ABSTRACT
Successful software maintenance is becoming increasingly critical due to the increasing dependence of our society and economy on software systems. One key problem of software maintenance is the difficulty in understanding the evolving software systems. Program workflows can help system operators and administrators to understand system behaviors and verify system executions so as to greatly facilitate system maintenance. In this paper, we propose an algorithm to automatically discover program workflows from event traces that record system events during system execution. Different from existing workflow mining algorithms, our approach can construct concurrent workflows from traces of interleaved events. Our workflow mining approach is a three-step coarse-to-fine algorithm. At first, we mine temporal dependencies for each pair of events. Then, based on the mined pair-wise temporal dependencies, we construct a basic workflow model by a breadth-first path pruning algorithm. After that, we refine the workflow by verifying it with all training event traces. The refinement algorithm tries to find out a workflow that can interpret all event traces with minimal state transitions and threads. The results of both simulation data and real program data show that our algorithm is highly effective.

Keywords
Workflow mining, graphical behavior models, temporal properties

1 INTRODUCTION
With the increasing dependence on software at all levels of our society and economy, successful software maintenance is becoming increasingly critical. Software maintenance mainly tries to ensure that a software product performs correctly in a changed or changing environment, and try to maximize the performance. A key technical issue of software maintenance is the difficulty in understanding what happens to systems (and software) over time [18], e.g., about 50% of the maintenance cost is due to comprehension or understanding the code base. Program workflow can help operators and administrators to understand system behaviors, to verify the system execution, and to validate the running results. It can also be used in software verification [6] to diagnose system problems. Unfortunately, the knowledge of program workflow is not always available to hand. This is because software documents and specifications are often not complete, accurate and updated due to bad development management and tight schedule (e.g., hard product shipping deadlines and “short-time-to-market”). However, most software systems generate event traces (also referred as event logs) for problem diagnosis and performance analysis. These trace messages usually record events or states of interest and capture the underlying execution flow of the system components. The above facts motivate building automatic tools for mining program workflow from program event traces.

There are a set of previous research efforts [1, 2, 4, 5, 6, 8, 10] on mining program workflow models from event traces. Most of them are variants of the classical k-Tails algorithm [2]. They learn a finite state automaton (FSA) from event traces. However, these k-Tails based algorithms cannot perform well in a complex system, in which multiple independent threads/processes generate traces of interleaved events. Because the events can be interleaved in many different ways, and a k-Tails based algorithm tries to interpret a huge number of event sequencing patterns, the resulting FSA model often becomes very complex. In fact, all these k-Tails based algorithms have the same assumption that the traces are sequential traces [6]. For traces produced by a multi-thread program, these algorithms assume that the traces contain thread IDs and can be analyzed thread by thread using the thread IDs to distinguish traces generated by different threads [6]. Although most software programs produce traces with thread IDs, there are still some existing systems that do not record thread IDs in their logs or traces by default. More important, most advanced software programs are designed and implemented based on event driven architecture, in which the flow of the program is determined by events [11]. In most event driven systems, a task’s workflow is divided into several stages or subtasks, and each subtask is related to an event which is handled by an event handler. In these systems, a thread usually handles many different events from multiple concurrent tasks, and these events often interleave with each other. Therefore, even given the thread IDs, we still cannot learn a task workflow from the event traces of such an event driven system by a k-Tails based algorithm.

In this paper, we present an algorithm to discover a program workflow model from traces of interleaved events by a three-step coarse-to-fine algorithm. At first, we mine temporal dependencies for each event pair from event traces. The dependencies that we mined are always valid under any event interleaving patterns. Rather than directly constructing the workflow from the input event traces, we construct a basic workflow model from the mined dependencies. Then, based on the different properties between a loop/shortcut structure and a thread spawn/sync structure, we design an algorithm to refine the basic workflow. The refinement algorithm can find the simplest workflow with a minimal number of thread types to interpret all event traces. Our approach works quite well on traces of interleaved events. To the best of our knowledge, the paper is the first work to learn workflows from traces of interleaved events produced by concurrent programs.

The rest of the paper is organized as follows. In section 2, we briefly introduce the previous work that is closely related to ours. Section 3 provides the basic ideas and concepts of our approach including event traces, temporal dependencies and the definition of the workflow model. In section 4, we describe an algorithm to mine temporal dependencies from event traces. The detailed algorithm on constructing a workflow model based on mined temporal dependencies is presented in section 5. In section 7, we validate the approach with some experimental results on both simulation and real event traces. Finally, we conclude the paper in section 8.
2 RELATED WORK

There are a set of existing research efforts [1, 2, 4, 5, 6, 8, 10] on learning program workflow models from execution traces for software testing and debugging. In these algorithms, Finite State Automatons (FSA) are used as workflow models. These algorithms are mostly extended from the popular k-Tails algorithm proposed by Biermann and Feldman [9], which can automatically learns an FSA from sample symbol sequences. At first, the algorithm constructs an initial FSA from input traces, and then it progressively refines the FSA by merging equivalent states until no further merge operation is possible. Different algorithms [6, 10] use different equivalent criteria to derive different FSA depending on different desired degree of generalization. For example, in the k-strings algorithm [10], if two states $s_a$ and $s_b$ generate the same k-strings (i.e. symbol sequences of length up to $k$ that can be generated from the state), they are merged. The learner modifies the k-Tails algorithm by comparing how likely two states are to generate the same k-strings. L. Mariani et al [6] also proposed an incremental FSA learner, namely k-Learner, by modifying the k-Tails with a new merge operation. The algorithm identifies subsequences of a new trace in the current FSA, and augments the FSA to include the new trace. Besides the state equivalence conditions, some recent algorithms also introduce other pre-conditions for the merge operation. In [2], two equivalent states $s_a$ and $s_b$ can only be merged when the merged FSA does not break a set of temporal patterns that are automatically mined from the training traces. A similar steering idea is also proposed by Walkinshaw et al [8], in which user defined Linear Temporal Logic (LTL) rules are used to determine whether two equivalent states can be merged. All these k-Tails based algorithms assume that the event traces are sequential traces [6]. However, many workflows exhibit concurrent behavior, where a single event trace comprises a set of events produced by more than one thread of control. The sequential state machine cannot capture the concurrent behavior of a system, and the k-Tails based algorithms will create a very complex model as they try to interpret all event sequencing patterns generated by the interleaved events. Unlike these, our algorithm learns workflow models from traces of interleaved events.

