Plasma Physics and Controlled Fusion, Vol. 31, No. 3, pp. 431 to 434, 1989
Printed in Great Britain.
0741-3335/89 $3.00+ .00

ALPHA PARTICLE INDUCED MAGNETOACOUSTIC INSTABILITY IN A THERMONUCLEAR PLASMA

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(*Received* 22 *August* 1988)

Abstract—We show that in a thermonuclear plasma the magnetosonic wave propagating normal to the magnetic field can be unstable. The growth rate is small, however, and the wave is easily stabilized by perpendicular thermal conduction.

IN A FULLY IGNITED thermonuclear plasma the fusion energy production balances the various energy losses and the fusion reaction is self-sustaining. The product ions (alpha particles in the case of a deuterium–tritium plasma) carry a considerable amount of free energy and can therefore drive new plasma instabilities. JACKSON (1986) considered as a preliminary example the propagation of a sound wave through a self-sustaining *unmagnetized* fusion plasma and showed that such a wave would be thermally unstable. At the peaks of the pressure wave the fusion energy production is enhanced due to the increased density and temperature (the converse being true at the pressure troughs), and this leads to amplification of the wave. A corresponding dispersion relation was derived from a simple fluid model. However, although increased alpha production occurs at the peaks of the wave, the very high energy of the alphas means they have very low collisionality. The increased pressure of the alphas can only be communicated to the bulk plasma (in the absence of a magnetic field) on the alpha particle slowing down time which is of the order of a second for typical fusion reactor parameters. The pressure amplification will be averaged out unless the wave period is longer than the slowing down time. Since typical sound speeds in a Tokamak are $\approx 10^6$ m s^{-1}, this condition requires wavelengths of order 10^6 times the machine size. Such modes have the character of a global thermal instability and not that of a propagating sound wave. [Note that in the original astrophysics context of stellar cores as discussed by DILKE and GOUGH (1972), the very much larger collisionality allows this mechanism to be relevant.]

JACKSON (1986) went on to consider the modifications (within the single fluid theory) due to the presence of a magnetic field, and obtained a dispersion relation demonstrating instability of a magnetosonic wave. Such a wave can indeed be unstable but the mechanism does not involve collisional slowing down of alphas, as in a field-free model. It relies instead on the "frozen-in" condition for a magnetic field in a highly conducting fluid. The increase in α-particle energy density at the wave peak is now communicated to the wave in one *gyro-period* (the effective time for "collisions" with the magnetic field). An instability can therefore exist (as Jackson correctly conjectured in his discussion) even when there is no collisional coupling of the alpha particles to the bulk plasma. To show this unambiguously, however, requires a collisionless two-fluid treatment as will be given in this paper.

We consider a uniform magnetic field $\mathbf{B}$ and distinct electrically neutral fuel and alpha particle fluids (denoted by subscripts f and α, respectively). The α fluid satisfies continuity and momentum equations

$$\frac{\mathrm{D}n_\alpha}{\mathrm{D}t} + n_\alpha \nabla \cdot \mathbf{u}_\alpha = 0 \tag{1}$$

$$m_\alpha n_\alpha \frac{\mathrm{D}\mathbf{u}_\alpha}{\mathrm{D}t} = -\nabla \cdot \mathbf{\Pi}_\alpha + \mathbf{J}_\alpha \times \mathbf{B} \tag{2}$$

with analogous equations for the fuel. Here $\mathrm{D}/\mathrm{D}t$ denotes the convective derivative, $\mathbf{\Pi}$ is the pressure tensor, and n, m, T, $\mathbf{u}$ and $\mathbf{J}$ denote density, particle mass, temperature, velocity and current density, respectively. We neglect for the moment viscosity and thermal conduction since these are not central to the present discussion. The magnetic field satisfies

$$\nabla \times \mathbf{B} = \mathbf{J}_\alpha + \mathbf{J}_\mathrm{f}. \tag{3}$$

We shall consider a fast magnetosonic wave propagating perpendicular to $\mathbf{B}$. Since the fluids can be regarded as perfectly conducting the usual flux freezing property implies that

$$\mathbf{u}_\alpha^\perp = \mathbf{u}_\mathrm{f}^\perp. \tag{4}$$

Finally we require equations of state for the two fluids. Since the plasma is almost collisionless these are obtained from the CHEW, GOLDBERGER and LOW (CHEW *et al.*, 1956) equation for the perpendicular component of pressure

