Diagnosing Type Errors with Class

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Abstract
Type inference engines often give terrible error messages, and the more sophisticated the type system the worse the problem. We show that even with the highly expressive type system implemented by the Glasgow Haskell Compiler (GHC)—including type classes, GADTs, and type families—it is possible to identify the most likely source of the type error, rather than the first source that the inference engine trips over. To determine which are the likely error sources, we apply a simple Bayesian model to a graph representation of the typing constraints; the satisfiability or unsatisfiability of paths within the graph provides evidence for or against possible explanations. While we build on prior work on error diagnosis for simpler type systems, inference in the richer type system of Haskell requires extending the graph with new nodes. The augmentation of the graph creates challenges both for Bayesian reasoning and for ensuring termination. Using a large corpus of Haskell programs, we show that this error localization technique is practical and significantly improves accuracy over the state of the art.

Categories and Subject Descriptors D.2.5 [Testing and Debugging]: Diagnostics; F.3.2 [Semantics of Programming Languages]: Program analysis.

Keywords Error diagnosis; type inference; Haskell

1. Introduction
Type systems and other static analyses help reduce the need for debugging at run time, but sophisticated type systems can lead to terrible error messages. The difficulty of understanding these error messages interferes with the adoption of expressive type systems.

Even when a program error is detected statically, it can be difficult to determine where the mistake lies in the program. The problem is that powerful static analyses and advanced type systems reduce an otherwise-high annotation burden by drawing information from many parts of the program. However, when the analysis detects an error, the fact that distant parts of the program influence this determination makes it hard to accurately attribute blame.

Recent work by Zhang and Myers [36] made progress on this problem, demonstrating that a more holistic Bayesian approach to localizing errors can improve accuracy significantly, for at least some nontrivial type systems (OCaml and Jif). A key idea of that work is to represent constraints as a constraint graph that allows efficient reasoning about a possibly large number of counterfactual error explanations.

However, that graph representation cannot handle richer type systems in which the reasoning process requires a constraint solver that can handle quantified propositions involving functions over types. Type classes and type families, as supported by GHC [21], require such a solver, whereas simple polymorphic types as in ML do not [8, 28]. Better error localization would be very valuable for such type systems, because their error messages can be particularly inscrutable. In the constraint graph representation, however, a solver for such rich type systems needs to add new nodes to the constraint graph, posing challenges for soundness, completeness, termination, and efficiency of the analysis.

Our principal contribution is to show that an approach based on Bayesian reasoning can be applied even to such type systems. Specifically:

• We define a constraint language and constraint graph representation that can encode a broad range of type systems and other analyses. In particular, they add the ability to handle the features of the expressive type system of Haskell, including type classes, GADTs, and type families. (§3 and §4)

• We extend the constraint-graph solving technique of Zhang and Myers [36] to allow the creation of new nodes and edges in the graph and thereby to support counterfactual reasoning about type classes, type families, and their universally quantified axioms. We prove that the new algorithm always terminates. (§5)

• We develop a Bayesian model for programmer mistakes that accounts for the richer representation of constraints and the presence of derived constraints. (§6)

• We have implemented this technique as an extension to the publicly available SHErrLoc diagnostic tool [30], using GHC itself as the constraint generator so that we handle all of Haskell. (§7)

• Using a corpus of more than 300 Haskell programs, many written by students solving programming assignments, we show that mistakes are more accurately located than with prior techniques. Further, the performance of the diagnostic algorithm is acceptable. (§8)

2. The Challenge We Tackle
Type inference problems can generally be expressed in terms of solving a set of constraints on type expressions, and type inference succeeds when variables in the constraints can be assigned types that make all the constraints satisfiable.
When constraints are unsatisfiable, the question is how to identify the program point that is most likely to be the error source. The standard practice is to report the program point that generates the last failed constraint. Unfortunately, this simple approach often results in misleading error messages—the actual error source may be far from that program point.

As a motivating example, consider the following Haskell program from [18], which fails to type-check:

```haskell
1  fac n = if n == 0 then 1
   2  else n * fac (n == 1)
```

The actual mistake is that the second equality test (==, in line 2) should be subtraction (-), but GHC instead blames the literal 0, saying that Bool is not a numerical type. A programmer reading this message would probably be confused why 0 should have type Bool. Unfortunately, such confusing error messages are not uncommon.

The core of the problem is that most type checkers, GHC included, implement constraint solving by iteratively simplifying type constraints, making error reporting sensitive to the order of simplification. GHC here decides to first unify the return type of (n == 1), namely Bool, with the type of n, which is the argument of fac. Once the type of n is fixed to Bool, the compiler picks up the constraint arising from line 1, (expression n == 0), unifies the type of θ with Bool and reports misleadingly that literal 0 is the error source.

Rather than reporting the location of a single failed constraint, we might think to report all locations that might contribute to the error (e.g., as in [7, 10, 32, 35]). But such error reports are often very verbose and hard to understand [14], because many expressions can be at least partly involved in a given failure.

A more promising approach is described by Zhang and Myers [36], where the structure of the constraint system as a whole is described to have type types [27]. Consider the following program.

```haskell
1  class Eq a where
2    eq x y = x == y
3  f x = eq x True
```

The type signature for `f` introduces a constraint hypothesis `Eq a`, on the universally quantified variable a, and that constraint is necessary for using == at line 3.