Another set of research efforts [3, 7, 12, 13, 14, 15, 16, 17] from the area of Business Intelligence [12] are also related to our work. Rather than software system traces, they try to mine business processes (or workflows) from business activity logs. In business workflow mining algorithms, different from FSA models, a process is often represented by a graph in which nodes correspond to the activities to be performed, and arcs describe the precedence or dependent relationships among the activities. For example, the authors of [14] use a directed acyclic graph (DAG) to model a workflow process. In [3] and [7], a constrained DAG, namely an AND/OR workflow graph (AO graph), is proposed to model a workflow process. Aalst et al [15] use a special class of Petri-net model, namely workflow nets (WF-net), to model a workflow process. Among these models, the WF-net is the most powerful model that can be simplified to other models by adding some constraints. Although the business process mining and software workflow mining algorithms have different application backgrounds and models, they share the same basic idea, i.e. constructing constrained graphic activity models from traces [13], and they can leverage each other [17]. For example, we can construct a DFA to mimic the behaviors of a DAG based model. Almost all business workflow mining algorithms consider the parallel property of different tasks by introducing AND split/join nodes [3, 7, 13, 14], fork/join places [12, 15, 17], or Parallel operators [16]. In order to handle the concurrent task problem, similar to our approach, these algorithms reconstruct the workflow graph by utilizing the dependent or causal relationships mined from workflow activity logs. For example, in [15], Aalst et al only use the ordering relation “$>$” that describes whether one is directly followed by the other in a trace, to discover a WF-net. Cook et al [17] use more information, such as event type counts, event entropy, and event periodicity, and combine the information into a ranking score to discover the concurrent behavior in event streams. However, all of these approaches assume that an input log sequence is produced by a single case in isolation [15, 17], and they cannot handle an interleaved log sequence generated by multiple concurrent executions of the same workflow (e.g. an event trace is produced by a WF-net workflow if there are two or more tokens in the start place of a WF-net). In addition, their models also do not allow that multiple concurrent threads run the same sub-workflow [15, 17]. However, in a parallelized program, a job may often be divided into several concurrent sub-tasks, with each sub-task having the same workflow. For example, a MapReduce job is often split into many Map tasks, and each Map task follows the same execution logic. Our algorithm handles these problems by a coarse-to-fine approach, in which we first construct a basic workflow model from temporal dependencies that are mined from the event traces, and then refine it through verification.

3 PRELIMINARY

This section introduces the basic concepts and techniques used in our algorithm including our view of events, our workflow model, different types of dependency relationships involved in our algorithm, and the assumptions of our mining algorithm.

3.1 EVENT TRACE

Similar to other methods, we view the event logs in a log file as a trace of events being produced by a black-box system. We assume that the temporal orders of event logs faithfully reflect the temporal orders of the corresponding events. Here, events are used to characterize the dynamic behavior of a software program, for instance, a method call. In software systems, events are often recorded with attributes, such as timestamp, related resources and data, and any other information that can identify the specific occurrence of that type of event. The activities of a system recorded in a log file are represented by an event sequence, namely an event trace. As mentioned above, a system may have simultaneous executing threads of control, with each of them producing events that form a resulting event trace. Thus, an event trace contains interleaved event sequences from all the concurrent control threads. Note here that although the term “thread” means a sequential execution control path within the workflow. It may not directly map to a process/thread in the operating system in an event-driven system. Although in some systems, event attributes can help to distinguish the events from different control threads of the workflow, in this paper, we present a general approach that only utilizes the ordering of events.

In this paper, all event traces are generated by complete executions of the program on some use cases. They may contain some noise that is usually caused by some execution anomalies in a program comprehension scenario. For event traces that may contain noise, we can use the noise filtering method [19] or anomaly detection method [20] to filter out the noise before we use the algorithm proposed in this paper to learn workflow models. In addition, we assume that the temporal ordering patterns of differ-
ent events can be as diverse as possible. This is reasonable for event logs produced by independent concurrency. Some concurrent systems that may not display much randomness are not the targets of our method. As with most data driven approaches, we also assume that we have enough event logs to make our analysis meaningful. Because our method is based on the temporal orders of events in event traces, enough log data means that we can find enough instances of any possible ordering relationship of every event pair to make the mined dependencies statistically meaningful.

Formally, let $\Sigma$ be a set of distinct events that appear in all event traces. An event trace is an ordered sequence of events, denoted as $< e_1, e_2, ..., e_m >$, where $e_i$ is an event, i.e. $e_i \in \Sigma$ for $1 \leq i \leq m$. For brevity, we can also write it as $e_1, e_2, ..., e_m$. Given an event trace $l = e_1, e_2, ..., e_m$, for an event $e_i, 1 \leq i \leq m$, we call the subsequence $e_{i+1}, e_{i+2}, ..., e_m$ as the postfix of $e_i$ in $l$, denoted as $post(e_i)$, and subsequence $e_1, e_2, ..., e_{i-1}$ as the prefix of $e_i$ in $l$, denoted as $pre(e_i)$. If $i = 1$, then $pre(e_i)$ is a null sequence, denoted as $\emptyset$. Similarly, when $i = m$, we also have $post(e_i) = \emptyset$.

### 3.2 WORKFLOW MODEL

In this paper, our algorithm outputs a transition-labeled finite state machine to represent a workflow. In order to model the concurrent properties of workflows, we introduce a vector of active states instead of just a single active state so that the state machine can have multiple, concurrent threads. To create and destroy these threads, we define four new types of states to mimic the concurrent behavior of a Petri net: split/merge states and fork/join states. Similar to the AND split/merge nodes in papers [3, 7], a split state has multiple out transitions (i.e. transitions leaving from the state), and all these transitions are taken concurrently. A merge state has multiple entering transitions, and it can transit to another state only if all its entering transitions have occurred. Informally, split states represent the points where the system will spawn one thread for each leaving transition. As a counterpart, merge states are used to represent points of synchronization. That is, before the merge state can transit to another state, it has to wait for the completion of threads from all its entering transitions. Unlike a split state, a fork state only has one leaving transition. At a fork state, the system will fork several threads to execute a sub-workflow starting from the state. Similar to a merge state, a join state is also a point of synchronization, and it is specially designed to absorb all threads created by a fork state. In general, a fork state and a join state will exist in pairs. To facilitate the description, we call the ordinary states in a state machine as a switch state. Fig. 1 shows some sample workflows, where a square box “□” is used to represent a split/merge state node, a diamond box “○” is used to represent a fork/join state node, and a circle “●” is used to represent a switch state node. By introducing the split/merge and fork/join nodes, we can model the concurrent properties in a workflow. For example, a single run of the workflow of Fig.1(a) can generate event traces of “ABCD” or “ACBD” because B and C are two concurrent events. Furthermore, two concurrent runs of the workflow can generate many kinds of traces of interleaved events, such as “ABACDBCD”, “ACBCBBDCD”, and so on. The fork node in Fig.1(c) can also generate concurrent events, because the sub-workflow between the fork node to the join node will be run with multiple threads.

