Dimensioning Playout Buffers from an ATM Network

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The interaction of traffic streams within an ATM network raises several interesting questions concerning the size of buffers at various positions within the network. Even an initially deterministic input stream of cells will be perturbed by cross traffic, producing cell delay variation that varies across the network. In this paper we particularly focus on the dimensioning of playout buffers, and investigate how earlier work of van den Berg and Resing [5] and of D'Ambrosio and Melen [1] can be combined with classical bounds for the GI/G/1 queue to give insight into the output buffer required if a smooth playout of cells is to be achieved. We investigate the effect of both the number of stages across the network and the rate of the stream. A possibly surprising conclusion is that as the rate of the stream decreases the required output buffer size may actually increase. While the cell delay variation of small rate streams may cause no problems for buffers within the network, the cell delay variation introduced by the network may cause problems for the customer on output.

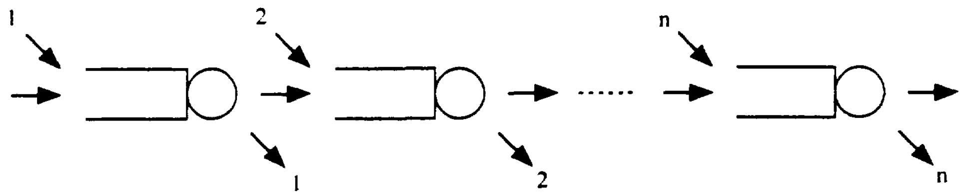

Figure 1. A series of queues

1. A series of heavily loaded $\cdot + M/D/1$ queues

Consider a series of FIFO queues, as illustrated in Figure 1. Each queue handles an independent Poisson cross-stream of rate λ, together with a reference flow of rate d^{-1} passing through each queue in turn. We suppose the common traffic intensity $\rho = \lambda + 1/d$ approaches 1, so that all queues are in heavy traffic.

Let D_n be the inter-exit time between two given reference cells from queue n. The important insight of van den Berg and Resing [5] is that in the heavy traffic limit $(D_n, n = 0, 1, \ldots)$ is a Markov chain, and a Markov chain which is independent of the inter-exit times between other pairs of successive reference cells. As shown by van den Berg and Resing [5], this allows straightforward calculation of many quantities of interest.

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Observe that D_n is distributed as $1 + P(\lambda D_{n-1})$, where $P(\mu)$ is a Poisson distribution of mean μ and $\lambda = 1 - 1/d$. Let $h_n(z)$ be the probability generating function of D_n. Then

$$\begin{aligned} h_n(z) &= \mathsf{E}[z^{D_n}] \\ &= \mathsf{E}\mathsf{E}[z^{D_n} \mid D_{n-1}] = \mathsf{E}[zg(z)^{D_{n-1}}] \end{aligned}$$

where

$$g(z) = e^{\frac{d-1}{d}(z-1)}, \tag{1}$$

the probability generating function of a Poisson random variable with mean $\lambda = (d-1)/d$. Thus

$$h_n(z) = zh_{n-1}(g(z)). \tag{2}$$

From this recurrence we can obtain recurrences for the mean and variance of D_n. Thus

$$\begin{aligned} h'_{n+1}(z) &= h_n(g(z)) + z'h'_n(g(z))g'(z) \\ h''_{n+1}(z) &= 2h'_n(g(z))g'(z) + zh'_n(g(z))g''(z) + zh''_n(g(z))g'(z)^2 . \end{aligned}$$

Hence

$$h'_{n+1}(z)\mid_{z=1} = 1 + h'_n(1)g'(1)$$

and so

$$\mathsf{E}D_{n+1} = 1 + \frac{d-1}{d}\mathsf{E}D_n . \tag{3}$$

Also

$$h''_{n+1}(z)\mid_{z=1} = 2h'_n(1)g'(1) + h'_n(1)g''(1) + h''_n(1)g'(1)^2,$$

and so, after some further manipulation,

$$\mathrm{Var}(D_{n+1}) = \left(\frac{d-1}{d}\right)^2 \mathrm{Var}(D_n) + \frac{d-1}{d}\mathsf{E}D_n . \tag{4}$$

From (3) and (4) we see that the mean and variance of D_n converge geometrically as n increases to limit values of

$$\mathsf{E}D_\infty = d \tag{5}$$

$$\mathrm{Var}(D_\infty) = \frac{d^2(d-1)}{2d-1} . \tag{6}$$

For example, if the reference flow is initially periodic (with $\mathrm{Var}(D_0 = 0)$), then

$$\mathrm{Var}(D_n) = \frac{d^2(d-1)}{2d-1}\left(1 - \left(\frac{d-1}{d}\right)^{2n}\right) .$$

Note that if d is large the convergence is slow, as illustrated in Figure 2.

