

# User-Specific Hand Modeling from Monocular Depth Sequences

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## 1. Contents

This supplementary material includes some additional detail on the linear blend skinning model, the subdivision surface model, and the parameter values used for the experiments.

## 2. Parameter settings

The following parameter settings, unless otherwise stated, define the final energy that we optimize in the experiments described in the paper.

$\sigma_x$	0.3
$\lambda_{\text{norm}}$	0.1
$\lambda_{\text{core}}$	3
$\lambda_{\text{inst}}$	2.5
$\lambda_{\text{skeleton}}$	0.5
$\lambda_{\text{L2}}$	0.05
$\lambda_{\text{prior}}$	10
$\lambda_{\text{motion}}$	300
$\lambda_{\text{scale}}$	1000

## 3. Linear Blend Skinning

In this section we specify the details of our chosen skinning function  $\mathcal{P}(V; \theta, \kappa)$ . The skeleton that we use consists of  $B$  bones organized into a tree structure in which the first bone is the root and  $\pi(b)$  indicates the parent of each node  $b \in \{2, \dots, B\}$ . Each bone  $b$  has an attached local coordinate system related to its parent's (or the world's in the case of the root) by a transformation  $T_b(\theta, \kappa)$  consisting of a rotation and a 3D translation. The rotation is specified using three exponential map coordinates contained in  $\theta$ . The translation is simply a scaling  $\beta_b(\kappa)$  of the translation  $\hat{t}_b$  in our template.

To define the function  $G_b(\theta, \kappa)$ , we then simply compose the transformations in a recursive manner up the skeleton as

$$G_1(\theta, \kappa) = T_1(\theta, \kappa) \quad (1)$$

$$G_b(\theta, \kappa) = G_{\pi(b)}(\theta, \kappa) * T_b(\theta, \kappa) \quad (2)$$

where the  $*$  operator indicates the composition of transformations. For our purposes, the global transformation

$G_{\text{glob}}(\theta, \kappa)$  is composed of an isotropic scaling encoded in  $\kappa$  and a rigid transform encoded in  $\theta$ .

## 4. Subdivision Surfaces

A modified Loop subdivision surface is used to *explicitly* model the surface of the hand. The advantages of an explicit surface representation are that the surface topology is fixed and that the surface is completely defined by a fixed number of control vertices  $\{\mathbf{v}_m\}_{m=1}^M$ . The advantages of subdivision surfaces are that they are continuous and have smoothly-varying normals.

A Loop subdivision surface is defined completely by a set of control vertices configured in a triangular mesh. The surface is described in sections by *patches*, where each patch is defined by a subset of the control vertices and the local patch topology. For patch  $p$  the surface patch is given by the set of points  $\{\mathcal{S}_p(\mathbf{u}; \mathbf{v}_{i_1^p}, \dots, \mathbf{v}_{i_{I_p}^p}) : \mathbf{u} \in \Delta\}$ , where  $\mathcal{S}_p$  is the patch position function,  $\Delta = \{(u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1 - u\}$  is the unit triangle, and  $i^p$  is the list of  $I_p$  vertex indices which contribute to patch  $p$ .

The local surface map  $\mathcal{S}_p$  depends on the patch topology. For a *regular* patch,  $\mathcal{S}_p(\mathbf{u}; \cdot) = \mathbf{b}(\mathbf{u})^\top V$ , where  $\mathbf{b}(\mathbf{u})$  are the basis functions for a regular triangular spline [4] and  $V \in \mathbb{R}^{12 \times 3} = [\mathbf{v}_{i_1^p} \dots \mathbf{v}_{i_{12}^p}]^\top$ . For the purposes of optimization, the essential property of  $\mathcal{S}$  is that it is linear in  $V$  and polynomial in  $\mathbf{u}$ .

For irregular or *extraordinary* patches the basis functions  $\mathbf{b}(\mathbf{u})$  cannot be applied directly. Instead, Loop subdivision [2] is used which defines  $\mathcal{S}_p(\mathbf{u}; \cdot)$  as a piecewise smooth function consisting of an infinite number of regular triangular patches [4]. The process of evaluating  $\mathcal{S}_p(\mathbf{u}; \cdot)$  can be understood by considering the extraordinary patch which contains a single extraordinary vertex with valency  $N = 5$ . By subdividing the control mesh, four child patches are created, three of which are regular and one of which has the same topology as the original patch. With reference to [4],  $\mathcal{S}_p(\mathbf{u}; \cdot)$  is given by:

$$\mathcal{S}_p(\mathbf{u}; \cdot) = \mathbf{b}(t_{k,n}(\mathbf{u}))^\top P_k \bar{A} A^{n-1} V \quad (3)$$

where  $\mathbf{u} = (u, v)$  and  $n = \lfloor -\log_2(u + v) + 1 \rfloor$  is the required level of subdivision,  $k \in \{0, 1, 2\}$  is the regular child patch index,  $t_{k,n}$  transforms  $\mathbf{u}$  to the child patch domain, and  $A$ ,  $\bar{A}$  and  $P_k$  are subdivision and “picking” matrices which are defined in [4]. In our implementation, code is automatically generated for up to 5 levels up subdivision, so this operation is no more expensive than the evaluation of ordinary patches.

A problem with (3) is that as  $\mathbf{u} \rightarrow \mathbf{0}$  first derivatives either vanish ( $N < 6$ ) or diverge ( $N > 6$ ) and are numerically unstable, which is problematic for continuous optimisation over  $\mathbf{u}$ . Similar behaviour has been noted for Catmull-Clark subdivision surfaces, but reparameterisation is computationally expensive [1]. Instead, we closely approximate extraordinary patches with quartic Bezier triangles. Similar approximations have been performed for Catmull-Clark subdivision surfaces using bicubic B-splines [3]. While the resulting surface is no longer  $C^1$  continuous between extraordinary patches, the discontinuities are minor and negligible in practice.

## References

- [1] I. Boier-Martin and D. Zorin. Differentiable parameterization of Catmull-Clark subdivision surfaces. In *Proc. Eurographics Symp. on Geometry processing*, 2004.
- [2] C. Loop. Smooth Subdivision Surfaces Based on Triangles. Master’s thesis, The University of Utah, Aug. 1987.
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