Formally, a workflow model is defined as:

**Definition 3.1** A workflow model is a tuple $W = (\Sigma, S, S_a, s_0, \delta, f)$, where:

- $\Sigma$ is the set of event types.
- $S$ is a finite and non-empty set of states including switch states, split/merge states and fork/join states.
- $s_0$ is an initial state.
- $S_a$ is a vector of active states.
- $\delta$ is the state-transition function that represents the transition from one state to other states, $\delta: S \times S \rightarrow \Sigma$. Each transition is labeled by an event type, which means the transition can generate an event of that type, and a transition can be uniquely determined by its event label. In this paper, we use these two terms interchangeably.
- $f$ is the end state.
- We also define a special symbol $\epsilon \in \Sigma$, which means an empty event. Also $\delta(q_1, q_2) = \epsilon$ means that the transition from state $q_1$ to state $q_2$ does not generate any event. We call such a transition a shortcut.

**Definition 3.2** Given a transition $\delta(q_1, q_2)$, we call $q_1$ and $q_2$ as the starting state and the ending state of the transition respectively. If one transition’s ending state is the starting state of another transition, we call the two transitions as neighboring transitions. Given a state $q$, we define all transitions that start from $q$ as $q$’s out transitions (denoted as $OutSet(q)$), and the transitions that ending at $q$ as $q$’s in transitions (denoted as $InSet(q)$).

For a realistic workflow, all states should be reachable from the initial state. It means that, for every state $s$, there is at least one transition path from the initial state to $s$. At the same time, each state also has a path to the end state. In this paper, a transition path from the initial state to the end state is defined as a route. A workflow often has several routes.

There are two reasons that we do not select Petri net, a more powerful model. At first, we focus on creating descriptive models to help operators to understand software behavior. Most software workflow mining algorithms use FSA as their workflow models because a state based representation is convenient to map to programming logic. We use a state based model to be consistent with representations of these related algorithms. On the other hand, a state based model is simpler than a Petri net model, which can simplify the operations and descriptions of our algorithm.

![Figure 1. Some simple workflows.](image)

### 3.3 EVENT DEPENDENCIES

In the context of workflow traces, the occurrence of an event may be dependent on the occurrence of another event in the trace. Our workflow model reconstruction depends on mining the temporal dependencies among events. Dependence means that the occur-
rence of one event type depends on the occurrence of another event type. In this paper, we define four types of temporal dependencies. **Forward dependency** (FD) describes that “Whenever an event A occurs, another event B must eventually occur after A’s occurrence”. **Forward dependency** is denoted as \( A \rightarrow_f B \). **Backward dependency** (BD) describes that “Whenever an event B occurs, another event A must have occurred before B’s occurrence”. **Backward dependency** is denoted as \( A \rightarrow_b B \). The third dependency we defined is **strict forward dependency** (SFD), which means “For each occurrence of event A, there must be at least one occurrence of event B after A’s occurrence.” We denote it as \( A \rightarrow_{sf} B \). Different from \( A \rightarrow_{sf} B \), which means one or more occurrences of event A will eventually cause the occurrence of event B, \( A \rightarrow_{sf} B \) means that each occurrence of event A will cause at least one occurrence of event B. Similarly, we also define **strict backward dependency** (SBD) with “For each occurrence of event B, there must be at least one occurrence of event A before B’s occurrence.”, which is denoted as \( A \rightarrow_{sb} B \). Unlike the dependencies defined in [15, 17], our dependencies do not require that one event is directly followed by another event in an event trace, which are not influenced by various interleaving patterns.

Among these four types of dependencies, FD and BD focus on the global temporal relationship between two event types, and SFD and SBD not only look at the temporal relationship but also take the event count information into account. It is obvious that: \( A \rightarrow_{sf} B \Rightarrow A \rightarrow_f B \), and \( A \rightarrow_{sb} B \Rightarrow A \rightarrow_b B \). Thus, in this paper, for simplicity, if two events have a strict dependency relationship, we will not list the corresponding non-strict dependency relationship. In addition, we use \( A \parallel B \) to denote that two events A and B do not have any defined temporal dependencies.

For each pair of events, we can determine whether there is a temporal dependency between the two events by verifying the relationship within all event traces. For example, for event A and B in Fig.1(a), we have \( A \rightarrow_{sf} B \) and \( A \rightarrow_{sb} B \) because these two dependencies are always true in the traces with any potential event interleaving pattern. For the workflow in Fig.1(c), we always have \( A \rightarrow_{sf} B \) and \( A \rightarrow_{sb} B \). For all simple workflows in Fig.1, we list all the dependencies between event type A and B in Table 1. Such dependencies are always true no matter how event logs interleave together. From the examples, we can see that different local logical structures in a workflow often have different types of dependencies.

**Table 1. Temporal dependencies of workflows in Fig.1**

<table>
<thead>
<tr>
<th>Workflow</th>
<th>Valid Temporal Dependencies between A and B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1(a)</td>
<td>( A \rightarrow_{sf} B ), ( A \rightarrow_{sb} B )</td>
</tr>
<tr>
<td>Fig. 1(b)</td>
<td>( A \rightarrow_{sf} B )</td>
</tr>
<tr>
<td>Fig. 1(c)</td>
<td>( A \rightarrow_{sf} B ), ( A \rightarrow_{sb} B )</td>
</tr>
<tr>
<td>Fig. 1(d)</td>
<td>( A \rightarrow_{sf} B ), ( A \rightarrow_{sb} B )</td>
</tr>
<tr>
<td>Fig. 1(e)</td>
<td>( A \rightarrow_{sf} B ), ( A \rightarrow_{sb} B )</td>
</tr>
<tr>
<td>Fig. 1(f)</td>
<td>( A \rightarrow_{sf} B ), ( A \rightarrow_{sb} B )</td>
</tr>
</tbody>
</table>

Pair-wise temporal dependencies describe the causal relationships of each event pair. They can provide information for workflow reconstruction. Our basic workflow model constructing algorithm is based on the properties of the mined temporal dependencies. Obviously, the dependencies have the following property:

**Property 3.1** Let \( W = (\Sigma, S, S_0, \delta, f) \) be a workflow. For any \( A, B \in \Sigma \):

- If \( A \rightarrow_f B \), there must be a transition path from A to B (denoted as \( A \rightarrow B \)) in the workflow, and the routes that pass through A must later pass through B.
- If \( A \rightarrow_b B \), then \( A \rightarrow B \), and the routes that pass through B must first pass through A.

