R_I_{N=B_r}]. \quad (7)
\]
where the last equality follows from (6).

**Case 2:** \( j = i \). In this case, we have
\[
\frac{\partial}{\partial x_{i,r}} p_{i,r}(x) = \mathbb{E}_x[I_{i,r,a} h'(x_{i,r}) + x_{i,r} h'(x_{i,r})]
\]
\[
= \mathbb{E}_x[I_{i,r,a} h'(x_{i,r})] + \mathbb{E}_x[I_{i,r,a} R_{i,r} h'(x_{i,r})]
\]
\[
= \mathbb{E}_x[I_{i,r,a} h'(x_{i,r})] + \mathbb{E}_x[I_{i,r,a} R_{i,r} h'(x_{i,r})] \quad (8)
\]
where \( (8) \) holds because \( R_{i,r} \) and \( I_{i,r,a} h'(x_{i,r}) \) are mutually independent random variables and \( (9) \) follows from (6).

The asserted result follows from (7) and (9) by turning back to the original notation.

### 3.4.2 Proof of Theorem 3.2

The result follows from Kushner and Yin [14][Chapter 12, Theorem 3.1] in view of the following facts. First, since we assume that the message publishing by sources and user mobility are stationary ergodic processes, so that we have for every relay \( r \),
\[
\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} (T_r(n) - T_r(n)) = \tau_r
\]
where recall \( T_r(n) \), \( n \geq 1 \), are instances at which feedback is received by a relay \( r \).

Second, we establish that the following holds for every channel \( i \) and relay \( r \),
\[
\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \sum_{j=1}^{N} \sum_{i \in \mathcal{U}} \frac{\lambda_j^r}{\lambda_r^j} Y_{i,r}(n) = \frac{\partial}{\partial x_{i,r}} \sum_{j=1}^{N} \sum_{i \in \mathcal{U}} w_{j,a} p_{j,a}(x)
\]
where \( c(n) \) and \( Y_{i,r}(n) \) are under \( x(n) \) fixed to \( x \), for every \( n \geq 1 \). Notice that
\[
\frac{1}{N} \sum_{n=1}^{N} \sum_{j=1}^{N} \frac{\lambda_j^r}{\lambda_r^j} Y_{i,r}(n) 1_{c(n) = j} = \sum_{j=1}^{N} \sum_{i \in \mathcal{U}} \frac{\lambda_j^r}{\lambda_r^j} Y_{i,r}(n) 1_{c(n) = j}
\]
where for each fixed channel \( j \), \( n'(l) \) is a subsequence at which \( c(n) = j \) and \( N_j \) is the length of this subsequence. Noting that for every channel \( j \), \( \lim_{N_j \to \infty} N_j/N = \lambda_j^r/\sum_j \lambda_j^r \) and
\[
\lim_{N_j \to \infty} \frac{1}{N_j} \sum_{i=1}^{N_j} Y_{i,r}(n'(l)) = \frac{\partial}{\partial x_{i,r}} \sum_{j=1}^{N_j} w_{j,a} p_{j,a}(x).
\]
The asserted result follows.

### 4. PERFORMANCE

We evaluate the performance of SCOOP by comparing with current state-of-the-art protocols based on realistic mobility scenarios. This section presents a brief characterization of SCOOP’s performance by examining the significance and effect of various parameters such as the buffer size, the publishing rate of messages, and the message expiration deadline. All results reported here are obtained for \( \epsilon = 0.01 \) (we varied \( \epsilon \) around this value and observed only negligible performance changes).

Since previous work does not support multicast delivery of streams, we have adapted RAPID [3] to support the delivery of messages from a source to multiple destinations. Previous evaluations of RAPID show that it outperforms other strategies and hence, it is the baseline used to compare SCOOP. In RAPID, a relay forwards messages greedily aiming at maximizing the marginal utility which is similar in spirit to our scheme. The utility might be, as in our system, the probability to deliver messages to destinations within specified deadlines. In order to estimate the marginal utility gain, a
relay has to ideally know which other nodes possess replicas of the message and when they expect to meet the destination. As this requires global knowledge, strategies to estimate the gain resort to approximation and simplifying assumptions about user mobility (e.g., statistically identical individual user movement, independence of delivery paths, and some Poisson approximations). In particular, RAPID assumes that delays through various relays are statistically independent, an assumption contradicted by the experimental results presented in Section 2.

