NON-LINEAR MODELING OF EYE GAZE PERCEPTION AS A FUNCTION OF GAZE AND HEAD DIRECTION

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ABSTRACT

In this paper, we further the characterization of a fundamental limit of human perception: the accuracy of human estimation of others' eye gaze directions. In particular, we introduce a non-linear model that describes how both the head direction and the gaze direction of a looker relative to an observer jointly affect the observer's perception of the looker's gaze direction. Ours is the first to explain in a single model the biases introduced by the looker's head direction, the relative accuracy of eye contact detection, and the relative accuracy of estimating gaze direction when the looker's head and gaze directions are aligned. We put our results into context with other perception studies.

***Index Terms*—** Psychophysics, Wollaston effect, Immersion

1. INTRODUCTION

Characterizing the fundamental limits of human perception is important not only scientifically, but also technologically, since it informs the design of engineered systems. Examples of fundamental limits of human perception that have affected the design of engineered systems include:

- Frequency sensitivity of hearing falls off at 15-20 KHz.
- Visual acuity falls off at 60 cycles/degree in the fovea.
- Sensitivity to delay in interactive voice conversations falls off below 150 ms.
- Sensitivity to audiovisual asynchrony (lip sync), falls off below 100-125 ms for audio lagging video, and below 25-45 ms for video lagging audio.
- Visual fusion occurs at frame rates below 60 Hz.

Watermarking (also known as fingerprinting or information embedding) is one area in which engineering takes full advantage of the limits of human perception [1]. For example, in [2] the authors use the property of visual fusion to embed information in the video signal that is invisible to humans at 60 Hz but interpretable by machine. A hugely impactful area of engineering that relies on the limits of human perception is coding or compression [3]. Essentially most of the speech and audio codecs, whether used for mobile telephony or entertainment, use masking models based on detailed models of the fundamental limits of human hearing. And of course,

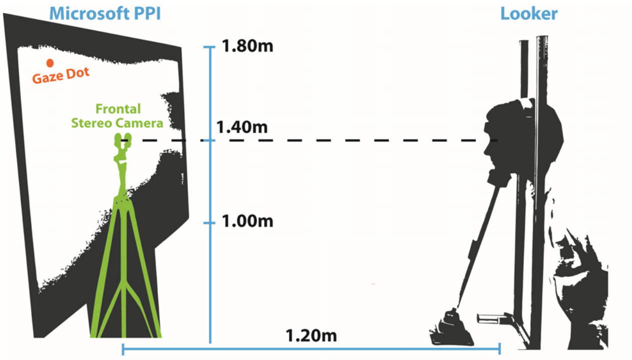

Figure 1. Capturing scenario. The looker seated at 1.20m in front of the PPI screen with the eye position aligned with the center of the screen and the cameras.

the evaluation of the quality of multimedia experience also relies directly on the limits of human perception. For example in [4] the authors used electro-encephalograms to detect whether participants noticed or not certain quality degradations while watching video clips. These examples show how scientific studies of the fundamental limits of perception can impact current and future technologies.

In the current paper we study the accuracy with which humans can estimate the direction of others' eye gaze, whether their gaze is directed toward the observer (i.e., eye contact) or elsewhere. The study of how humans perceive eye gaze is important both in the area of immersive communication between people and the area of human-computer interaction (HCI). Eye gaze is important for immersive communication because it is a major component of non-verbal interaction, used for inference of attention, intent, and desire [5]. In fact, eye gaze effects have been found relevant for transmitting non-verbal information even when telecommunicating using humanoid avatars in immersive environments [6]. Studies have shown that eye gaze may also be essential for self-recognition, as eye gaze synchrony induces strong body ownership illusions on avatars when looking at mirrors [7].