### 4 MINE TEMPORAL DEPENDENCY

In this section, we provide the details about the method of mining temporal dependencies. As with most classical algorithms of sequence pattern mining, we measure the significance of a temporal dependency between two events by computing the statistical metrics of support and confidence. For event types A and B, when we mine the relationships of \( A \rightarrow_{sf} B \), the support is defined as the number of times that event A appears in the traces. In contrast to the support of \( A \rightarrow_{sf} B \), the support of \( A \rightarrow_f B \) is computed as the number of traces that contain event A. As a counterpart, the support of \( A \rightarrow_{sb} B \) is counted as the number of times that event B appears in the traces, and the support of \( A \rightarrow_b B \) is the number of traces that contain event B. The confidence values of the dependencies are defined by the corresponding conditional probabilities. For example, the confidence of \( A \rightarrow_f B \) is calculated by

\[
\text{conf}(A \rightarrow_f B) = \frac{\text{No. of traces that have } B \text{ after the last } A}{\text{No. of traces that contain } A}
\]

Similarly, the confidence of \( A \rightarrow_b B \) can be calculated by

\[
\text{conf}(A \rightarrow_b B) = \frac{\text{No. of traces that have } B \text{ before the first } A}{\text{No. of traces that contain } B}
\]

The computing of \( A \rightarrow_{sf} B \) and \( A \rightarrow_{sb} B \)’s confidence values is a little bit complex. We take \( A \rightarrow_{sf} B \) as an example to describe the computing procedure. For each event trace \( t \), we find all occurrences of event \( e_i \) that satisfy \( e_i = A \) and \( |[e \in A \cap \text{post}(e_i)]| < |[e \in B \cap \text{post}(e_i)]| \) (i.e. the number of B is larger than the number of A in post(e_i)). Denoting the total number of such events \( e_i \) in all traces \( |[e \in (A \rightarrow_{sf} B)]| \), we can calculate the confidence by

\[
\text{conf}(A \rightarrow_{sf} B) = \frac{|[e \in (A \rightarrow_{sf} B)]|}{\text{No. of A in all traces}}
\]

As a counterpart, the dependency of \( A \rightarrow_{sb} B \) is

\[
\text{conf}(A \rightarrow_{sb} B) = \frac{|[e \in (A \rightarrow_{sb} B)]|}{\text{No. of A in all traces}}
\]

where \( |[e \in (A \rightarrow_{sb} B)]| \) is the number of events that satisfy \( e_i = B \) and \( |[e \in A \cap \text{pre}(e_i)]| < |[e \in B \cap \text{pre}(e_i)]| \) (i.e. the number of B is larger than the number of A in pre(e_i)).

**Algorithm 1.** Pseudo Code of Mining Forward Dependencies

**Inputs:**
- \( L \): the set of all event traces
- \( s_p \): support threshold
- \( c_f \): confidence threshold

**Output:**
- \( FDs \): a set of mined forward dependency
- \( SFDs \): a set of mined strict forward dependency

1. Set \( d = |\Sigma| \);
2. Let \( conf_f[d][d], sup_f[d], conf_s[d][d], \) and \( sup_s[d] \) to zero for all \( d \);
Algorithm 1 describes the procedure of mining forward dependency and strict forward dependency, where \( d \) is the number of distinct events in all traces. \( \text{conf}_f[d][d] \), \( \text{sup}_f[d][d] \), \( \text{conf}_{sf}[d][d] \), and \( \text{sup}_{sf}[d][d] \) are vectors used to record the support numbers and confidence values. The time complexity of the algorithm is \( O(Nd) \), where \( N \) is the cumulative length of all event traces. Generally, \( d = |\Sigma| \) (i.e., the number of distinct event types) is constant for a program, and is always significantly smaller than \( N \). Thus, the algorithm possesses linear complexity with respect to \( N \).

Unlike the scenarios of traditional sequence (or frequent item set) mining, where some meaningless patterns can happen by chance, in our context any occurrence of an event ordering in event traces is meaningful and reflects an aspect of the execution behavior of the software system. In this paper, we set the support threshold as 5 (It means the number of observations should be at least five to make our analysis results statistically meaningful [21]), and all our events in our experiments can meet this requirement. In addition, a dependency relationship is valid only if it has a perfect confidence (conf=100%).

5 WORKFLOW RECONSTRUCTION

In this section, we provide our main algorithm of constructing workflow from mined temporal dependencies. We first construct an initial workflow by recovering all connections (defined in section 5.1) based on the mined temporal dependencies. During the basic workflow construction, our approach does not consider shortcut transitions and loop structures, thus, the learned basic workflow does not contain such workflow structures. In order to recover the missing structures, we refine the workflow by verifying with event traces. The aim of refinement is to find the simplest workflow with a minimum number of threads to interpret all training event traces.

5.1 CONSTRUCT A BASIC MODEL

From section 3, we can see that, given dependencies \( A \rightarrow_f B \) or \( A \rightarrow_h B \), we can conclude that there is a path from event \( A \) to event \( B \) (denoted as \( A \rightarrow B \)). In addition, for two neighboring events \( A \) and \( B \), if \( A \rightarrow B \), we can determine a connection between \( A \) and \( B \) in the original workflow, i.e. the ending state of \( A \) is the starting state of \( B \). In this paper, we call the dependency between two neighboring events as a direct dependency. Furthermore, supposing that we have a pair-wise dependency for each pair of neighboring events, we can recover all connection relationships.

Although a mined temporal dependency from event traces shows that there is a path between two events, we cannot directly establish a connection between them because many dependencies are not direct dependencies (i.e. they are indirect dependencies). An indirect dependency does not correspond to a connection between two events. For example, in Fig 1 (d), we have a temporal dependency of \( A \rightarrow_f C \). However, there is no direct connection between \( A \) and \( C \). Here, the path from \( A \) to \( C \) is composed by a path from \( A \) to \( B \) and a path from \( B \) to \( C \). In order to handle such problems, we try to construct a compact basic workflow in which there is at most one transition path between every two events. We use a pruning strategy to remove indirect dependencies during the basic workflow construction. For each event pair \((\alpha, \beta)\) that satisfies \( \alpha \rightarrow_f \beta \) or \( \alpha \rightarrow_h \beta \), we denote \( \beta \) as \( \alpha \)'s successor, and \( \alpha \) as \( \beta \)'s predecessor. For the simplicity of implementation, we first use a graph data structure to store the obtained paths, in which each event has a predecessor list and a successor list. The algorithm starts from the events that do not have any preceding event. Then, we add events into the graph and construct preceding/succeeding relations according the mined dependencies. For any pair of events \( A \) and \( C \) where \( A \) is a predecessor of \( C \), if a successor event of \( A \) (e.g. \( B \)) is also a predecessor of \( C \), we remove \( C \) from \( A \)'s successor list. In the resulting graph, all indirect dependencies are removed. By converting the remaining preceding/succeeding relations to event connections, we can construct a transition-labeled workflow, namely basic workflow. The algorithm is shown in Algorithm 2. In the algorithm, the function \( \text{Find}_\text{Root}(NV) \) returns a set of events in which each event does not have any predecessor in the set \( NV \).