**Our approach** We develop a rich constraint language (§3) that can encode all type constraints generated by GHC [34]. We use GHC itself to generate type constraints for Haskell programs with all sophisticated features above, and then translate these constraints to our constraint language. To simplify the presentation, we collapse these two steps into one, by generating constraints in our language directly. Our tool handles all GHC constraints, but for illustration we use a sufficiently rich Haskell subset (§4).

The set of constraints is then transformed into a constraint graph (§5.1). For example, part of the graph for the motivating (factorial) example is depicted in Figure 1, where nodes αn, αθ, and αa represent the types of n, θ, 1 and the first parameter of the type family, respectively, and each bidirectional edge represents type equality between the end nodes. In this figure, each edge is annotated with the expression that generates it. For example, the edge between αa and Bool is generated since the return type of (n == 1), namely Bool, must equal the type of n, the argument of fac.

Besides equality constraints, Haskell also generates type class constraints. Type classes introduce, in effect, relations over types. For example, the type of literal θ can be any instance of the type class Num, such as Int and Float. We use a directed edge, encoding a partial ordering, to express a type class constraint. For example, the edge from αθ to Num in Figure 1 means that αθ must be an instance of Num.

The constraint graph is then saturated and expanded so that all possible deductions are represented as graph edges (§5.2). For example, the dashed edges in Figure 1 are derived by transitivity. Each edge in the saturated graph is then classified as satisfiable or unsatisfiable (§5.3). For example, the edges marked with a red X are unsatisfiable, since Bool is not an instance of Num.

Finally, we use the classification of constraint edges to assign the most likely error source, according to Bayesian principles (§6). Taken together, the chosen error locations should: 1) explain all
Unification variables $\alpha, \beta, \gamma$  
Constructors $\text{con}$  
Skolem variables $a, b, c$  
Functions $\text{fun}$  
Quantified variables in hypothesis $a, b, c$  

$G ::= A_1 \land \ldots \land A_n \quad (n \geq 0)$  
$A ::= H \vdash I$  
$H ::= Q_1 \land \ldots \land Q_n \quad (n \geq 0)$  
$Q ::= \forall \tau. C \Rightarrow I$  
$C ::= I_1 \land \ldots \land I_n \quad (n \geq 0)$  
$I ::= E_1 \leq E_2$  
$E ::= aE | aE | a \mid \text{con } \tau | \text{fun } \tau$

**Figure 2.** Syntax of SCL constraints.

unsatisfiable paths, 2) be small, and 3) not appear often on satisfiable paths. In accordance with these three principles, we correctly determine expression $(n = 1)$ to be the most likely cause of the error.

**What is new** The general plan of graph generation, saturation, and classification follows prior work [36]. The new aspects are these: first, a rich constraint language that can encode the expressive type system of Haskell (§3), including type class constraints and type families; second, the encoding of type class constraints as inequalities (§4.2); third, a new graph-saturation algorithm, which handles type classes and type families by generating new nodes and edges in the constraint graph (§5.2); fourth, an edge-classification algorithm that correctly handles nested quantifiers (§5.3); finally, a modified Bayesian model that takes the creation of new nodes and edges into account (§6).

3. The SCL Constraint Language

We substantially modified and extended the constraint language of Zhang and Myers [36] in order to handle the rich type system of Haskell. The most significant new features of the new constraint language are quantified axioms, nested universally and existentially quantified variables, and type-level functions.

3.1 Syntax of the SCL Constraint Language

Figure 2 presents the syntax of the new constraint language, which we call SCL (for SHErrLoc Constraint Language). A top-level goal $G$ is a conjunction of assertions $A$. An assertion has the form $H \vdash I$, where $H$ is a hypothesis (an assumption) and $I$ is an inequality to be checked under the assumption $H$.

**Constraints** A constraint $C$ is a conjunction of inequalities $E_1 \leq E_2$ over elements from the constraint element domain $E$ (typically types of the source language), where $\leq$ defines a partial ordering on elements$^1$. Throughout, we write equalities $(E_1 = E_2)$ as syntactic sugar for $(E_1 \leq E_2 \land E_2 \leq E_1)$, and $(H \vdash E_1 = E_2)$ is sugar for two assertions, similarly.

**Quantified axioms in hypotheses** Hypotheses $H$ can contain (possibly empty) conjunctions of quantified axioms, $Q$. Each axiom has the form $\forall \tau. C \Rightarrow I$, where the quantified variables $\tau$ may be used in constraints $C$ and inequality $I$. For example, a hypothesis $\forall a. a \leq A \Rightarrow a \leq B$ states that for any constraint element $a$ such that $(a \leq A)$ is valid, inequality $a \leq B$ is valid as well. When both $\forall \tau$ and $C$ are empty, an axiom $Q$ is written simply as $I$.

**Handling quantifiers** To avoid notational clutter associated with quantifiers, we do not use an explicit mixed-prefix quantification notation. Instead, we distinguish universally introduced variables

$^1$ The full constraint language of SHErrLoc also supports lattice joins and meets on elements. We omit them here since 1) they are not needed to represent Haskell constraints, and 2) adding them is straightforward.