To adapt RAPID for a multicast scenario, we examine the aggregate utility of the probability of delivering a message across all destinations for every message. We further compare SCOOP against an optimized version of RAPID, that will henceforth be referred to as R-OPT. The optimizations include the following: (1) Each relay node has complete knowledge of the dissemination state, i.e., at any point in time each node knows exactly which messages are carried by all nodes and (2) Each relay knows the complete matrix of mean pairwise inter-contact times for all nodes. As discussed above, in the original RAPID algorithm, these quantities are approximated since it is practically infeasible for all nodes to have a complete view of the whole network.

Fig. 5 presents how SCOOP performs against R-OPT, for the DieselNet trace that was initially used to evaluate RAPID in [3]. For comparison purposes, we further highlight R-OPT’s performance by restricting it to two-hop relay-paths only. Fig. 5 highlights the message delivery ratio as we vary the node buffer sizes and the source publishing rate. Each point represents mean value computed based on ten runs where five sources and five destinations were chosen randomly, and the rest of the nodes are relays. The average delivery ratio are reported at a confidence level of 95% (confidence intervals are omitted for better visualization purposes). Message deadline is set to 1 week. Finally, SCOOP’s initial relaying probabilities are set to 0.5.

Despite its decentralized nature, SCOOP’s performance is at least comparable to R-OPT in most scenarios. For small and intermediate buffer sizes (i.e., 1 and 10 respectively), SCOOP appears to outperform R-OPT, with the exception of high publishing rates (i.e., scenarios with sources that generate messages at inter-publish time of one hour). We further observe that as the publishing rate decreases, SCOOP’s performance improves. This is because under lower publishing rates there are more opportunities to identify better paths through which messages of interest can be delivered to their respective destinations. On the other hand, when the buffer size is large enough (i.e., buffer size equal to 100), mobility is the most significant factor determining the performance of the algorithms; that is, no message drops exist since buffers are large enough, and hence all messages of interest are replicated at node meeting instances. Thus, R-OPT performs best, and essentially presents the limit for any algorithm since it is not restricted to two hop paths. Note that in this case, SCOOP’s performance is comparable to the two-hop version of R-OPT.

Fig. 6 examines similar scenarios but in this case with message deadlines varying from 1 to 14 days, with intermediate buffer sizes (equal to 10 messages). As expected, increasing message deadlines improves the performance for all algorithms. In agreement with Fig. 5, as the publishing rate becomes lower, SCOOP’s delivery ratio improves, and may outperform R-OPT by more than 10%. This is despite R-OPT’s complete knowledge of the dissemination state, and the fact that SCOOP only uses local information.

5. RELATED WORK

Several proposals for routing or disseminating messages in DTNs have been made and we refer to [26, 27] and [3] for overview of the state-of-the-art. Routing protocols in DTNs are usually classified into two broad categories: (1) forwarding protocols that keep a single copy of the message.
to deliver in the network, see e.g. [11]; (2) epidemic routing protocols that replicate messages at transfer opportunities to find a path to the destination. We are interested in the second category of protocols since our system goal is to disseminate the channel contents to multiple interested users.

In most of the algorithms proposed so far, nodes limit the number of times they forward a message using various kinds of information. For example, in [24, 5], the routing uses the number of replicas already generated by nodes to decide whether new replicas should be created; most of the protocols use the history of node encounters to infer likelihood of message delivery if forwarded to a particular node (e.g. [17, 6, 5, 20, 3]); replication algorithms may also try to leverage the social structure of the network for message forwarding decisions [8] as socially-related nodes are more likely to meet. Some routing algorithms account for storage limits at nodes, see e.g., [9, 17, 16, 6, 25]. Only a few papers, e.g. [5, 3], propose algorithms that in addition, try to cope with transmission or bandwidth constraints (the amount of information that can be exchanged per contact is limited).