Algorithm 2. Pseudo Code of Basic Workflow Construction

Inputs:
\( L \): the set of all event traces
\( D \): the set of all dependencies

Output:
\( T \): learned basic workflow

1. \( N = NV = \) the set of all log keys;
2. \( Q = \) an empty FIFO queue;
3. while \( NV \) is not empty
4. \( S = \text{Find}_\text{Root}(NV) \);
5. \( \text{Add} S \) into \( T \);
6. \( \text{Push}_\text{back}(Q, S) \);
7. while \( Q \) is not empty:
8. \( i = \text{Pop}_\text{front}(Q) \);
9. if \( i \) is not in \( NV \)
The following theorem shows that the remaining transitional paths obtained by the above algorithm must exist in the original workflow. In other words, our algorithm can obtain a basic workflow skeleton.

Theorem 5.1. Let $W = (\Sigma, S, S_0, \delta, f)$ be a workflow, with at least one temporal dependence between every two neighboring events. For any $A, B \in \Sigma$ that $A \rightarrow_B B$ or $A \rightarrow_B B$, if $[\{A \rightarrow C \land C \rightarrow B\}] = \emptyset$, there must be a connection from $A$ to $B$ in the original workflow.

Proof. Obviously, there is a path from $A$ to $B$ (i.e. $A \rightarrow B$). In addition, because $\{A \rightarrow C \land C \rightarrow B\} = \emptyset$, then $A$ and $B$ are a pair of neighboring events. Therefore, there must be a connection from $A$ to $B$ in the original workflow.

The above algorithm does not consider a special case where two events have dependencies with different directions. For example, from the event traces generated by the workflow in Fig.1(f), we can learn both dependencies of $B \rightarrow_B C$ and $C \rightarrow_B B$ at the same time. We call it a bidirectional dependence, denoted as $C \leftrightarrow B$. If we directly run the basic workflow construction algorithm on such dependencies, the algorithm will run into an endless loop. In order to overcome this problem, we first check whether there are bidirectional dependencies in the mined dependencies. If there is a bidirectional dependence, e.g. $C \leftrightarrow B$, we create a new virtual event type $B'$ to replace the events of type $B$ in all forward dependencies. Then, we run our basic construction algorithm to reconstruct the basic workflow. After that, we merge the virtual events (e.g. $B'$) with their corresponding events (e.g. $B$) in the basic workflow.

Adding initial\&end state: Each workflow contains an initial state and an end state. Thus, we need to add an initial state and an end state into the basic workflow. Obviously, the first event and the last event of an event trace are potentially an initial event and an end event respectively. In this paper, we first find out all events that have appeared as the first event in event traces. If the support number of an event appearing as the first event in event traces is larger than a certain level (we use 5% in experiments because we assume the noise level is less than 5%), we add a shortcut transition from the initial state of the workflow to the starting state of the event. Similarly, if the support number of an event appearing as the last event in event traces is larger than a certain level, we add a shortcut from the ending state of the event to the end state of the workflow.

Determining state types: According to the definition of the workflow model in section 3.2, there are five types of states. In the above basic workflow construction algorithm, we do not identify the type of each state. Given an event type that has several event types following it, we have to make a decision on whether the program behavior at this point is a sequential selection (i.e. a switch state) or a concurrent splitting (i.e. a split/fork state). In this subsection, we determine the state types by utilizing the information of event type counts. As studied in our previous work [20], the linear relationships between the occurrence times of different event types can also provide cues for the workflow structure. For example, for a switch state $q$, it is always true in every event trace that:

$$\sum_{A \in \text{InSet}(q)} \text{Occur}(A) = \sum_{B \in \text{OutSet}(q)} \text{Occur}(B)$$ \hspace{1cm} (1)

On the other hand, if $q$ is a split state, then for any $A \in \text{OutSet}(q)$ and $B \in \text{OutSet}(q)$,

$$\text{Occur}(A) = \text{Occur}(B)$$ \hspace{1cm} (2)

Similarly, a merge state also has a property that $\text{Occur}(A) = \text{Occur}(B)$ for any $A \in \text{InSet}(q)$ and $B \in \text{InSet}(q)$. However, fork and join states do not have such regular properties on the counts of event types. If a state satisfies equation (1), it must be a switch state. In this subsection, we first find out the split/merge states by verifying whether a state satisfies equation (2), and then find out the switch states that can be identified by equation (1). Because our model allows shortcut transitions, some switch states cannot be easily identified by equation (1). For all remaining states with their state types undetermined, we will determine their state types through a workflow refinement process based on event traces (refer to section 5.2). The default state type is switch.

5.2 REFINING THE WORKFLOW MODEL

The basic workflow obtained by Algorithm 2 does not contain any shortcut transitions or loop structures, because we only keep one transition path between every two dependent events. However, in a real workflow, there may be some shortcut transitions and loop structures. In addition, the above algorithm cannot identify the fork/join state types, thus, the types of some states in the basic workflow are not determined. In this subsection, we identify fork/join states and recover loop structures and shortcut transitions to refine the workflow by verifying with event traces.

We recover loop structures or shortcut transitions based on the statistical properties of these structures. Here, we use a simple example to describe the basic idea behind our algorithm. Fig.2(a) presents a simple program workflow containing a loop structure.
Fig. 2(b) shows two event traces. Both of them are generated by a
two-thread program with different interleaving patterns in which
each thread runs along the workflow in Fig.2(a). Our basic
workflow construction algorithm can only construct a basic
workflow without a loop (see Fig.2(c)). When we use the basic
workflow in Fig.2(e) to interpret the first event trace in Fig. 2(b),
we find that the first five events of the event sequence are gener-
ated by two threads (denoted as $T_1$ and $T_2$) running along the basic
workflow. When the 6th event of the trace (i.e. $B$) is being ver-
ified, the active states of $T_1$ and $T_2$ are $s_3$ and $s_2$ respectively,
and both threads cannot produce event $B$ from their active states (i.e. this
event $B$ is an un-interpretable event by the basic workflow.).
The reason why some events cannot be interpreted is that some
transitions are missing in the basic workflow. Specifically, event
$B$ is a part of the recurrence of the loop in Fig.2(a). However, we
do not have knowledge about the loop structure and the original
workflow. In order to interpret the first event trace, we now have
two possible solutions: the event is either generated by $T_1$ or gen-
nerated by $T_2$. If it is generated by $T_1$, then there is a loop from $s_3$
to $s_1$ in the workflow, which is the workflow in Fig.2(a). If the
event is generated by $T_2$, then there is a loop from $s_2$ to $s_1$ in the
workflow, which is the workflow in Fig.2(b). Similarly, when we
try to interpret the 8th event of the second event trace in Fig.2(b),
the active states of the threads are $s_3$ and $s_4$. One can interpret the
second event trace either by the workflow in Fig.2(a) or by that in
Fig.2(e). Here, we observe that, for both event traces, when we try
to interpret an event $B$ that is a part of the recurrence of the loop,
there is a thread at state $s_3$. In general, for any training event se-
quence with a different interleaving pattern, when we verify an
un-interpretable event of type $B$, which is a part of the recurrence
of the loop, there is at least one thread whose active state is $s_3$. On
the contrary, there is a thread with active states of $s_2$ or $s_4$ only by
chance. Therefore, if we vote for threads’ active states over all
event traces once we encounter an un-interpretable event, we will
find that $s_3$ has the highest vote value. Although the example in
Fig.2 is a simple case, this statistical property is widely valid for
workflows with loop or shortcut structures. This property can help
us to detect the loop structures. Similarly, we can also recover the
missing shortcuts.