Term variables $x, y, z$  
Type variables $a, b, c$  
Type families $D, F$  

Expressions $e ::= x | \lambda x. e | e_1 e_2$  
$| \text{let } x :: \sigma = e_1 \text{ in } e_2$  

Constraints $P ::= P_1 \land P_2 \mid \tau_1 = \tau_2 \mid D \tau$  

Signatures $\sigma ::= \forall \tau. P \Rightarrow \tau$  

Monotypes $\tau ::= a | \text{Int } | \text{Bool } | [\tau] | \text{T } \tau | F \tau$  

Axioms schemes $Q ::= P \mid Q_1 \land Q_2 \mid \forall \tau. P \Rightarrow D \tau \mid \forall \tau. F \tau = \tau'$

**Figure 3.** Syntax of a Haskell-like language.

(a, b, . . . ) and existentially introduced variables $(\alpha, \beta, . . . )$; further, we annotate each variable with its level, a number that implicitly represents the scope in which the variable was introduced. For example, we write the formula $a_1 = b_1 \vdash (a_2, b_2) = \alpha_2$ to represent $\forall a, b. \exists \alpha. a = b \vdash (a, b) = \alpha$. Any assertion written using quantifiers can be put into prenex normal form and therefore can be represented using level numbers.

**Constructors and functions over constraint elements** As well as a variable, an element $E$ may be an application $\text{con } \tau$ of a type constructor $\text{con} \in \text{Con}$, or an application $\text{fun } \tau$ of a type-function $\text{fun} \in \text{Fun}$. Constants are nullary constructors, with arity 0. Since constructors and functions are global, no levels are associated with them. Our full constraint language and implementation support contravariant and invariant constructors as well, but in order to keep this paper focused on the key challenges and contributions, we assume all constructors are covariant hereafter.

The main difference between a type constructor $\text{con}$ and a type function $\text{fun}$ is that functions are not necessarily injective (i.e., $\text{fun } \tau = \text{fun } \tau' \neq \tau = \tau'$), but constructors can be decomposed (i.e., $\text{con } \tau = \text{con } \tau' \Rightarrow \tau = \tau'$).

3.2 Validity and Satisfiability

An assertion $A$ is satisfiable if there is a level-respecting substitution $\theta$ for $A$’s free unification variables, such that $[\theta][A]$ is valid. A substitution $\theta$ is level-respecting if the substitution is well-specified. More formally, $\forall \alpha \in \text{dom}(\theta). a_m \in \text{fvs}(\theta(a_l)). m \leq l$. For example, an assertion $a_1 = b_2 \vdash (a_3 = a_2 \land a_2 = b_1)$ is satisfiable with substitution $[a_2 \mapsto a_1]$. But $a_1 = b_2$ is not satisfiable because the substitution $[a_1 \mapsto b_2]$ is not level-respecting. The reason is that with explicit quantifiers, the latter would look like $\forall \alpha \forall \beta. \vdash \alpha = b$ and it would be ill-specified to instantiate $\alpha$ with $b$.

A unification-variable-free assertion $H \vdash I$ is valid if $I$ is entailed by $H$. Since the entailment rules, available in the associated technical report [37], are entirely standard, we omit them in this paper. A variable-free goal $G$ is valid if all assertions it contains are valid.

4. Generating Constraints from a Type System

The SCL constraint language is powerful enough to express advanced type system features in GHC. We demonstrate this constructively, by giving an algorithm to generate suitable constraints directly from a Haskell-like program.

4.1 Syntax

Figure 3 gives the syntax for a Haskell-like language. It differs from a vanilla ML language in four significant ways:

- A let-binding has a user-supplied type signatures ($\sigma$) that may be polymorphic. For example,
One further point of departure is that using the algorithm in Figure 4.2 Constraint Generation parts from implicit generalization anyway [33].

Following prior work on constraint-based type inference [25, 28, 34], we formalize type inference as constraint solving, generating SCL constraints using the algorithm in Figure 4.

The constraint-generation rules have the form $H; \Gamma \models e : \tau \leadsto G$, read as follows: “given hypotheses $H$, in the typing environment $\Gamma$, we may infer that an expression $e$ has type $\tau$ and generates assertions $G$”. The level $\ell$ associated with each rule is used to track the scope of unification (existential) and skolem (universal) variables. Here, both $H$ and $G$ follow the syntax of SCL.

Rule (VARCON) instantiates the polymorphic type of a variable or constructor, and emits an instantiated constraint of that type under the propagated hypothesis. Rule (ABS) introduces a new unification variable at the current level, and checks $e$ with an increased level. Rule (APP) is straightforward. Rule (SIG) replaces quantified type variables in type signature with fresh skolem variables. Term $e_1$ is checked under the assumption ($H'$) that the translated constraint in the type signature ($P$) holds, under the same replacement. The assumption is checked at the uses of $x$ (Rule (VARCON)). The quantifier level is not increased when $e_2$ is checked, since all unification/skolem variables introduced for $e_1$ are invisible in $e_2$.

Figure 4. Constraint generation.

let id :: (forall a . a -> a) = (\x -> x) in ... declares an identity function with a polymorphic type.

- A polymorphic type $\sigma$ may include constraints ($P$), which are conjunctions of type equality constraints ($\tau_1 = \tau_2$) and type class constraints ($D \tau$). Hence, the language supports type-multiclass type parameters. The constraints in type signatures are subsumed by SCL, as we see shortly.

- The language supports type families: the syntax of types $\tau$ includes type families ($F \tau$). A type can also be quantified type variables ($\alpha$) and regular types (Int, Bool, $[\tau]$) that are no different from some arbitrary data constructor $T$.

- An axiom scheme ($Q$) is introduced by a Haskell instance declaration, which we omit in the language syntax for simplicity. An axiom scheme can be used to declare relations on types such as type class instances, and type family equations. For example, the following declaration introduces an axiom ($\forall a . Eq a \Rightarrow Eq [a]$) into the global axiom schemes $Q$.