Our framework differs from all previous proposals. First, it addresses multi-point (channel sources) to multi-point (interested users) communication. The closest related work are the protocols RAPID [3] and the one proposed in [13]. As discussed for RAPID in Section 4, these protocols are based on simplifying assumptions regarding user mobility (e.g. statistically identical individual user movements, independence of delivery paths). It remains unclear whether they perform well under general mobility models. In contrast, our framework identifies decentralized relaying strategies that provably converge to optimal solutions of a global system objective, and allow for general user mobility, and thus alleviates to resorting to any simplifying assumptions that may not be met in practice. Finally, we remark that aiming at a global system objective underlie some other work on the design of protocols for opportunistic communications. For example, [22] but the problem therein is optimizing caching of content and is thus different.

6. CONCLUDING REMARKS

We presented SCOOP, a relaying strategy that supports multicast delivery of information streams to interested users in mobile networks and that aims at optimizing a well-defined global system objective. SCOOP is fully decentralized, requires only local observations, allows for general user mobility, and provably converges to optimal points of the underlying global system objective. It is simpler than current state-of-the-art relaying schemes, and yet provides similar or better performance. SCOOP also seems, by design, more robust than existing relaying schemes, whose principles are based on specific user mobility assumptions, e.g. statistical independence of delays through different paths. We believe that the performance improvements obtained with SCOOP could be significant in many practical scenarios where user mobility exhibits strong correlations.

Although we have established the convergence of SCOOP towards optimal regimes, for future work, it could be interesting to understand analytically how fast is the convergence, and how it depends on user mobility, publishing rates, and algorithm parameters.

7. REFERENCES


APPENDIX

A. GLOBAL OBJECTIVE WITH TRANSMISSION COSTS

We discuss how to derive a sub-gradient algorithm for solving SYSTEM (1) with the objective function replaced with (2). We consider the gradient of the objective function

\[ F(x) = \sum_{j \in I, u \in U} V_j(u)(p_{j,u}(x)) - \sum_{g \in U} C_g(a_g(x)). \]

We note that for every channel \( i \) and relay \( r \),

\[
\frac{\partial}{\partial x_{i,r}} F(x) = \sum_{j \in I, u \in U} V'_j(u)(p_{j,u}(x)) \frac{\partial}{\partial x_{i,r}} p_{j,u}(x) - \sum_{g \in U} C'_g(a_g(x)) \frac{\partial}{\partial x_{i,r}} a_g(x). 
\]

We have already derived \( \partial p_{j,u}(x)/\partial x_{i,r} \) in Theorem 3.1. Therefore, in this section, we focus on the second summation element that requires to compute \( \partial a_g(x)/\partial x_{i,r} \), for every relay \( g \), relay \( r \), and channel \( i \).

The communication rate \( a_g(x) \) for a relay \( g \) is a sum of the download rate \( d_g(x) \) and upload rate \( u_g(x) \), i.e. \( a_g(x) = d_g(x) + u_g(x) \).
In the rest of this section, we consider any path from a source to user \( u \) the prevailing case. Using similar steps as in the proof of Theorem 3.1, one can notice that 

\[
\delta_t u_g(x) = \sum_{j \in I, u \in \mathcal{U}} \lambda_j \partial_x \mathcal{E}_x[I_{A_{j,u},u} \leq A_{j,u}\partial_x] + \mathcal{E}_x[I_{A_{j,u},u} A_{j,u}\partial_x].
\]

For the download rate \( d_g(x) \), it is immediate that

\[
\frac{\partial}{\partial x_{i,r}} d_g(x) = \lambda_i \gamma_{i,r}.
\]

In the rest of this section, we consider

\[
\frac{\partial}{\partial x_{i,r}} u_g(x) = \sum_{j \in I, u \in \mathcal{U}} \lambda_j \frac{\partial}{\partial x_{i,r}} \mathcal{E}_x[I_{A_{j,u},u} \leq A_{j,u}\partial_x].
\]

We separately consider the following two cases.