(a). A simple workflow with a loop.

(b). Sample event traces of a two-thread program (a)

(c). The result workflow of Algorithm 2.

(d). A possible workflow of the first trace in (b).

(e). A possible workflow of the second trace in (b).

Figure 2. An example for the depiction of our refinement idea.

Unlike loop and shortcut structures, fork/join states do not expose
any unique statistical property. We cannot use the above statistical
method to identify a fork/join structure. If we perform the above
method forcibly on the event traces generated by a fork/join struc-
ture, then the resulting workflow is often very complex, which is
caused by various event interleaving patterns. On the other hand,
all event traces generated by a loop structure can always be inter-
preted by a fork/join structure. For example, all event traces pro-
duced by the workflow in Fig.2(a) can always be interpreted by the
workflow in Fig.2(f). Formally, the observation is described as:

**Property 5.1** Event traces that can be interpreted by a workflow $W_1$
with loop structures can also be interpreted by a workflow $W_2$
which is created based on $W_1$ by replacing the loop structures with
fork/join structures, and $cp(W_2) \leq cp(W_1)$, but not vice versa.

Here, $cp(W)$ is the complexity of a workflow $W$, which is de-
fined as the sum of transition number and the number of thread
types (note: a thread type means a pair of thread starting and end-
ing points.). Based on this property, we introduce a loop favorite
rule in our algorithm. If event traces can be interpreted by either a
workflow with a loop structure or a workflow with a fork/join
structure, and both of them have the same complexity, we prefer
the former. In our algorithm, we try to use the simplest workflow
with a minimal number of threads to interpret all event sequences.
In other words, for two workflows that can interpret the same
event traces, we prefer the workflow with less complexity. If two
workflows have the same complexity, we prefer the workflow that
interprets all event traces with a minimal thread number.

Because we have no information about when a new thread starts,
an un-interpretable event can be interpreted as an event log pro-
duced by either a missing workflow structure component (i.e. short-
cut or loop) or a newly started thread (i.e. fork state). In the
algorithm, we have to make a decision to select one structure be-
tween them (i.e. loop decision or fork decision) whenever a new
un-interpretable event is encountered. A workflow has a Markov
property that states that the current state is what determines the
next step in its behavior. Thus, an early decision will influence the
later decisions, but the converse is not true. At each decision point,
we first create two temporary workflows. One (denoted as $W_1$ in
algorithm) is constructed by a procedure in which we make a loop
decision at the current decision point and make fork decision at all
following decision points. The other (i.e. $W_2$) is constructed
through a procedure with all fork decisions. Then, we select a
decision at the current decision point based on the loop favorite
rule. Similarly, we also make the next decision with the same
procedure. Note: here, the temporary workflows are only con-
structed for decision making, and they are not output as the results
of the algorithm. The detailed algorithm is presented in APPEN-
dix A. In the algorithm, we do not count the active states with
their neighboring events having strict forward dependencies, be-
cause such a state neither has an out-shortcut transition nor is a
join state. For example, we do not have $B \rightarrow s_f C$ in Fig. 1(c) and
(f). The following theorem shows that the workflow learned by our
refinement algorithm is optimal in the sense of complexity
defined above.
Theorem 5.2. The refinement algorithm finds out the workflow with a minimal complexity to interpret all event traces.

Proof: According to property 5.1, a workflow created by a fork decision always has a value of complexity not larger than that of a workflow created by a loop decision at the same decision point. Therefore, the workflow constructed by making fork decision at all decision points has the minimal value of complexity among all workflows that can interpret all event traces. In the algorithm, we only make a loop decision when the decision does not increase the workflow’s complexity, thus, the resulting workflow will have the minimal complexity. □

6 EMPIRICAL EVALUATION

To validate and evaluate our proposed workflow algorithm, we performed a set of experiments on simulated event traces and case studies on real event traces generated by some open source programs (Hadoop and JBoss). We use open source programs because they are publicly available for download. The results on Hadoop and JBoss are easy to be verified and reproduced by third parties. The results demonstrate the usefulness of our workflow mining technique in recovering the underlying procedure that the system carries out, thus aid program comprehension. The simulator and the code of our algorithm will be available soon at http://research.microsoft.com/apps/pubs/default.aspx?id=118640 (currently under the code post review of LCA.).

6.1 SIMULATION

To construct a controlled experimental environment, we designed a simulator that can generate synthetic program event traces according to a software workflow model. The design of the simulator follows the principles proposed in QUARK [22] including the guarantee of “code and branch coverage” and locality of reference, and so on. Unlike QUARK, our simulator can generate traces of interleaved events based on a concurrent workflow model. In this experiment, we measure the performance of our workflow miner by discovering workflows from the synthetic program event traces.

Simulation Models: In our simulation experiments, several real application models are used to generate the event traces, which include (1) the IBM® WebSphere® Business Integration processes from WebSphere® Commerce provided in [24], (2) a workflow process of reviewing a conference paper similar to that provided in [17]. The models are shown in Fig. 3 and Fig. 4, and are referred to as WS(a), WS(b), and Rev respectively. In [24], Zou et al. presented two workflows in the form of automata: the release of expired allocations (Fig. 3(a)) and the processing of backorders (Fig. 3(b)). In [17], the authors use a paper reviewing process (Fig.4) to demonstrate their workflow mining algorithm. By using these models, users can evaluate and compare our algorithm with other algorithms in [17] [24]. On the other hand, these typical real application workflows are complex enough to demonstrate the capability of our algorithm: the models in Fig. 3 contain several loops and many shortcut transitions, and the model in Fig. 4 has a loop embraced by a fork/join structure. We run these models with several threads (we randomly start 1-3 threads) in our simulator to generate traces of interleaved events.

Evaluation Metric: In order to carry out a quantitative evaluation of the workflow miner, we adopt two metrics to measure the similarity from the mined workflow X and the simulator model Y in terms of their generated traces. The first metric is known as recall, the percentage of event traces produced by workflow X that can be interpreted by workflow Y.