$\text{instance Eq a => Eq [a] where \{ ... \}}$

Implicit let-bound polymorphism One further point of departure from Hindley-Milner (but not GHC) is the lack of let-bound implicit generalization. We decided not to address this feature in the present work for two reasons: 1) Implicit generalization brings no new challenges from a constraint-solving perspective, the focus of this paper, 2) It keeps our formalization closer to GHC, which departs from implicit generalization anyway [33].

4.2 Constraint Generation Following prior work on constraint-based type inference [25, 28, 34], we formalize type inference as constraint solving, generating SCL constraints using the algorithm in Figure 4.

Type classes How can we encode Haskell’s type classes in SCL constraints? The encoding is shown in Figure 4: we express a class constraint ($D \tau$) as an inequality ($\tau \leq D$), where $D$ is a unique constant for class $D$. The intuition is that $\tau$ is a member of the set of instances of $D$. For a multi-parameter type class, the same idea applies, except that we use a constructor $\text{ tup}_a$ to construct a single element from the parameter tuple of length $n$.

For example, consider a type class $\text{Mul}$ with three parameters (the types of two operands and the result of multiplication). The class $\text{Mul}$ is the set of all type tuples that match the operators and result types of a multiplication. Under the translation above, $[\text{Mul} \tau_1 \tau_2 \tau_3] = (\text{tup}_a \tau_1 \tau_2 \tau_3 \leq \text{Mul})$.

4.3 Running Example We use the program in Figure 5 as a running example for the rest of this paper. Relevant axiom schemes and function signatures are shown in comments. Here, the type family $F$ maps $[a]$, for an arbitrary type $a$, to a type family $(F \tau)$. A function h is called only when $a = [b]$. Hence, the type signature is equivalent to $\forall b . (b, b) \rightarrow b$, so the definition of h is well-typed. On the other hand, expression (g $\tau$) has a type error: the parameter type $\tau$ of Char is not an instance of class Num, as required by the type signature of g.

The informal reasoning above corresponds to a set of constraints, shown in the centre column of Figure 5. The shaded constraints are generated for the expression (g $\tau$) in the following ways. Rule (VARCON) instantiates $d$ in the signature of $g$ at type $\delta_0$, and generates the third constraint (recall that (Num $\delta_0$) is encoded as ($\delta_0 \leq \text{Num}$)). Instantiate the type of character $\alpha$ at type $\alpha_0$; hence $\alpha_0 = \text{Char}$. Finally, using (APP) on the call (g $\alpha$) generates a fresh type variable $\gamma_0$ and the fifth constraint ($\alpha_0 \rightarrow \gamma_0$) = ($\delta_0 \rightarrow \text{Bool}$). These three constraints are unsatisfiable, revealing the type error for g $\tau$. On the other hand, the first two (satisfiable) constraints are generated for the implementation of function g. The hypotheses of these two constraints contain $\alpha_0 = [\delta_0]$, added by rule (SIG).
Graph Generation

A constraint graph is generated from assertions $G$ as follows. As a running example, Figure 5, excluding the white node and the dotted edges, shows part of the generated constraint graph for the constraints in the centre column of the same figure.

1. For each assertion $H \vdash E_1 \leq E_2$, create nodes for $E_1$ and $E_2$ (if they do not already exist), and a edge $\text{LEQ}(H)$ between the two. For example, nodes for $\delta_0 \rightarrow \text{Bool}$ and $[\alpha_0] \rightarrow \gamma_0$ are connected by $\text{LEQ}(H)$.

2. For each constructor node $\text{(con \, \mathcal{E})}$ in the graph, create a node for each of its immediate sub-elements $E_i$ (if they do not already exist); add a labeled constructor edge $\text{con}^i$ from the sub-element to the node; and add a labeled decomposition edge $\text{cons}^i$ in the reverse direction. For example, $\delta_0$ and $\text{Bool}$ are connected to $(\delta_0 \rightarrow \text{Bool})$ by edges $\rightarrow \mathcal{E}$ and $\leftarrow \mathcal{E}$ respectively; and in the reverse direction by edges $\rightarrow \mathcal{E}$ and $\leftarrow \mathcal{E}$ respectively.

Graph Saturation

The key ingredient of graph-based constraint analysis is graph saturation: inequalities that are derivable from a constraint system are added as new edges in the graph. We first discuss the challenge of analyzing Haskell constraints, and then propose a new algorithm that tackles these challenges.

Figure 7. Graph saturation rules. New edges (left) are inferred based on existing edges (right).

Limitations of previous approach

Graph saturation can be formalized as a context-free-language (CFL) reachability problem [3, 24, 36]. For example, Zhang and Myers formalized a graph saturation algorithm for a subset of our constraint language as the first three rules in Figure 7. The first rule infers a new LEQ edge given two consecutive LEQ edges, reflecting the transitivity of $\leq$. This rule also aggregates hypotheses made on existing edges to the newly inferred edge. The second rule infers a new LEQ edge when a constructor edge is connected to its dual decomposition edge, reflecting the fact that constructors can be decomposed. Given $n_i \leq n'_i$ for all parameters of $\pi$ and $\pi'$, the third rule infers an LEQ edge from $\text{con}(\pi)$ to $\text{con}(\pi')$, reflecting the fact that constructors are covariant.