Eye gaze is important for HCI, both for designing humanoid computer interfaces and for creating machines that can capture and interpret a human's gaze direction, which can be considered a special form of gesture [8]. To date this research has focused on gaze only as an input signal to a machine and has kept aside the fundamentals of human gaze perception. However, it would be of potential interest for the HCI community to understand the accuracies necessary to match human performance. Furthermore, taking into account how a real human user would perceive eye gaze is important for designing virtual assistants, agents, and other humanoid characters in games and virtual worlds.

Measuring the accuracy with which humans estimate eye gaze direction has received significant attention within the perception community as well as among vision scientists, over a period of decades [9]–[14]. Psychophysics has been the methodology of choice for studying eye gaze estimation, since it attempts to eliminate effects of subjective bias, opinion, and mental state and instead attempts to measure only fundamental limits of human perception [15]. Human estimation of eye gaze direction is complex, involving the decoding of several phenomena such as the perception of the head orientation, eyes within the head, convergence of the eyes, potential objects of attention, etc. Given these complexities, studies to date, including our own, focus at most on the joint effect of head direction and gaze direction on the perceived gaze direction. The influence of head direction on gaze perception is known as the Wollaston effect [12], [13], [16].

Gibson and Pick (1963) studied *detection* of eye contact. They found that the standard deviation of the horizontal gaze angle at which eye contact is detected is about 0.9º, whether the looker's head is frontal (directed toward the viewer) or turned 30º to the left or right. However, they found that when the looker's head is turned 30º, there is a "constant error" (or *bias*) in the horizontal gaze angle at which eye contact is detected of about 2.9º in the direction of the head turn [9].

Cline (1967) studied *estimation* of the gaze direction, whether the gaze is directed towards the viewer (the eye contact case) or elsewhere. He reported that when the looker's head is frontal and the looker's gaze is directed towards the viewer, the viewers' gaze direction estimates have low bias (0.14º) and low standard deviation (1.55º). Similarly if the looker's head and gaze direction are aligned, even if not directed towards the viewer, the viewers' estimates have low bias (0.21-1.4º), though moderate standard deviation (4.02-6.75º). However, in most other conditions, the viewers' estimates have a significant bias (2.64-5.26º) as well as moderate standard deviation (5.09-6.60º) [10].

Anstis et al. (1969) used regression to develop a linear model of the perceived gaze direction y as a function of the actual gaze direction x in degrees: $y = 1.50x - 0.05$ when the looker's head is frontal, $1.65x + 3.85$ when the head is turned 30º to the left, and $y = 1.69x - 3.48$ when the head is turned 30º to the right. Noting that the linear coefficient is greater than 1.0, they claim a consistent overestimation of the true gaze direction [11]. However, their models are unable to account for the case when the looker's head and gaze direction are aligned, in which Cline observed low bias.

Todorović (2006, 2009) studied the effect of face and eye eccentricity (defined as the fraction of offset of a visual feature towards the outline of the object), on gaze direction perception, using schematics, or cartoons [12], [13]. However cartoons offer no ground truth gaze direction, only ground truth eccentricity. Other researchers have used the accuracy of eye gaze perception estimation to evaluate the efficacy of communication systems [14].

In the current paper we extend the work of Anstis et al. by developing a non-linear model of gaze perception. Our model is the first to explain the major features of gaze perception as influenced by head direction (i.e., the Wollaston effect), observed by Cline: human estimates for gaze are accurate in the case of eye contact and in the case of head-aligned gaze, and is otherwise biased towards the head direction. Our model is mathematical, and can be used for prediction. In our psychophysical study, we leverage modern technologies unavailable in the 1960s: a computer-controlled high-resolution stereoscopic fisheye camera, an immersive virtual environment, and machine learning.

2. MATERIALS AND METHODS

2.1. Capture

In order to present a repeatable stimulus to all subjects, we captured high-resolution stereoscopic images of a person, called the *looker*, gazing at different targets (gaze directions) while holding different head poses (head directions). These same images were later presented to all subjects in turn.

