A Linear Approximation Based Method for Noise-Robust and Illumination-Invariant Image Change Detection

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Abstract. Image change detection plays a very important role in real-time video surveillance systems. To deal with the illumination, a category of linear algebra based algorithms were designed in the literature. They have been proved to be effective for surveillance environment with lighting and shadowing. In practice, other than illumination, the detecting process is also influenced by the noises of cameras and reflections. In this paper, analysis is made systemically on the existing linear algebra detectors, showing their intrinsic weakness in case of noises. In order to get less sensitive to noises, a novel method is proposed based on the technique of linear approximation. Theoretical and experimental analysis both show its robustness and high performance for noisy image change detection.

Keywords: Video surveillance, change detection, linear algebra.

1 Introduction

It has been a long-time studied topic in computer vision to detect changes in images taken at the same scene but at different times by a static camera. It serves as the basis of a large number of applications including video surveillance, medical diagnosis, civil infrastructure and so forth. However, varying illumination conditions, shadows, reflections and the noises of cameras make the veracious and robust detection a hard work. If the algorithm is not well designed, these influences will lead to false alarms, even when there are no changes at all.

In the past twenty years, many approaches have been proposed for image change detection. In [1], the calculation of changes between two images was performed on predefined sliding windows over each pixel. A statistical description of the ensemble of pixels was given and the decision about the change was made by statistical hypothesis test. Similar methods based on likelihood ratio tests

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were developed in [2] and [3]. Hsu et al. [4] fit the intensity values in each sliding window to a polynomial function of the pixel coordinates, and compared different likelihood tests using constant, linear and quadratic models. The polynomial model used in [4] was further extended to be illumination-invariant by introducing the partial derivatives on the quadratic model [5]. Besides, many other methods were also proposed in the literature. Among these methods, a series of linear algebra based approaches [6,7] attracted great attention because they are illumination-invariant. This is an important improvement as compared to the previous solutions. However, as analyzed in the following sections of this paper, the performance of these algorithms drops significantly in cases of noises. As we know, in practical environments, there always exist additive noises in the images. So it is a must for a practical change detection algorithm to be robust to noises. To tackle this problem, we proposed a novel method based on linear approximation. Theoretical analysis and experiments both show that this method outperforms the previous linear algebra based methods greatly in case of noises.

The rest of this paper is organized as follows. In Section 2 the general theories of linear algebra based image change detectors are described and the noise analysis is made. In Section 3 the new method is presented. Experimental results are discussed in Section 4. Then the conclusion remark is drawn in the last section.

2 Noise Analysis on Existing Linear Algebra Methods

As mentioned above, there have been several change detection techniques employing linear algebra in the literature. Almost all of them are closely related to the shading model [5] and working with the concept of linear dependence. The basic idea is that if there is no change, the pixel intensities in the current and the reference images should be linear dependent in spite of the illumination. In this sense, when there is no noise, different algorithms are equivalent. However, the situation may change when noises exist. To make it clear, in this section, we will focus on the theoretical analysis of such algorithms' performance by taking noise into consideration.

2.1 Shading Model

In the shading model, the intensity $F(x, y)$ of a pixel(x, y) can be modelled as the product of the illumination $I(x, y)$ from the light sources and the reflectance coefficient $R(x, y)$ of the object surface [8]:

$$F(x, y) = I(x, y)R(x, y). \tag{1}$$

Such a model covers most of the influences mentioned in the introduction: illumination, shadowing and reflection. By applying it to both the current and the reference images, we have,

$$F_r(x, y) = I_r(x, y)R_r(x, y), \quad F_c(x, y) = I_c(x, y)R_c(x, y), \tag{2}$$

where r and c represent the reference and current images respectively. Since the reflectance coefficient depends only on the physical structure of the object surface, $R_c(x, y)$ and $R_r(x, y)$ should be equal for the same pixel. Then we have:

$$F_c(x, y)/F_r(x, y) = I_c(x, y)/I_r(x, y) = k(x, y). \tag{3}$$

When working on a small area, it is reasonable to approximate that the illuminations $I_c(x, y)$ and $I_r(x, y)$ are independent of the pixel positions (x, y). Accordingly $k(x, y)$ will become a constant. In other words, $F_c(x, y)$ and $F_r(x, y)$ are linear dependent. That is just the key point of the shading model.

2.2 Linear Dependence Models

In the linear dependence detector (LDD) proposed by Durucan and Ebrahimi [6], a vector model as illustrated in Fig. 1 was used: a center pixel and its neighbors form a sliding window, and then the center pixel is represented by a vector made up of all pixels in the window. The windows usually take a size of 3×3, 5×5, 7×7 or 9×9. For the example shown in Fig. 1, the vector representation of pixel x_5 is $\boldsymbol{X} = (x_1, x_2, ..., x_9)^T$.