However, graph saturation is insufficient to handle SCL. We can see this by considering the constraint graph of the running example, in Figure 5. Excluding the white nodes and the edges leading to and from them, this graph is fully saturated according to the rules in Figure 7. For example, the dotted edges between $\delta_0$ and $[\alpha_0]$ can be derived by the second production. However, a crucial inequality (edge) is missing in the saturated graph: $\text{(Char} \leq \text{Num})$, which can be derived from the shaded constraints in Figure 5. Since this inequality reveals an error in the program being analyzed (that $\text{Char}$ is not an instance of class $\text{Num}$), failure to identify it means an error is missed. Moreover, the edges between $\{\xi_2, F a_2\}$ and $F a_0$ are mistakenly judged as unsatisfiable, as we explain in §5.3.
Expanding the graph

The key insight for making the algorithm more sound and complete is to expand the constraint graph during graph saturation. Informally, nodes are added to the constraint graph so that the third and fourth rules in Figure 7 can be applied.

The (full) constraint graph in Figure 5 is part of the final constraint graph after running our new algorithm. The algorithm expands the original constraint graph with a new node that triggers expansion. First, both elements of constraint G must be black nodes. Second, a trace to a new node T is required that a single substitution cannot be applied twice (line 1). When a white node is added, a substitution (E ⇔ E') is appended to the trace of T(Enew) (line 2).

Returning to our running example in Figure 5, the LEQ edge from a0 to Char, as well as the node [α0], match the pattern in Figure 9. In this example, the white node [Char] is added to the graph. As an optimization, no constructor/decomposition edges are added, since these edges are only useful for finding α0 = Char, which is in the graph already. Moreover, T([Char]) = ([α0], α0 = Char).

Termination

The algorithm in Figure 8 always terminates, because the number of nodes in the fully expanded and saturated graph must be finite. This is easily shown by observing that |T(Enew)| = |T(Eold)| + 1, and trace size is finite (elements in a substitution must be black).

5.3 Classification

Each LEQ edge LEQ(H) (E1 ⇔ E2) in the saturated constraint graph corresponds to an entailment constraint, H ⊢ E1 ≤ E2, that is derivable from the constraints being analyzed. For example, in Figure 5, the LEQ edge from b0 to b1 corresponds to the following entailment:

\[(∀a, F(a) = (a, a)) ∨ (|Int| ≤ \text{Num}) ∨ (a0 = b0) \vdash (b1, b0) ≤ F(a0)\]

Now, the question is: is this entailment satisfiable?

Hypothesis graph

For each hypothesis H shown on LEQ edges in the saturated constraint graph, we construct and saturate a hypothesis graph so that derivable inequalities from H become present in the final graph.

The construction of a hypothesis graph is shown in Figure 10. For an entailment H ⊢ E1 ≤ E2, the constructed graph of H includes both E1 and E2. These nodes are needed as guidance for graph saturation. Otherwise, consider an assertion a0 = b0 ⊢ [a0] = [b0]. Without nodes [a0] and [b0], we face a dilemma: either we need to infer (infinite) inequalities derivable from a0 = b0, or we may miss a valid entailment. As an optimization, all nodes (but not edges) in the constraint graph (N) are added to the constructed graph as well. The benefit is that we need to saturate a hypothesis graph just once for all edges that share the hypothesis graph.

The function \(Q[H]\) translates a hypothesis H into a graph representation associated with a rule set R. Hypotheses in the degenerate form (I) are added directly; others are added to the rule set R, which is part of a hypothesis graph. Returning to our running example,

\[Q[H] = Q[\text{Char}] ∪ Q[\text{Int}] ∪ Q[\text{Bool}]\]
example, Figure 11 (excluding the white node and dotted edges) shows (part of) the constructed hypothesis graphs for hypotheses \( H \) and \( H' \).

The hypothesis graph is then expanded and saturated in a similar way as the constraint graph. The difference is that axioms are applied during saturation, as shown in Figure 12. At line 3, an axiom \( \forall x. G \Rightarrow I \) is applied when it can be instantiated so that all inequalities in \( G \) are in \( G' \) already (i.e., \( H \) entails these inequalities).

Then, an edge corresponding to the inequality in conclusion is not be identified in the hypothesis graph, so the edges from and to \( a \) are both in \( G \) if not in \( G \) already.

![Figure 11. Hypothesis graphs for the running example.](image)

### Procedure: \( \text{saturate}(G : \text{Graph}) \)

1. Add new edges to \( G \) according to the rules in Figure 7
2. foreach \( H = (\forall x. I_1 \land \ldots \land I_n \Rightarrow E_1 \leq E_2) \in \mathcal{R} \) do
3. \[ \text{if } \exists \theta : \mathcal{N} \Rightarrow \mathcal{G} \forall 1 \leq i \leq n. \theta[I_i] \in G \text{ then} \]
4. \[ \text{if } \theta[E_1] \text{ and } \theta[E_2] \text{ are both in } G \text{ then} \]
5. \[ \text{add edge from } E_1 \text{ to } E_2 \text{ if not in } G \text{ already} \]

![Figure 12. Hypothesis graph saturation for axioms.](image)

Classification

An entailment \( H \vdash E_1 \leq E_2 \) is classified as satisfiable iff there is a level-respecting substitution \( \theta \) such that the hypothesis graph for \( H \) contains an LEQ edge from \( \theta[E_1] \) to \( \theta[E_2] \). Such substitutions are searched for in the fully expanded and saturated hypothesis graph.