<table>
<thead>
<tr>
<th>Simulation Models</th>
<th>WS(a)</th>
<th>WS(b)</th>
<th>Rev</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-Learner (k=1)</td>
<td>0.511</td>
<td>1.000</td>
<td>0.069</td>
</tr>
<tr>
<td>k-Learner (k=2)</td>
<td>0.255</td>
<td>1.000</td>
<td>0.080</td>
</tr>
<tr>
<td>Our Algorithm</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Results: From these generated traces (2000 event traces for each case), we learn workflow models through the algorithm provided in the above sections. We compare the effectiveness of a k-Learner algorithm [6] and our algorithm by measuring the precision and recall of the resulting state machines. We repeat each experiment 10 times with 10 different set of traces, and computing the average that shown in Table 2. Here, we round the results to keep three numbers after the decimal point. For these three workflows, our algorithm can exactly rediscover the original workflow model, thus, both the recall and precision are 100%. On the other hand, the precisions of the models produced with k-Learner are very poor (2 models have a precision less than 0.1). This indicates that k-Learner cannot perform well when events are interleaved.

Computational cost: Our algorithm is efficient, which only uses 9.9, 22.3 and 72.0 seconds (with a CPU of 2.33GHz, the code is not fully optimized) to learn the models of WS(a), WS(b), and Rev from event traces (2000 event traces for each respectively).

Figure 3. WebSphere® Commerce Processes.

A1: Find State Order Items; A2: Verify State Order Items; A3: Is Using ATP; A4: Deallocate Existing Inventory Cmd; A5: Deallocate Expected Inventory Cmd.

B1: Find Locked Orders with StatusB; B2: Verify Locked Orders with StatusB; B3: Allocate Inventory Cmd; B4: Reprepare Order Cmd; B5: Process Order Cmd

(a) The release of expired allocations.

(b) The processing of backorders.
6.2 CASE STUDY: HADOOP

Hadoop [23] (our version: 0.19) is a well-known open-source implementation of Google’s Map-Reduce computing framework and distributed file system (HDFS). It enables distributed computing of large scale, data-intensive and stage-based parallel applications. JobTracker acts as a task scheduler that decomposes a job into smaller tasks and assigns the tasks to different TaskTrackers. The logs produced by Hadoop are not sequential log message sequences. Even in the log messages of the same Map task, some messages (e.g. messages about data shuffling) are also interleaving.

We run different Hadoop jobs of some sample applications, such as WordCount and Sort, and collect log data after each job is finished (Note: we enable the logging tool at the info level). At first, we use the log preprocessing method presented in [20, 25] to parse event log messages and to group log messages according to log parameters. For example, all log messages that contain a parameter called MapTask ID are grouped into an event sequence. Because several MapTasks are running simultaneously, these events are highly interleaved with each other. Then, we use the error detection algorithm in [20] to filter out event traces that contain errors. After that, we learn workflow from event traces with our proposed algorithm. Fig. 5 is an example of the resulting workflow that is learned from the event traces related to the parameter MapTask ID. By carefully checking with Hadoop source code and documents, we find that the workflow can reflect the real process of MapTask with very high precision. A task is launched and then it processes the input data read from HDFS. After the task is done, it sends the resulting data to many Reducers concurrently (This is represented by a fork/join structure in the workflow), and finally cleans up the resources by removing temporary files. During the running, the task may be killed by the scheduler. Some killed tasks can also report their status due to thread race conditions, therefore, events of $H_{10}$, $H_{13}$ and $H_{14}$ are triggered.

6.3 CASE STUDY: JBOSS APP. SERVER

Another case study is performed on an application called JBoss Application Server (JBoss AS) [26]. JBoss AS is a commonly used open source J2EE application server. This case study demonstrates the usefulness of our workflow mining on discovering behavior of a transaction procedure of JBoss AS. We develop a prototype of online book store, through which people can buy books by submitting an order over a website. We use jBoss jBPM to design a business procedure to handle these orders. JBoss jBPM is a flexible, extensible process design framework for SMB and large enterprise applications. The workflow repeatedly checks whether there exists an unordered pair of events. For each order in the queue, we will spawn a new order handling process to handle it. The order handling process will wait till the pay of the order has been received. Then it progresses to check if there are enough books in the storehouse. If there aren’t, we must get a supply from the publishing house as soon as possible. After the checking procedure, the books are delivered to corresponding client. When the deliver is completed, the order handling process goes to the end.

We run the program and invite the visiting students and our colleagues to test the system. At the same time, we record the event logs to form event traces. We collect 200 event traces for our experiment. The mined workflow is shown in Fig. 6, which can correctly reflect the process flow of our program. Note the end state is also a join state in this workflow.

6.4 DISCUSSION

As with the results of all other workflow mining algorithms [2][17][22], some resulting workflow models of our algorithm are over-generalized (i.e. having more possible routes than the real workflow). For example, in the case in section 6.3, there is a path from $s_{10}$ to $s_{14}$, which does not exist in the real program. There are two main reasons for the over-generalization problem. At first, we only consider the first-order of event dependencies in our current algorithm, i.e. the dependencies of neighboring events. In some real programs, there are some high-order dependencies, e.g. the occurrences of $H_{10}$, $H_{13}$ and $H_{14}$ depend on the occurrence of $H_{7}$ in Fig. 5. Second, our approach assumes that there is at most one transition referring to the same event type in a workflow model. For a workflow in which there are several transitions labeled as the same event type, our current algorithm will over-generalize the resulting workflow. For example, from the event traces generated by the workflow in Fig.7(a) (This is the workflow of XI1 [19]), we learned a workflow in Fig.7(b) which more general than the original one. The workflow in Fig.7(b) can generate event traces such as $<B,E,...,E>$ and $<A,D,E,...,E>$ that cannot be generated by the original workflow Fig.7(a). We will leave it for future work to deal with these problems.
7 CONCLUSION

Most existing techniques for mining program workflow models can only learn models from sequential event traces. They cannot be applied to interleaved logs which are prevalent in distributed or parallel programs (or some event driven programs). In this paper, we proposed an approach to automatically discover program execution workflows from traces of interleaved events. We extend the traditional state machine to support concurrency by introducing split/merge states and fork/join states. Our mining approach is based on the statistical inference of temporal dependency relations from traces of interleaved events. We then use such dependency relations to construct a basic workflow by building the connections among neighboring events. After that, we further refine the workflow by validating it with event traces. During the validation procedure, we add the shortcut transitions, loop structure, and fork/join states into the workflow model to make sure that all event traces can be interpreted by the workflow model. To the best of our knowledge, the paper is the first work that learns the workflow from interleaved logs produced by concurrent programs. The experimental results on both simulated event traces and real program traces demonstrate that our approach can learn the workflow with a high precision.

Although our work is motivated by the purpose of software comprehension, workflow mining is a basic research topic that has a wide range of application fields other than software engineering. We believe our approach can be widely applied in many applications, such as business intelligence. Future research directions include integrating high order temporal dependencies, incorporating domain or existing knowledge about a program, allowing for a workflow model having an event type at multiple points.