During the image capture session, the looker was seated 1.20m in front of a large Microsoft Perceptive Pixel (PPI) screen. The screen resolution and size were 1920 x 1080 pixels and 133 x 80 cm. At the center of the screen facing the looker were a pair of Point Grey Flea3-U3 cameras attached to Fujinon FE185C fisheye lenses mounted in parallel at an interocular distance of 6.69 cm, located 7 cm away from the PPI (Figure 1). Each camera resolution was 4096 x 2160 pixels.

A target was shown to the looker in the form of a dot on the screen and the looker was asked to fixate her eye gaze on the dot. The target moved to different positions on an 8x15 grid, in steps of 120 pixels covering the whole PPI in a total of 120 points. Images of the looker gazing in all 120 gaze directions were captured for each of three head directions: 0º, 15º, and 30º (Figure 2). Images were captured by computer and stored for later rendering.

Figure 2. The three conditions of head rotation that were captured for the experiment.

2.2. Rendering

For the experiment, the images previously captured were rendered inside a Cave Automatic Virtual Environment CAVE™ [17] system. In our setup the Cave had three 70 × 155 cm back-projected screens: front, left, and right. The projection system was driven by an Intel Xeon 3.1 GHz with two Nvidia Quadro K5000 graphics cards with a display resolution of 1920 × 1080 pixels for each screen.

The participant, or *subject*, had to estimate where the looker was looking at while fitted with Crystal Eyes shutter glasses that were synchronized with the projectors, delivering active stereo at 68 Hz each eye (Figure 3). Head-tracking was performed with an Optitrack V120: Trio 6DoF tracking device; the markers were mounted as rigid bodies into the shutter glasses. The head tracker was used to render the stereoscopic images to the subject from the point of view of the subject's head position. An Xbox 360 joystick was used to capture feedback from the subject.

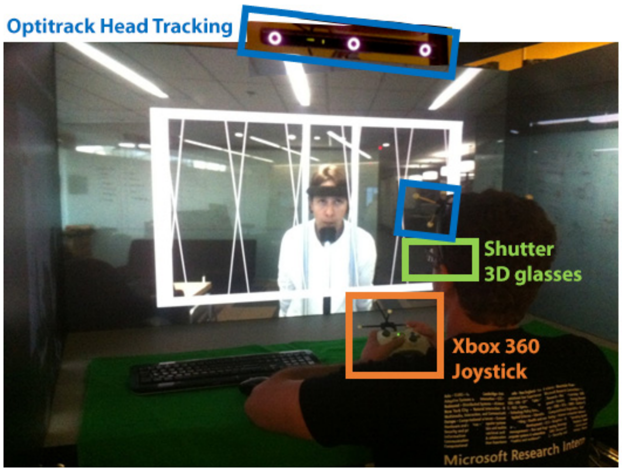

Figure 3. Rendering: the participant wears the shutter glasses with rigid body markers for the head tracking, using an Xbox 360 for interacting with the CAVE.

2.3. Experimental Design

Twelve healthy volunteers with normal or corrected vision (age=31±7.9, 2 females) participated in this study. The exclusion criteria were color or stereo blindness and effects of any medication that could influence the perceptual system. All participants gave consent according to the declaration of Helsinki and were given a small gratuity for participating.

As a group the 12 subjects were exposed to all six permutations of the three different head rotation conditions to remove order effects. During each condition the 120 images of the looker were randomly presented as the stimuli. The study was designed to be within-subjects so differences among conditions could always be evaluated in the context of the skills of the particular subject. Conditions were counterbalanced.

2.4. Measures

As in previous eye gaze studies [9]–[14], in our study, subjects were exposed to a stimulus and had to report where they thought the looker eye gaze was directed.