Returning to the example in Figure 5, our algorithm correctly classifies the LEQ edges between \( b_0, b_0 \) and \( (F \circ a_0) \) as satisfiable, since the corresponding edges are in Figure 11(b). Our algorithm correctly classifies LEQ edges between \( (E_2, E_2) \) and \( (F \circ a_0) \) as satisfiable as well, with substitution \( \xi_2 \rightarrow b_0 \). On the other hand, the LEQ edge from \( \text{Char} \) to \( \text{Num} \) is (correctly) judged as unsatisfiable, since the inequality is not present in the fully expanded and saturated hypothesis graph for \( H \).

To see why the level-respecting substitution requirement is needed, consider the following example, adapted slightly from the introduction:

\[(\lambda x. \ let \ g :: (\forall a. a \rightarrow (a, a)) = \lambda y. (x, y) \ in \ldots)\]

This program generates an assertion \( \emptyset \vdash (\beta_2 \rightarrow (\alpha_0, \beta_2)) = (a_1 \rightarrow (a_1, a_1)), \) which requires that the inferred type for the implementation of \( g \) be equivalent to its signature. The final constraint graph for the assertion contains two LEQ edges between nodes \( \alpha_0 \) and \( a_1 \). These edges are correctly classified as unsatisfiable, since the only substitution candidate, \( \alpha_0 \rightarrow a_1 \), is not level-respecting.

If the signature of \( g \) is \( (\forall a. a \rightarrow \text{Int} \Rightarrow a \rightarrow (a, a)) \), the program is well-typed, since the parameter of \( g \) must be \( \text{Int} \). This program generates the same assertion as the previous example, but with a hypothesis \( a_1 = \text{Int} \). This assertion is correctly classified as satisfiable, via a level-respecting substitution \( \alpha_0 \rightarrow \text{Int} \).

### Informative edges

When either end node of a satisfiable LEQ edge is an unification variable, its satisfiability is trivial and hence not informative for error diagnosis. Also uninformative is an LEQ edge derived from unsatisfiable edges. Only the informative edges are used for error diagnosis.

### 6. Bayesian Model for Ranking Explanations

When unsatisfiable edges are detected, we are interested in inferring the program expressions that (generated the constraints that) most likely caused the errors. To do this, we extend the Bayesian model of Zhang and Myers [36].

The observed symptom of errors is a fully analyzed constraint graph (§5), in which all informative LEQ edges are classified as satisfiable or unsatisfiable. For simplicity, in what follows we write "edge" to mean "informative edge".

Formally, an observation \( o \) is a set \( \{o_1, o_2, \ldots, o_n\} \), where \( o_i \in \{\text{unsat}, \text{sat}\} \) represents satisfiability of the \( i \)-th edge. Let \( E \) be the set of all expressions in a program, each occurring in a distinct source location and giving rise to a typing constraint. We are looking for a set \( E \subseteq \mathcal{E} \) that maximizes \( \text{P}(E|o) \), the posterior probability that the expressions \( E \) contain errors. By Bayes’ theorem, this term has an easier, equivalent form: \( \text{P}(E) \times \text{P}(o|E) / \text{P}(o) \), where \( \text{P}(E) \) is a prior probability that expressions in \( E \) contain errors, and \( \text{P}(o) \) is a prior distribution on observations. Since \( \text{P}(o) \) does not vary in \( E \), the goal of error diagnosis is to find:

\[
\text{arg} \max_{E \subseteq \mathcal{E}} \text{P}(E) \times \text{P}(o|E)
\]

### Redundant edges

To further simplify the term \( \text{P}(E) \times \text{P}(o|E) \), Zhang and Myers [36] assume that the satisfiability of informative edges is independent. However, the introduction of white nodes undermines this assumption. In Figure 5, the satisfiability of the edge between \( (\alpha_0) \) and \( \text{Char} \) merely repeats the edge between \( \alpha_0 \) and \( \text{Char} \); the fact that end-nodes can be decomposed is also uninformative because white nodes are constructed this way. In other words, this edge provides neither positive nor negative evidence that the constraints it captures are correct. It is redundant. We can soundly capture a large class of redundant edges:

**Definition 1.** An edge is redundant if 1) both end-nodes are constructor applications of the same constructor, and at least one node is white; or 2) both end-nodes are function applications to the same function, and for each simple edge along this edge, at least one of its end-nodes is white. Otherwise, an edge is non-redundant.

The following lemma (see the associated technical report [37] for the proof) shows that if an edge is redundant according to the previous definition then it does not add any positive or negative information in the graph—it is equivalent to some other set of non-redundant edges.

**Lemma 1.** For any redundant edge from \( E_1 \) to \( E_2 \), there exist non-redundant edges say \( P_1 \) from \( E_1 \) to \( E_2 \), so that \( E_{12} \land \ldots \land E_{n1} \leq E_{12} \Leftrightarrow E_1 \leq E_2 \).

### Calculating likelihood

Let \( \hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_n) \) be all non-redundant edges. Lemma 1 implies that \( \text{P}(E) \times \text{P}(o|E) = \text{P}(E) \times \text{P}(\hat{\alpha}|E) \). We make two simplifying assumptions:
1. All expressions are equally likely to be wrong (with fixed probability \( P_1 \)), and
2. Remaining paths in \( \hat{\delta} \) are independent\(^2\).