8 ACKNOWLEDGMENTS

We thank David Lo from Singapore Management University for providing the executable file of his QUAK simulation tool.

9 REFERENCES


10 APPENDIX A

In this appendix, we present the detail of our refinement algorithm. Supposing that there are a set of threads \( \xi \) running in the program and each thread \( \xi_i \) runs along an instance \( W_i \) of the program's workflow, we denote the current state of \( W_i \) as \( q_i \). For every event type \( a \), and state \( q \), we define two integer values \( N[a] \) and \( C[a, q] \). Here, \( N[a] \) denotes the number of times that event type \( a \) in the training event traces cannot be inferred by the workflow instances in \( \xi \) during refinement. \( C[a, q] \) records the number of workflow instances whose current state is \( q \) when the input event \( a \) cannot be inferred by the workflow instances. In addition, we denote the current input event as \( l \) and the input sequence of events as \( L \). \( f_1 \) and \( f_2 \) are two flags used to indicate whether current workflow model \( W \) should be refined by a loop decision or a fork decision. Their initial values are both false. \( W_1 \) and \( W_2 \) are two workflow models created by a procedure that we make a loop decision and a fork decision at the current decision point respectively, and all following decisions are fork decisions. The whole refinement process contains the following steps.

**Step 0.** Let \( C[a, q] = 0, K[q] = \{ q \text{ is an empty set} \} \), and \( N[a] = 0 \) for any \( a \in (\Sigma \setminus a) \) and \( q \in S \); Set \( \xi = \{ q \text{ is an empty set} \} \), and \( p_{ass} = \text{true} \); 

**Step 1.** For an input event trace \( T \), we use the method presented in Appendix B to interpret the trace. Once there is an event \( t \) in \( L \) that cannot interpreted, we increase \( N[\xi] \) by 1, and set \( p_{ass} = \text{false} \). At the same time, for each workflow instance \( W_k \) in \( \xi \), we increase \( C[t, q_k] \) by 1 where \( q_k \) is the current state of \( W_k \).

**Step 2.** For each event trace in the training set, we carry out the process of **Step 1**.

**Step 3.** For each event \( a \) with a non-zero value of \( N[a] \), we find out a state \( q \) that satisfies \( C[a, q] = \max_{q_i \in S}(C[a, q_i]) \), and add \( a \) to \( K[q] \).

**Step 4.** If \( p_{ass} = \text{false} \), find an arbitrary element \( q \in \text{Find Root}(\xi) \) (refer to section 5.1), and find event \( a \) that does not have any predecessor in \( K[q] \), then goto **Step 5**. Otherwise, goto **Step 6**.

**Step 5.** We denote \( q' \) as the preceding state of \( a \) in the current workflow model \( W \). If \( f_1 = \text{false} \), we set \( W = W \), \( f_1 = \text{true} \), and update \( W \) by adding a shortcut transition from \( q \) to \( q' \) and update \( W \) by setting \( q' \) as a fork state. After that, back to **Step 0**.

**Step 6.** If \( p_{ass} = \text{True} \) and \( f_1 = \text{false} \), we mark all join states and terminate the execution of the algorithm. Otherwise, if \( f_2 = \text{false} \), we set \( W_1 = W \) and goto **Step 7**; else \( W_2 = W \) and go to **Step 8**.

**Step 7.** Set \( W = W' \); \( f_2 = \text{true} \), and then back to **Step 0**.

**Step 8.** Set \( f_1 = f_2 = \text{false} \) and \( W = W' \). If \( cp(W_1) \leq cp(W_2) \), we update \( W \) by adding a shortcut transition from \( q \) to \( q' \) else update \( W \) by setting \( q' \) as a fork state. After that, back to **Step 0**.

11 APPENDIX B

In the refinement procedure, we use the current workflow model to interpret the event traces. Here, interpretation means that, given a workflow model and an event trace, we try to explain how the events in the trace are generated one by one through running a set of threads of the workflow. More specifically, each thread has an active state at any point of time. We need to update the threads' active states according to the workflow so as to interpret the event trace. This is a non-trivial problem when there are multiple concurrent threads. The complexity arises from that there may be several different choices to update the active states of the threads when we try to generate the same event. Furthermore, due to the Markov property of a workflow (refer to section 5.2), the current choice will influence the interpretations of following events in the same trace. In the paper, we adopt a dynamic programming based algorithm to find the optimal interpretation choice which can interpret the events in the event trace as long as possible. In details, we define an active state vector to represent the active states of all the threads, and maintain a valid state set in which each element is an active state vector. The valid state set is initialized as empty, and is updated as the interpretation of the event in the trace one by one. For a new input event, we remove the active state vectors from the valid state set, which can’t generate the new event. Then, we update the active state vectors in the valid state set according to the workflow model so as to generate the event. Especially, when an active state vector can generate the event by different ways, we replace the vector by adding multiple state vectors according to different choices respectively. Sometime, there may be some updated vectors are equivalent, i.e. for each state \( q \), there are the same number of threads with active state \( q \). For multiple equivalent active state vectors, we only keep one in the valid state set. The pseudo code of the algorithm is presented in Algorithm 3.

In Algorithm 3, the total number of state in the workflow is denoted as \( SN \), and the \( i \)-th state is denoted as \( S_i; 1 \leq i \leq SN \). We denote an active state vector as \( SV \), which is an \( SN \) dimension vector. The \( i \)-th element in \( SV \), i.e. \( SV[i] \), represents the number of threads with active state of \( i \). The valid state set is denoted as \( SS \).

**Algorithm 3. Pseudo Code of Event Trace Interpretation**

**Inputs:**
1. \( l \): an event trace
2. \( T \): the workflow model

**Output:**
1. \( \hat{\epsilon} \): the first event that can’t be interpreted
2. \( SS \): the set of active state vectors that can interpret until \( \hat{\epsilon} \)
3. \( SV^0 = [0, .., 0]; SS = SV; SS' = SS \)
4. for each event \( e \) in \( l \)
5. \( SS' = SS \)
6. for each active state vector \( SV \) in \( SS \)
7. \( SS' = SS' - SV \)
8. for \( i = 1 \) to \( SN \)
9. if \( SV[i] > 0 \)
for $j = 1$ to $SN$
for each path $p$ from $S_i$ to $S_j$
  if $p$ generates $e$
    $SV' = SV$;
    $SV'[j] = SV'[j] + 1$;
    if $p$ is NOT forked or spitted
      $SV'[t] = SV'[t] - 1$;
    endif
    $SS' = SS' \cup SV'$;
  endif
endfor
endfor

if $SS' = \emptyset$
  $e = e$;
  $SS = SS'$;
  break;
endif

if $SS' \neq \emptyset$
  $e = null$;
  $SS = SS'$;
endif

return $e, SS$;