The reporting was done using the Xbox 360 joystick that moved a red cursor over a synthetic plane that was drawn for this specific purpose (Figure 3). This plane was transparent with only five lines forming an oblique grid that enhanced the correct depth perception of the plane during the stereo; the plane was located 1 m away from the looker, resting between the subject and the looker. Subjects received the following instructions: "There is a synthetic plane between you and the looker; as you can see the cursor moves on that plane. The looker was originally staring somewhere behind you, so you won't find a convergence towards this plane, but the eye-rays cross this plane. Your task is to find the point where the eye-rays cross the plane." The synthetic plane had a size of 35 in (90 cm) with the aspect ratio of the original PPI screen 16:9. The cursor was 1 cm in diameter. After each trial the subjects saw the ground truth for 1 s.

In order to homogenize the data for the final analysis, the subjects' estimation errors in pixels were converted to degrees based on the geometry of the setup shown in Figure 4. With the looker at distance d to the center (x_c, y_c) of the synthetic plane, the ground truth gaze target at (x_t, y_t), and the subject's estimate of the gaze at (x, y), we computed the signed angular error in degrees in both horizontal and vertical directions $(\delta\theta, \delta\phi)$ and magnitude of the angular error δ as:

$$\theta = \arctan\left(\frac{x - x_c}{d}\right);\ \phi = \arctan\left(\frac{y - y_c}{\sqrt{(d^2 + (x - x_c)^2)}}\right) \quad (1)$$

$$\theta_t = \arctan\left(\frac{x_t - x_c}{d}\right);\ \phi_t = \arctan\left(\frac{y_t - y_c}{\sqrt{(d^2 + (x_t - x_c)^2)}}\right) \quad (2)$$

$$\delta\theta = \theta - \theta_t;\ \delta\phi = \phi - \phi_t \quad (3)$$

$$\delta = \sqrt{(\delta\theta)^2 + (\delta\phi)^2} \quad (4)$$

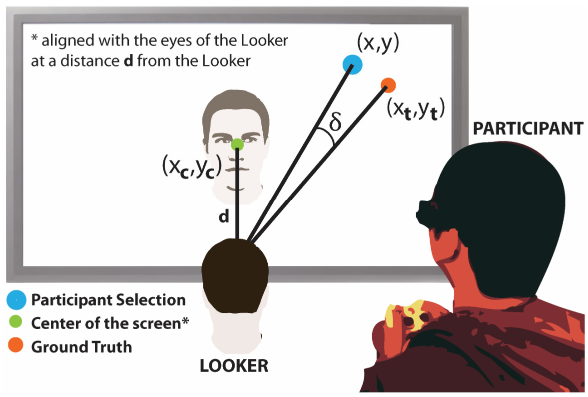

Figure 4. This sketch illustrates how knowing the distance where the looker is from the plane we can easily calculate the deviation in degrees from the ground truth.

3. RESULTS

3.1. Bias

For each of the 120 gaze directions and each of the three head directions, the 12 subjects provide 12 different estimates of gaze direction, resulting in a point cloud containing 12 points. The point cloud has a mean and a covariance. The 2D vector from the ground truth gaze target to the mean of the point cloud is the average error, or *bias*, of the subjects' estimates. The biases for the 120 gaze directions and the three head directions are shown in Figure 5.

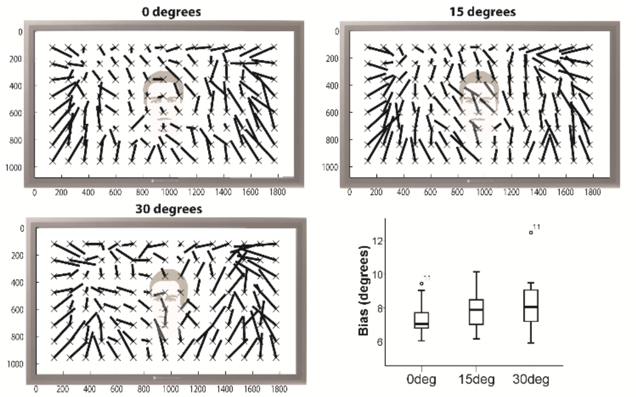

Figure 5. Grand average of the 120 points selected by the 12 participants for each condition. The x locates the ground truth, and the vector indicates the average error.