These assumptions allow us to rewrite \( P_{\hat{\delta}}(E) \times P(\hat{\delta}|E) \) as \( P_{\hat{\delta}^{(E)}}(E) \times \prod_{i} P(\hat{\delta}_i|E) \). The term \( P(\hat{\delta}|E) \) is calculated using two heuristics:

1. If \( \hat{\delta}_i = \text{unsat} \), at least one constraint that gives rise to the edge must be wrong. Therefore, we only need to consider the expressions that generate constraints along unsatisfiable edges in \( \hat{\delta} \). We denote this set by \( \mathcal{G} \).
2. If \( \hat{\delta}_i = \text{sat} \), it is unlikely (with fixed probability \( P_2 < 0.5 \)) that expressions in \( E \) give rise to the edge.

Assume that constraints generated for \( E \) appear on \( k_E \) of satisfiable edges. Using the previous heuristics, the likelihood is maximized at:

\[
\arg\max_{E \subseteq \mathcal{G}} P_{\hat{\delta}^{(E)}}(E) = \arg\max_{E \subseteq \mathcal{G}} P_{\hat{\delta}^{(E)}}(E) (P_2/(1 - P_2))^{k_E}
\]

If \( C_1 = -\log P_1 \) and \( C_2 = -\log (P_2/(1 - P_2)) \), maximizing the likelihood is equivalent to minimizing the ranking metric \( |E| + C_2 \). An intuitive understanding is that the cause must explain all unsatisfiable edges; the wrong entities are likely to be small (\( |E| \) is small) and not used often on satisfiable edges (since \( C_2 \) > 0 by heuristic 2). We use the efficient A* search algorithm in [36] to find a set of expressions minimizing this metric. Moreover, empirical results show that the ranking of expression sets according to this metric is insensitive to the value of \( C_2/C_1 \) (§8.5).

7. Implementation

We built our error diagnostic tool based on the open-source tool SHErrLoc [30]. Our diagnostic tool reads in constraints following the syntax of Figure 2, and computes constraints most likely to have caused errors in the constraint system being analyzed. The extension includes about 2,500 lines of code (LOC), above the 5,000 LOC of SHErrLoc.

Generating constraints from Haskell type inference involved little effort. We modified the GHC compiler (version 7.8.2), which already generates and solves constraints during type inference, to emit unsimplified, unsolved constraints. The modification is minimal; only 50 LOC are added or modified. Constraints in GHC’s format are then converted by a lightweight Perl script (about 400 LOC) into the syntax of our error diagnosis tool.

8. Evaluation

8.1 Benchmarks

To evaluate our error diagnosis tool, we used two sets of previously collected Haskell programs containing errors. The first corpus (the CE benchmark) contains 121 Haskell programs, collected by Chen and Erwig [5] from 22 publications about type-error diagnosis. Although many of these programs are small, most of them have been carefully chosen or designed in the 22 publications to illustrate important (and often, challenging) problems for error diagnosis.

The second benchmark, the Helium benchmark [11], contains over 50k Haskell programs logged by Helium [13], a compiler for a substantial subset of Haskell, from first-year undergraduate students working on assignments of a programming course offered at the University of Utrecht during course years 2002–2003 and 2003–2004. Among these programs, 16,632 contain type errors.

8.2 Evaluation Setup

To evaluate the quality of an error report, we first need to retrieve the true error locations of the Haskell programs being analyzed, before running our evaluation.

The CE benchmark contains 86 programs where the true error locations are well-marked. We reused these locations in evaluation. Since not all collected programs are initially written in Haskell, the richer type system of Haskell actually makes 9 of these programs type-safe. Excluding these well-typed programs, 77 programs are left.

The Helium benchmark contains programs written by 262 groups of students taking the course. To make our evaluation objective, we only considered programs whose true error locations are clear from subsequences of those programs where the errors are fixed. Among those candidates, we picked one program with the latest time stamp (usually the most complex program) for each group to make our evaluation feasible. Groups were ignored if either they contain no type errors, or the error causes are unclear. In the end, we used 228 programs. The programs were chosen without reference to how well various tools diagnosed their errors.

We compared the error localization accuracy of our tool to GHC 7.8.2 and Helium 1.8 [15]; both represent the state of the art for diagnosing Haskell errors. A tool accurately locates the errors in a program if and only if it points at least one of the true error locations in the program.

The difference from GHC and Helium is that sometimes, our tool reports a small set of top-ranking source locations, with the same likelihood. For fairness, we ensure that the majority of suggestions are correct when we count our tool as accurate. Average suggestion size is 1.7, so we expect a limited effect on results for offering multiple suggestions.

8.3 Error Report Accuracy

Figure 13 shows the error report accuracy of our tool, compared with GHC and Helium. For the CE benchmark, our tool provides strictly more accurate error reports for 43% and 26% of the programs, compared with GHC and Helium respectively. Overall, GHC, Helium and our tool finds the true error locations for 48%, 68% and 88% of programs. Clearly, our tool, with no Haskell-
specific heuristics, already significantly improves accuracy compared with tools that do.

On the Helium benchmark, the accuracy of GHC, 68%, is considerably better than on the CE benchmark; our guess is the latter offers more challenging cases for error diagnosis. Nevertheless, our tool still outperforms GHC by 21%. Compared with Helium, our tool is strictly better for 21% of the programs. Overall, the accuracy of our tool is 89% for the GHC benchmark, a considerable improvement compared with both GHC (68%) and Helium (75%).