**Case 1**: \( g = r \). In this case, we have

\[
\frac{\partial}{\partial x_{i,r}} u_g(x) = \sum_{j \in I, u \in \mathcal{U}} \lambda_j \partial_x \mathcal{E}_x[I_{A_{j,u},u} \leq A_{j,u}\partial_x].
\]

Notice that

\[
\mathcal{E}_x[I_{A_{j,u},u} \leq A_{j,u}\partial_x] = \mathcal{E}_x[I_{A_{j,u},u} A_{j,u}\partial_x] R_j, N_{j,u} \leq B_r.
\]

Using similar steps as in the proof of Theorem 3.1, one can readily establish that

\[
\frac{\partial}{\partial x_{i,r}} \mathcal{E}_x[I_{A_{j,u},u} \leq A_{j,u}\partial_x] = \mathcal{E}_x[I_{D_{j,u},u} A_{j,u}\partial_x] R_j, N_{j,u} \leq B_r, I_{j,u} = i.
\]

This characterizes the gradient of the upload rate in (10) for the prevailing case.

**Case 2**: \( g \neq r \). In this case,

\[
\frac{\partial}{\partial x_{i,r}} \mathcal{E}_x[I_{A_{j,u},u} \leq A_{j,u}\partial_x] = \frac{\partial}{\partial x_{i,r}} \mathcal{E}_x[I_{A_{j,u},u} A_{j,u}\partial_x].
\]

where \( A_{j,u}^{r-1} \) is the age of the channel-\( j \) message along any path from a source to user \( u \) other than those traversing either relay \( g \) or relay \( r \) and by definition

\[
I_{j,u},u = \sum_{j \in I, u \in \mathcal{U}} I_{j,u, u} A_{j,u} A_{j,u}^{r-1}.\]

Notice that \( I_{j,u, r} A_{j,u} A_{j,u}^{r-1} \) is true if and only if relay \( g \) provided the second best path for the channel-\( j \) message from a source to user \( u \) with respect to the message age and this age is smaller or equal to the deadline.

Again, proceeding along similar steps as in the proof of Theorem 3.1, one can readily establish that

\[
\frac{\partial}{\partial x_{i,r}} \mathcal{E}_x[I_{A_{j,u},u} \leq A_{j,u}\partial_x] = -\mathcal{E}_x[I_{j,u, u} A_{j,u} B_r] I_{j,u} = i
\]

\[
+ \mathcal{E}_x[I_{j,u, u} A_{j,u} A_{j,u} B_r + K_{j,u}]].
\]

This completes the characterization of the gradient of the upload rate in (10).

**A.1 Stochastic Approximation**

The stochastic approximation algorithm can be derived straightforwardly by augmenting the approach in § 3.2.2 with the elements of the gradient that account for the transmission costs, which we derived in the preceding section. Notice that the algorithm uses only local information communicated at device encounters. Notice that any feedback sent from a user to a relay is only for a message for which this relay could have delivered the message first or second among all relays, and thus the amount of the feedback is limited.

**B. Structural Properties of the Global Optimization**

In this section we remark some structural properties of the optimal relaying probabilities under SYSTEM defined in (1). We first note the following property of any optimal solution.

**Proposition B.1.** For every optimal relaying probabilities \( x_{i,r}, i \in I, r \in \mathcal{U} \), with respect to SYSTEM, there exists a set of positive values (shadow prices) \( \mu_r \), one for each relay \( r \), such that for every relay \( r \) and every channel \( i \), the following holds:

\[
x_{i,r} = 0 \quad \text{and} \quad \theta_{i,r}(x) \leq \mu_r,
\]

\[
0 < x_{i,r} < 1 \quad \text{and} \quad \theta_{i,r}(x) = \mu_r,
\]

\[
x_{i,r} = 1 \quad \text{and} \quad \theta_{i,r}(x) \geq \mu_r,
\]

where \( \theta_{i,r}(x) = \sum_{u \in \mathcal{U}} V_{j,a}(p_{i,u}(x)) \mathcal{E}_x[A_{j,a} > t_i \wedge t_i \leq t_j] \).

**Proof.** Follows by using (4) along with the method of Lagrange multipliers. \( \square \)

The result tells us that some of the optimal relaying probabilities \( x_{i,r} \) may take extreme values 0 or 1. Notice that if for any given relay \( r \), and two distinct channels \( i \) and \( j \), the relaying probabilities \( x_{i,r} \) and \( x_{j,r} \) are in the interior of the interval \([0,1]\), then for these two channels, the marginal utilities \( \theta_{i,r}(x) \) and \( \theta_{j,r}(x) \) are equal.