Visual inspection shows that the bias is low in the 0deg (frontal head) condition particularly towards the center (the eye contact direction), and that the bias radiates out from there. In the 15deg and 30deg head direction conditions, the bias is low not only in the center, but there also seem to be basins of attraction, where there is low bias, on either side of the center. However, it is not immediately clear whether the magnitudes of the bias or the patterns of bias are different in the three conditions.

We began with a global analysis of the bias per condition through a repeated measures ANOVA with factor *Condition* (0deg, 15deg, 30deg). A significant main within-subjects effect was found for *Condition* (F(2,22)=5.290, p=0.013). Further post hoc pairwise comparisons showed that the error was significantly higher in 15deg (Bias=7.83°±0.3°) than in the 0deg (Bias=7.35°±0.3°) (t=4.517, df=2, p=0.046) and also 30deg (Bias=8.20°±0.5°) was higher than 0deg (t=4.517, df=2, p=0.212). The differences in bias between the 15deg and the 30deg conditions were not significant (t=4.517, df=2, p=0.013). The data was tested for normality using Shapiro-Wilk test and p-values were higher than 0.05.

Even though we already had designed a fully counterbalanced study, we wanted to study whether any order effects were present, especially since it is a skill based experiment in which we provided the ground truth after each trial. The data in this case was not found normally distributed therefore a non-parametric test was chosen. The related-samples Friedman's two-way analysis of variance by ranks showed that the distributions were the same across the three orders (test statistic= 0.667, df=2, p=0.717).

3.2. Variance

Each of the 120x3 point clouds has a covariance matrix

$$\Sigma = V \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} V^{-1}, \tag{5}$$

where $V = [\boldsymbol{v}_1, \boldsymbol{v}_2]$ is the matrix of eigenvectors of Σ, and σ_1^2 and σ_2^2 are their corresponding eigenvalues. In Figure 6 we plot the axes $\sigma_1 \boldsymbol{v}_1$ and $\sigma_2 \boldsymbol{v}_2$ of the major and minor components, and their unit standard deviation ellipse ($(\theta, \phi)\Sigma^{-1}(\theta, \phi)^T = 1$), for a subset of the 120 possible target positions, in each of the three conditions.

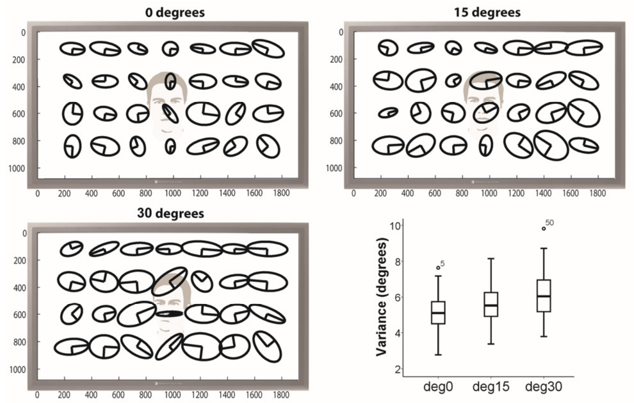

Figure 6. Using a Principal Components Analysis we can study the distribution of the selected point cloud for all the participants. The two vectors in each ellipse represent the principal components.

Visual inspection suggests that the covariance in the 0deg (frontal head) condition is lowest, particularly for 0 degrees of gaze azimuth (the eye contact direction). To determine whether the covariances in all conditions are statistically different, we did an analysis of the standard deviation

$$\sigma = \sqrt{(\sigma_1^2 + \sigma_2^2)/2} \tag{6}$$

as a function of condition through a repeated measures ANOVA with factor *Condition* (0deg, 15deg, 30deg). A significant main within-subjects effect was found for *Condition* (F(2,238)=30.880, p<0.001) (Figure 6). Further post-hoc comparison Paired Samples t-test showed significant differences across all the conditions (t=26.330, df=118, p<0.001). The variance was higher in 30deg (σ=6.13°±0.1°), and in 15deg (σ=5.54°±0.1°) than in 0deg (σ=5.11°±0.1°). The data was tested for normality using Shapiro-Wilk test and p-values were higher than 0.05.