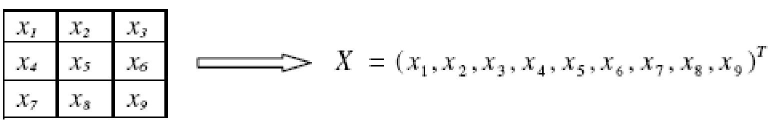


Fig. 1. A central pixel and its neighbors are illustrated on the left and its substituted vector is illustrated on the right.

More generally, let x_i denote the intensities of the pixels in the sliding window of the reference image, and y_i denote those of the pixels in the corresponding window of the current image. Then the vector representations are $\boldsymbol{X} = \{x_i, i = 1, 2, ..., n\}$ and $\boldsymbol{Y} = \{y_i, i = 1, 2, ..., n\}$ respectively. As indicated by the shading model, if there is no change, $\boldsymbol{X}$ and $\boldsymbol{Y}$ are linear dependent. Accordingly, it is easy to prove that the variance σ^2 in the sliding window expressed as below should be zero.

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left[\frac{y_i}{x_i} - \mu \right]^2, \quad \mu = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{x_i}. \tag{4}$$

Following this work, many other test criteria for linear dependence were also proposed. For instance, in [7] the criterion is designed on top of the determinants of Wronskian matrices (we call it Wronskian detector in the following discussions). If there is no change, no matter with or without illumination, this test should be some constant $k_0(k_0 - 1)$ (see Section 2.3 for the definition of k_0).

$$W = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i^2}{x_i^2} - \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{x_i}. \tag{5}$$

2.3 Noise Analysis

Although the above linear dependence based methods are expected to give illumination invariant change detection results, this is only true when no noise is present. In practical situations, the digital images are usually interfered by the noises of cameras and reflections. In such cases, as shown below, these algorithms will encounter problems.

Here we use a popular noise model, in which the noise has additive Gaussian distribution. When there is no change, the image intensities can be written as:

$$\boldsymbol{X} = \boldsymbol{S} + \boldsymbol{\delta_1}, \quad \boldsymbol{Y} = k_0\boldsymbol{S} + \boldsymbol{\delta_2}, \tag{6}$$

where $\boldsymbol{S}$ denotes the vector of the underlying image which is not affected by noise, and k_0 is a scalar factor representing the linear dependence. $\boldsymbol{\delta_1}, \boldsymbol{\delta_2}$ are two noise vectors in which the distribution of each element is $N(0, \sigma_d^2)$(i.i.d). By working out $\boldsymbol{S}$ from the first equation in (6) and substituting it to the second one, we can get the following model:

$$\boldsymbol{Y} = k_0\boldsymbol{X} + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} = \boldsymbol{\delta_2} - k_0\boldsymbol{\delta_1}, \tag{7}$$

where $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, ..., \epsilon_n)^T$ is the combined noise vector in which the distribution of each element is $N(0, \sigma_s^2)$ (i.i.d). Substitute the first equation of (7) to (4), there flows:

$$\mu^* = k_0 + \frac{1}{n}\sum_{i=1}^{n}\frac{\epsilon_i}{x_i}; \tag{8}$$

$$(\sigma^*)^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left[\frac{y_i}{x_i} - \mu^*\right]^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}\frac{\epsilon_i^2}{x_i^2} - \frac{1}{n}\left(\sum_{i=1}^{n}\frac{\epsilon_i}{x_i}\right)^2\right]. \tag{9}$$

As the elements of $\boldsymbol{\epsilon}$ are independently identically distributed, there holds:

$$E[\epsilon_i\epsilon_j] = \begin{cases} E[\epsilon_i]E[\epsilon_j] = 0, & i \neq j \\ E[\epsilon_i^2] = \sigma_s^2, & i = j \end{cases} \tag{10}$$

Hence, the expectation of $(\sigma^*)^2$ is:

$$E[(\sigma^*)^2] = \frac{1}{n-1}\left[\sum_{i=1}^{n}\frac{E[\epsilon_i^2]}{x_i^2} - \frac{1}{n}E\left[\left(\sum_{i=1}^{n}\frac{\epsilon_i}{x_i}\right)^2\right]\right] = \frac{\sigma_s^2}{n}\sum_{i=1}^{n}\frac{1}{x_i^2}. \tag{11}$$

Similar noise analysis can be done to the criterion of the Wronskian detector (WD) and its expectation is:

$$E[W^*] = k_0(k_0 - 1) + \frac{\sigma_s^2}{n}\sum_{i=1}^{n}\frac{1}{x_i^2}. \tag{12}$$

From the above derivations, we could find that even if there is no change, the expectations of the test criteria used in LDD and WD vary along with the image attributes and the noise variances. As a result, it is difficult to select a reasonable threshold. Most likely, the selection can only be done empirically and adaptive to different images. To tackle this problem, an unbiased criterion is required. In order to do that, we propose a new method in the next section, where the technique of linear approximation is adopted.