Our tool sometimes does miss error causes identified by other tools. For 14 programs, Helium finds true error locations that our tool misses. Among these programs, most (12) contain the same mistake: students confuse the list operators for concatenation (+) and cons (:). To find these error causes, Helium uses a heuristic based on the knowledge that this particular mistake is common in Haskell. It is likely that our tool, which currently uses no Haskell-specific heuristics, can improve accuracy further by exploiting knowledge regarding common mistakes. However, we leave integration of language-specific heuristics to future work.

Comparison with CF-typing Chen and Erwig [5] evaluated their CF-typing method on the CE benchmark. For the 86 programs where the true error locations are well-marked, the accuracy of their tool is 67%, 80%, 88% and 92% respectively, when their tool reports 1, 2, 3 and 4 suggestions for each program; the accuracy of our tool is 88% with an average of 1.62 suggestions. When our tool reports suboptimal suggestions, the accuracy becomes 92% with an average suggestion size of 3.2.

Comparison with SHErrLoc Zhang and Myers evaluated their error diagnosis algorithm using a suite of OCaml programs collected from students by Lerner et al. [20]. We checked that our extensions to their SHErrLoc tool did not harm accuracy. Using their benchmark data, in which true errors are already labeled, and their constraint generation process, we found that accuracy is unaffected to their SHErrLoc tool did not harm accuracy. Using their benchmark data, in which true errors are already labeled, and their constraint generation process, we found that accuracy is unaffected by our extensions. This result is expected since OCaml programs use none of the advanced features that this paper targets.

8.4 Performance

We evaluated the performance of our tool on a Ubuntu 14.04 system with a dual-core 2.93GHz Intel E7500 processor and 4GB memory. We separate the time spent into that taken by graph-based constraint analysis (§5) and by ranking (§6).

The CE benchmark Most programs in this benchmark are small. The maximum constraint analysis and ranking time for a single program are 0.24 and 0.02 seconds respectively.

The Helium benchmark Figure 14 shows the performance on the Helium benchmark. The results suggest that both constraint analysis and ranking scale reasonably with increasing size of Haskell program being analyzed. Constraint analysis dominates the running time of our tool. Although the analysis time varies for programs of the same size, in practice it is roughly quadratic in the size of the source program.

Constraint analysis finishes within 35 seconds for all programs; 96% are done within 10 seconds, and the median time is 3.3 seconds. Most (on average, 97%) of the time required is used by graph saturation rather than expansion. Ranking is more efficient: all programs take less than one second.

8.5 Sensitivity

Recall (§6) that the only tunable parameter that affects ranking of error diagnoses is the ratio between $C_2$ and $C_1$. To see how the ratio affects accuracy, we measured the accuracy of our tool with different ratios (from 0.2 to 5). The result is that accuracy and average suggestion size of our tool change by at most 1% and 0.05 respectively. Hence, the accuracy of our tool does not depend on choosing the ratio carefully.

If only unsatisfiable paths are used for error diagnosis (i.e., $C_2 = 0$), the top-rank suggestion size is much larger (over 2.5 for both benchmarks, compared with $\sim 1.7$). Hence, satisfiable paths are important for error diagnosis.

9. Related Work

The most closely related work is clearly that of Zhang and Myers [36]. In order to handle the highly expressive type system of Haskell, it was necessary to significantly extend many aspects of that work: the constraint language and constraint graph construction, the graph saturation algorithm, and the Bayesian model used for ranking errors.

Error diagnoses for ML-like languages Efforts on improving error messages for ML-like languages can be traced to the 80’s [16, 35]. Most of these efforts can be categorized into three directions.

The first direction, followed by [14, 16, 19, 22, 26, 36] as well as most compilers for ML-like language, attempts to infer the most likely cause. One approach is to alter the order of type unification [6, 19, 22]. But any specific order fails in some circumstance, since the error location may be used anywhere during the unification procedure. Some prior work [12, 14, 16, 26, 36] also builds on constraints, but these constraint languages at most have limited support for sophisticated features such as type classes, type signatures, type families, and GADTs. Most of these approaches also use language-specific heuristics to improve report quality.

The second direction [7, 9, 10, 29, 31, 32, 35], attempts to trace everything that contributes to the error. Despite the attractiveness of feeding a full explanation to the programmer, the reports are usually verbose and hard to follow [14].

A third approach is to fix errors by searching for similar programs [20, 23] or type substitutions [5] that do type-check. Unfortunately, we cannot obtain a common corpus to perform direct comparison with [23]. On the suite of OCaml programs used in [36], our tool improves on accuracy of [20] by 10%. The results on the CE benchmark (§8.3) suggests that our tool localizes true error locations more accurately than in [5]. Although our tool currently does not provide suggested fixes, accurate error localization is likely to provide good places to search for fixes.
Constraints and graph representations for type inference  
Modelling type inference via constraint solving is not a new idea. The most related work is on set constraints [1, 2] and type qualifiers [9]. Like SCL, this work has a natural graph representations, with constraint solving strongly connected to CFL-reachability [17, 24]. However, neither set constraints nor type qualifiers handle the hypotheses and type-level functions essential to representing Haskell constraints.

Probabilistic inference  
More broadly, other work in the past decade has explored various approaches for applying probabilistic inference to program analysis and bug finding. This work is summarized by Zhang and Myers [36].

10. Conclusion

We have shown how to use probabilistic inference to effectively localize errors for the highly expressive type system of Haskell. This contribution is clearly useful for Haskell programmers. However, because Haskell is so expressive, success with Haskell suggests that the approach has broad applicability to other type systems.

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