3.3. Mathematical Model

To model the 2D patterns of the bias as a function of head and gaze direction, we leveraged Eureqa's Symbolic Regression [18], a groundbreaking machine learning technique, which performs simultaneous regression and model selection. Eureka uses genetic algorithms to search for the model that has an optimal combination of high correlation with the data and low description complexity (e.g., model order), among a large family of models of increasing order including all combinations of the independent variables, a set of dyadic operators (+, -, *, /, ^) and monadic operators (sin, cos, sqrt), as shown in Table 1. The aim of Eureka is to discover the underlying law from data, in our case a perceptual law.

Table 1. Summary of the Symbolic Regression

Solution Population Size	564801
Solution Encoding	Operation List (graph)
Operator Set	+,-,*,/,sin,cos,^,sqrt
Total runtime	8 core hours (combining the 4 searches)
Error Minimization	Correlation coefficient

We used Eureka to find optimal non-linear models of horizontal and vertical estimation biases $\delta\theta$ and $\delta\phi$, each as a joint function of horizontal and vertical gaze directions θ and ϕ, for each head angle (0,15, and 30 degrees). The model for the vertical bias $\delta\phi$ is relatively simple: only affected by the original ϕ without dependences on the horizontal θ or the head angle. In contrast, the model for the horizontal bias $\delta\theta$ is relatively more complex since it depends on the head angle. Eureka finds no dependence of $\delta\theta$ on ϕ, i.e., the horizontal bias is the same regardless of vertical elevation ϕ. The models discovered by Eureka are found in Table 2, and are plotted in Figure 7.

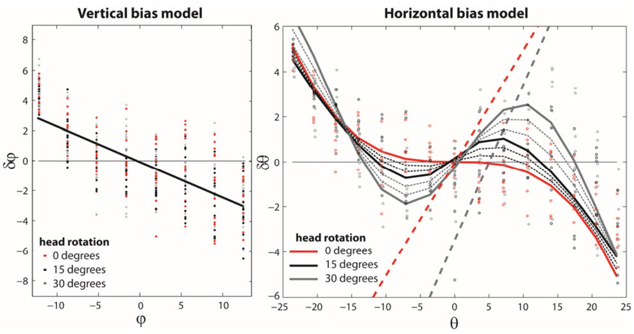

Figure 7. Non-linear model of human gaze estimation. Vertical bias $\delta\phi$ as a function of true gaze direction ϕ. Horizontal bias $\delta\theta$ as a function of true gaze direction θ, for head directions of 0º, 15º and 30º and their interpolation every 5º of head rotation. Estimation is unbiased when $\delta\theta = 0, \delta\phi = 0$; which for each head direction occurs near the eye contact direction ($\theta = 0, \phi = 0$) and again near the head direction. The dots represent the real data to which the equations were fitted. In the horizontal bias graph, the linear model of Anstis et al. is shown in dashed lines for comparison: red for the 0º of head direction ($\delta\theta = 1.50\theta - 0.05$), and blue for the 30º ($\delta\theta = 1.69\theta - 3.48$).