3 Linear Approximation Detector (LAD)

3.1 The Proposed Method

For the purpose of applying linear approximation, an assistant plane is used with an orthogonal coordinate system. The components of vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ are reorganized as points $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ in the assistant plane. Considering that the above n points might coincide with each other, an extra helper point $(x_0, y_0) = (0, 0)$ is added to the points set. The linear approximation of these points is calculated using least square algorithm [9], the corresponding result line of which is as below,

$$y = kx + b; \tag{13}$$

$$k = \Big[(n+1)\sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i\Big] \Big/ \Big[(n+1)\sum_{i=0}^{n} x_i^2 - \Big(\sum_{i=0}^{n} x_i\Big)^2\Big]; \tag{14}$$

$$b = \frac{1}{n+1}\Big[\sum_{i=0}^{n} y_i - k\sum_{i=0}^{n} x_i\Big]. \tag{15}$$

Following the idea of shading model, if there is no change, vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ should be linear dependent. That is, we could use b as a test criterion: there is no change when $|b| = 0$; otherwise, a change is detected. We call the proposed method as linear approximation detector (LAD).

3.2 Noise Analysis

When the same noise exists as described in section **2.3**, we could get the following derivations (the details are removed due to limitation of the paper length).

$$k^* = k_0 + (n+1)\sum_{i=0}^{n} x_i \epsilon_i \Big/ \Big[(n+1)\sum_{i=0}^{n} x_i^2 - \Big(\sum_{i=0}^{n} x_i\Big)^2\Big]; \tag{16}$$

$$b^* = \frac{1}{n+1}\Big[\sum_{i=0}^{n}(k_0 x_i + \epsilon_i) - k^*\sum_{i=0}^{n} x_i\Big] \tag{17}$$

$$= \sum_{i=1}^{n} x_i \epsilon_i \Big[\sum_{i=1}^{n} x_i \Big/ \Big((n+1)\sum_{i=1}^{n} x_i^2 - \Big(\sum_{i=1}^{n} x_i\Big)^2\Big)\Big]. \tag{18}$$

We could further get the conclusion that the expectation of the criterion b^* is zero and it can be hardly affected by both the noise and the local pixel intensities.

$$E[b^*] = \sum_{i=1}^{n} x_i E[\epsilon_i] \Big[\sum_{i=1}^{n} x_i \Big/ \Big((n+1)\sum_{i=1}^{n} x_i^2 - \Big(\sum_{i=1}^{n} x_i\Big)^2\Big)\Big] = 0. \tag{19}$$

In this sense, the proposed LAD algorithm will be more robust to noise than the aforementioned LDD and WD methods. This does make sense because in fact the noise always exists. And for some practical cases, especially when the environment is dark, the influence of the noise is much more significant.

4 Experimental Results

In our experiments, the algorithms of LDD, WD and LAD were implemented on the frames of *PETS2001*[1]. The size of the original color images in CAMERA1 is 768×576 pixels. We convert them to gray images and resize them to 384×288 pixels. For the experimental settings, we used a fixed sliding window of size 7×7.

Firstly, we tested the algorithms' performance on detecting changes between real images. Fig. 2(a) is the reference image (FRAME 0001), and Fig. 2(b) is the current image (FRAME 1220, where the car in the street corner moves back a little and an extra car moves in). Fig. 3(a) to (c) are the detection results by LDD, WD and LAD respectively. From these results, we can see that LDD and WD lead to some false alarms, while LAD can well wipe off these influences. However, the objects detected by LAD are a bit smaller than their actual sizes due to the operation of linear approximation.

Secondly, we would like to test the illumination sensitivity of the above algorithms. Fig. 2(c) is gained by adding illumination artificially to Fig. 2(b) with the scalar factor k_0 increasing gradually from the rightmost column ($k_0 = 0.9$) to the leftmost column ($k_0 = 1.3$). The change detection results between Fig. 2(a) and Fig. 2(c) are shown in Fig. 3(d) to (f). From them, we can see that all of these three algorithms are illumination-invariant. This is just the design purpose of utilizing linear algebra.

Thirdly, we examined the detectors' performance with camera noises added. For this purpose, we generate Fig. 2(d) by adding zero-mean Gauss noises to Fig. 2(b). Fig. 3(g) to (i) show that the noises are sensitively detected as moving objects by LDD and WD, while LAD successfully wipes off the influence of the noises.