In the remainder of this section, we note some further structural properties under some additional assumptions. Suppose that utility functions are linear, i.e., \( V_{j,a}(p_{j,a}(x)) = w_{j,a} p_{j,a}(x) \), for some constant \( w_{j,a} \geq 0 \), for every channel \( j \in I \) and every relay \( r \in \mathcal{U} \). It is not difficult to note that solving (1) is equivalent to solving the following optimization problem, for an arbitrarily fixed relay \( r \):

\[
\text{maximize } f_r(x) \quad \text{over } x \in [0,1]^{I \times |\mathcal{U}|}
\]
where
\[ f_i(x) := \sum_{j \in I, u \in \mathcal{U}} w_{j,u} \mathbb{P}_x[A_j^{j,u} \leq t_j] + \sum_{j \in I} x_j f_{j,r}(x) \tag{11} \]
and
\[ f_{j,r}(x) := \sum_{u \in \mathcal{U}} w_{j,u} \mathbb{P}_x[A_{j,u}^{j,u} > t_j, A_{j,j,r,u} \leq t_j | R_{j,r} = 1]. \]

Suppose that for every message \( m \) of channel \( j \), relay \( r \), and user \( u \), \( N_{j,r,u} \) is a Poisson random variable with mean \( \sum_{i \in I} \lambda_i x_i (D_{j,r,u} - D_{j,r}) \), conditional on the values of \( D_{j,r,u} \) and \( D_{j,r} \) and any other state. This assumption can be justified as follows. Suppose that each source \( i \) publishes messages at instances of a Poisson process at rate \( \lambda_i \). Furthermore, if for any \( i \neq j \), \( 0 < x_{i,r} < 1 \) and \( 0 < x_{j,r} < 1 \), then \( f_{i,r}(y_r) = f_{j,r}(y_r) \).

We conclude this section by noting that under prevailing assumptions, there exist cases under which optimal relaying probabilities \( x_{j,r} \) would tend to assume extreme values 0 or 1, i.e., a relay either admits or does not admit messages of a channel with probability 1.

For the prevailing case, it is easily observed that \( f_{i,r}(x) \) is a function of \( x \) only through \( y_r(x) := \sum_{i \in I} \lambda_i x_i \). Therefore, we consider the above optimization problem where with a slight abuse of notation we can replace \( f_{i,r}(x) \) with \( f_{i,r}(y_r(x)) \). The structure of the solution of this problem is as follows. Suppose that \( y_r(x) \) is fixed to arbitrary feasible values, for every relay \( r' \), and \( x_{i,r'} \) is fixed to an arbitrary feasible value for every channel \( i \) and every relay \( r' \neq r \). The first term in the definition of function \( f_i \) in (11) does not depend on \( (x_{i,r}, i \in I) \). Therefore, the relaying probabilities \( (x_{i,r}, i \in I) \) are a solution of the following linear program:

\[
\begin{align*}
\text{maximize} & \quad \sum_{i \in I} x_{i,r} f_i(y_r) \\
\text{over} & \quad x_{i,r} \in [0, 1], \ i \in I \\
\text{subject to} & \quad \sum_{i \in I} \lambda_i x_{i,r} = y_r.
\end{align*}
\]

This is a fractional Knapsack problem whose solution is as follows. Without loss of generality, let channels be enumerated such that
\[
\frac{f_1(y_r)}{\lambda_1} \geq \frac{f_2(y_r)}{\lambda_2} \geq \cdots \geq \frac{f_{|I|}(y_r)}{\lambda_{|I|}}.
\]

The optimum solution is \( x \) such that: (1) \( \sum_{i \in I} x_{i,r} = y_r \), and (2) there exist \( 1 \leq i_1 \leq i_2 \leq |I| \) such that \( x_{i,r} = 1 \) for \( 1 \leq i \leq i_1 \), \( 0 < x_{i,r} < 1 \) for \( i_1 < i \leq i_2 \) and \( x_{i,r} = 0 \), otherwise. Furthermore, if for any \( i \neq j \), \( 0 < x_{i,r} < 1 \) and \( 0 < x_{j,r} < 1 \), then \( f_{i,r}(y_r) = f_{j,r}(y_r) \). 

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