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{p_\perp}{nB}\right) = 0 \tag{5}$$

which corresponds to the conservation of magnetic moment in a guiding centre picture. (Note that a simple adiabatic equation of state would lead to essentially the same results.) The parallel component of pressure does not enter for waves propagating normal to $\mathbf{B}$. For the alpha fluid there is an additional contribution to the equation of state due to thermonuclear energy production. We therefore have, using equation (5),

$$n_\alpha B\frac{\mathrm{D}T_\alpha}{\mathrm{D}t}-n_\alpha T_\alpha\frac{\mathrm{D}B}{\mathrm{D}t}=BE(n_\mathrm{f},T_\mathrm{f}) \tag{6}$$

$$B\frac{\mathrm{D}T_\mathrm{f}}{\mathrm{D}t}-T_\mathrm{f}\frac{\mathrm{D}B}{\mathrm{D}t}=0 \tag{7}$$

where E is the net rate of change of alpha energy density (fusion energy generation minus loss terms), and we have used $p_\perp = nT$ and the fact that the effective ratio of specific heats $\gamma = 2$. Linearizing the above equations for perturbations proportional to $\exp\{i\omega t-i\mathbf{k}\cdot\mathbf{x}\}$ we obtain the following dispersion relation

$$\omega = kC_\mathrm{m}-\frac{i}{2(m_\alpha n_\alpha+m_\mathrm{f}n_\mathrm{f})C_\mathrm{m}^2}\left\{\frac{\partial E}{\partial T_\mathrm{f}}T_\mathrm{f}+\frac{\partial E}{\partial n_\mathrm{f}}n_\mathrm{f}\right\} \tag{8}$$

where C_m is the magnetosonic wave velocity

$$C_\mathrm{m}^2=\frac{2(n_\alpha T_\alpha+n_\mathrm{f}T_\mathrm{f})+B^2}{(m_\alpha n_\alpha+m_\mathrm{f}n_\mathrm{f})}. \tag{9}$$

Thus if the derivatives in equation (8) are positive the wave will be unstable. If for instance thermal equilibrium were established by a balance of alpha energy production and Bremsstrahlung then since both of these processes are proportional to n^2 we would have $\partial E/\partial n = 0$ in the dispersion relation. This highlights the basic result that the stability properties depend on the signs of

$$\frac{\partial}{\partial n}(\text{Source}-\text{Sink})$$

and

$$\frac{\partial}{\partial T}(\text{Source}-\text{Sink}).$$

Even if these terms are positive the growth rate of the wave will be very small and the wave will be easily stabilized by viscosity, particle diffusion and thermal conduction effects. Using the classical expression for the viscosity one finds that the wave can only be unstable for wavelengths $\lambda > 0.64$ m. [We note here a correction to equation (2) of JACKSON (1986); the final term, which is destabilizing, should contain a factor $(C_\mathrm{s}/C_\mathrm{m})^2$. As a result the critical wavelength, $\lambda \simeq 0.14$ m, found by Jackson is increased to $\lambda = 0.64$ m.] If instead we use a realistic experimental value for the anomalous thermal conductivity $\chi = 4\ \mathrm{m^2\ s^{-1}}$ (LEIWER, 1985), we find the wave will be damped unless $\lambda > 10$ m. Anomalously large perpendicular viscosity (which is also observed in current Tokamaks) would provide yet more damping, so this instability seems unlikely to present a problem to any realistic fusion reactor.

CONCLUSIONS

By considering a two fluid model (fuel plus α-particle fluids) we have shown that a magnetosonic wave propagating normal to the magnetic field could be weakly unstable in a thermonuclear plasma and that the instability mechanism does not involve the deposition of α-particle energy in the fuel. However, when transport processes are included in the model (e.g. perpendicular viscosity or perpendicular thermal conduction, Jackson, 1986) only long wavelength modes remain unstable. With classical transport processes, wavelengths greater than half a metre could be weakly unstable, but this dimension increases to about 10 m when realistic Tokamak transport coefficients are employed. We conclude that the instability does not provide a threat to proposed fusion reactor designs.

Acknowledgement—The authors would like to thank Dr J. B. Taylor for helpful comments.

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