Fourthly, we enumerated different thresholds for LDD, WD and LAD algorithms and got the *recall-precision* curves by comparing the detection results to the manually labelled ground truth in Fig. 4(a). The *recall* and *precision* are calculated as:

$$recall = D/(D+M), \quad precision = D/(D+F), \tag{20}$$

where D, M, and F denote the pixel numbers of the accurate detections, the missed detections and the false alarms respectively. Fig.4.(b) tells us that LAD outperforms LDD and WD by much in most of the cases. For example, when the precision is 80%, LAD resulted in 50% higher recall than LDD and WD.

Lastly, we fixed the thresholds for each algorithm and got the $F1$-*noise* curves by changing the variance of the added Gaussian noises from 0 to 10, where $F1$ is defined as:

$$F1 = 2 \times (recall \times precision)/(recall + precision). \tag{21}$$

We can see from Fig. 4(c) that the $F1$ curves of LDD and WD drop quickly when the noises become heavier. However, the curve of LAD is not influenced much by the noises, indicating to some extend that LAD is quite robust to noises.

[1] http://peipa.essex.ac.uk/ipa/pix/pets/PETS2001/DATASET1/TRAINING/

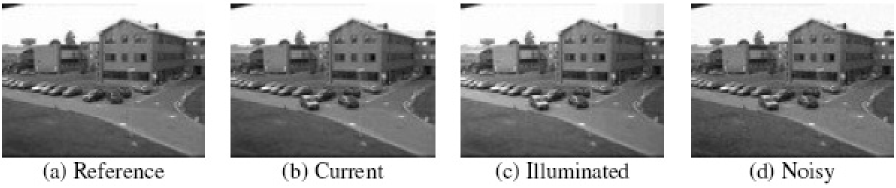


Fig. 2. Test images used in the experiments.

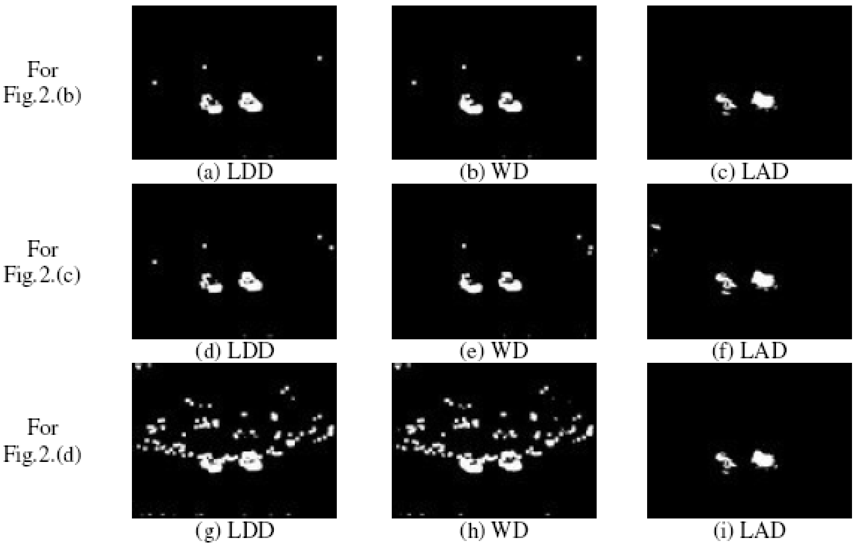
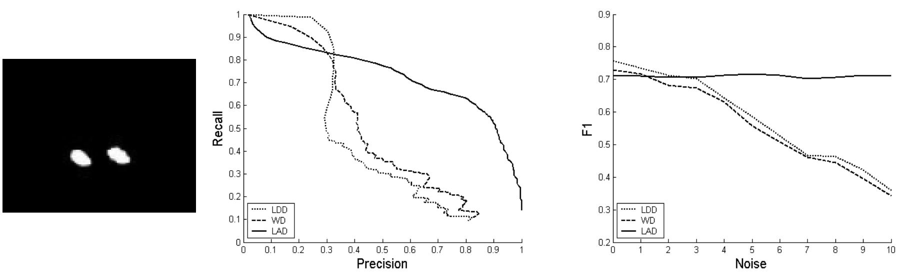


Fig. 3. Detection results of LDD, WD and LAD algorithms (here we manually adjusted the threshold to get best compromises between *recall* and *precision*).



(a)Ground truth (b)*recall-precision* curves (c)$F1$-*noise* curves

Fig. 4. The ground truth, *recall-precision* and $F1$-*noise* curves for LDD, WD and LAD.

5 Conclusion

This paper introduces a new linear algebra based approach for image change detection by adopting the linear approximation technique. All linear dependence based methods in the paper are analyzed theoretically with a popular noise model, which shows that the proposed method can avoid the intrinsic weakness of the other congener methods. Experimental results also verify that the proposed algorithm is much more robust to camera noises and reflections than other linear algebra based methods.

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