Table 2. Equations Found

$\delta\theta$ **equation**	**Head Angle**	R^2 **goodness of the fit**	**Correlation Coefficient**
$\delta\theta = -0.0003861\,\theta^3$	**0**	0.72	0.85
$\delta\theta = \sin(1.99\,\theta) - 0.00034\,\theta^3$	**15**	0.65	0.80
$\delta\theta = 3.49\sin(6.05 + 0.16\,\theta) - 0.14\,\theta$	**30**	0.59	0.77
$\delta\phi$ **equation**			
$\delta\phi = -0.237\phi$	**all**	0.51	0.71

4. DISCUSSION

The presented experiment explores how the looker's head direction affects the viewer's ability to estimate the gaze direction. A simple analysis of the estimation bias shows that viewers' estimates are more accurate when the looker's head direction is frontal to (i.e., facing) the viewer.

A more detailed analysis of the estimation bias using machine learning reveals non-linear models that describe the relevant properties of the Wollaston effect, showing not only that head direction influences eye gaze estimation, but also how it affects the estimation qualitatively and quantitatively.

Specifically, as illustrated in Figure 7, the models show that for the vertical bias there is a tendency to under-estimate towards the looker's head direction. The bias follows a more complex equation in the horizontal model. When the looker's head is at 0 degrees, and the true gaze direction is modest (+ or - 5 degrees), the bias is low. As the true gaze direction gets more extreme, the bias gets larger in the opposite direction of the gaze, meaning that we tend to under-estimate the looker's gaze direction, favoring instead the direction of the head (which is frontal). In the extremes the bias can reach up to five degrees towards the head direction. This extreme error is similar also in both the 15 degree and 30 degree cases.

Furthermore, when the looker's head is directed to the side (h=15 and 30 degrees), subjects are still good at estimating when the looker is looking at them directly (i.e., detecting eye contact), but when the gaze is between 0 and h' degrees (where h' is slightly less than h), the bias is positive, meaning that there is an over-estimation of the gaze in the direction of the head. If the gaze direction goes beyond h', then the bias becomes negative, meaning that we under-estimate the gaze direction here, though again the bias is in the direction of the head. Interestingly, when the gaze direction is between 0 and $-h'$, there is an over-estimation of the gaze towards the complement of the head direction. In this case the viewer feels as if the looker is gazing far beyond the target. At $-h'$, there is no estimation bias. Past $-h'$, the viewer starts to under-estimate the gaze direction again.

The linear model of Anstis et al. [11] is also shown in Figure 7, for comparison. While it captures eye contact for

$h = 0$ degrees, and generally predicts over-estimation when the gaze direction is between $-h$ and h, the amount of over-estimation becomes too large, and is not credible, when the gaze direction is aligned with the head direction. At that point, as measured by Cline, estimation bias should be quite low. The reason for this weakness of the model of Anstis et al. is linearity. A non-linear equation is required to model these features of Wollaston effect. Moreover, our model discovers not only two, but three gaze directions at which viewers' gaze direction estimation has zero bias:

1. When the looker is gazing at the participant (eye contact), regardless of the looker's head direction.
2. When the looker is gazing in a direction aligned with the head direction (head and eye alignment).
3. When the looker is gazing in a direction complementary to the head direction.

One characteristic of this model is that even though the bias is minimal in those specific points, the full equation obeys a continuous first derivative, probably a consequence of our gaze estimation mechanism being smooth. The perceptual system does not make sudden changes.

5. CONCLUSIONS

The current paper advances knowledge of the fundamental limits of human eye gaze perception, by proposing a mathematical model to explain the Wollaston effect: how head direction affects the perception of gaze direction. We find that there is a natural bias towards the head direction when estimating eye gaze direction. Moreover, we find three gaze directions at which the viewer's gaze estimation is unbiased: the direction towards the viewer (eye contact), the direction of the head, and the direction complementary to the head direction, with respect to the viewer. The model was validated using experimental data showing high correlation coefficients (over .70). These scientific results have technical implications for the design of both immersive communication systems and HCI.

Future work, besides applying the model to the design and evaluation of systems for immersive communication and HCI, could include studying the effect of resolution. It is expected that distant, blurred, or low-resolution views of the looker's eyes would bias gaze estimation even more strongly in the direction of the head.

6. ACKNOWLEDGEMENTS

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