After an infinite series of heavily loaded queues, the output reference stream will be a renewal process, with interarrival time variance $d^2(d-1)/(2d-1)$, *whatever* the input reference stream. The reference stream may become more or less bursty as it passes through the series, depending on whether it started out more or less bursty. The limit renewal process is more regular than a Poisson process: in particular its interarrival times have variance $d^2(d-1)/(2d-1)$ rather than d^2, about half the variance. If we consider a large window of size τ, the number of arrivals in the window will have approximately a normal distribution

$$N\left(\frac{\tau}{\mu}, \frac{\tau\sigma^2}{\mu^3}\right) = N\left(\frac{\tau}{d}, \frac{\tau(d-1)}{d(2d-1)}\right) . \tag{7}$$

(Here μ, σ^2 are the mean and variance of the interarrival time of a general renewal stream, and the left hand side of equation (7) is the central limit for the number of arrivals of a general renewal stream in a large window.) Thus the variance of the number of arrivals in the window is again about half the variance for a Poisson process.

2. The large deviation bound

Consider the limit renewal process, after a reference stream has passed through an infinite series of heavily loaded queues. The probability generating function $h(z)$ of the interarrival times is, from (5), given by the implicit relation

$$h(z) = zh(g(z))$$

where g is given by equation (1) and d^{-1} is the rate of the reference stream. How large should a buffer be in order to smooth this arrival process? To address this question we first recall the estimates and bounds for the tail behaviour of a GI/G/1 queue provided by Kingman [3] and Ross [4]. If A is a random variable with the interarrival time distribution, X a random variable with the service time distribution, and κ a positive constant such that

$$\mathsf{E}(e^{\kappa X})\mathsf{E}(e^{-\kappa A}) = 1 \tag{8}$$

then the stationary distribution of W, the unfinished work found by an arriving customer, satisfies

$$a_1 e^{-\kappa w} \leq \mathsf{P}\{W > w\} \leq a_2 e^{-\kappa w} \quad w \geq 0$$

for constants $a_1, a_2 \leq 1$.

n	$d = 2$	$d = 5$	$d = 10$	$d = 15$	$d = 50$	$d = 100$
1	103	42	21	14	5	3
2	104	43	22	15	5	3
5	104	44	24	16	5	3
10	104	45	24	17	6	3
∞	104	45	25	18	8	6

Table 1. Bucket depth bounds ($\gamma = 20, w = 200$)

Suppose the buffer is modelled by a leaky bucket of depth b and output rate a^{-1}; what conditions on a and b must be imposed? Using the above bound we find that to ensure

$$\mathsf{P}\{W > w\} \leq e^{-\gamma}$$

it is sufficient that

$$a = -\frac{w}{\gamma} \log h(e^{-\gamma/w}). \qquad (9)$$

For given γ and w this equation fixes a, and then $b = w/a$. The last line of Table 1 shows the values of b when $\gamma = 20$ and $w = 200$, corresponding to a fixed cell delay variation [6]. Other choices are, of course, possible: if $\gamma = 20$ and $a = 0.9d$ then values of b are as in the last line of Table 2. Earlier lines of both Tables are calculated with h replaced by h_n in equation (9), where $h_0(z) = z^d$ (h_n thus corresponds to the renewal stream that results after an initially periodic reference stream is passed through n heavily loaded queues). Figure 3 shows how the ratio a/d decreases as d increases when $\gamma = 20$ and $w = 200$. This ratio can be interpreted as a measure of efficiency, whilst its reciprocal d/a gives the required speed-up of the leaky bucket.

n	$d = 2$	$d = 5$	$d = 10$	$d = 15$	$d = 50$	$d = 100$
1	24	17	10	7	3	2
2	30	27	17	12	4	2
3	31	33	23	17	6	3
4	31	37	27	21	8	4
5	31	39	31	24	10	5
6	31	40	34	27	11	6
7	31	41	36	30	13	7
8	31	42	38	32	14	8
9	31	42	40	34	16	9
10	31	43	41	36	17	10
∞	31	43	46	47	48	49

Table 2: Bucket depth bounds on output from a network

These results have consequences for the buffer necessary on output from the network if a smooth playout of cells is to be achieved. Table 1, illustrated in Figure 4, corresponding to a fixed cell delay variation on output, shows buffer size related in a natural way to the rate of the stream. However Table 2, illustrated in Figure 5, corresponding to a fixed speed up, shows that as the rate of the stream decreases the required output buffer size may need to *increase* if a smooth playout of cells is to be achieved. The situation is worse (larger buffers will be necessary) if the output stream is Bernoulli (see Section 3.2). Then equation (8) becomes

$$e^{\kappa a} \frac{pe^{-\kappa}}{1 - e^{-\kappa}(1-p)} = 1$$

where $p = d^{-1}$. This becomes

$$a = 1 + \frac{1}{\kappa} \log[d - e^{-\kappa}(d-1)]. \qquad (10)$$

To lose less than one in e^{γ} cells corresponds to the choice $\kappa = \gamma/(ab)$.

Note that equations (9) and (10) give the essential relationship between the various parameters a, b, d, γ and w. Another natural use of the relationship might be to bound the cell delay variation, $w = ab$, when the output is smoothed through a leaky bucket of rate a^{-1} and depth b cells. Figure 6 graphs the cell delay variation w for Table 2, where $\gamma = 20$ and $a = 0.9d$.

If it is required that the playout buffer have $a = d$ then it becomes reasonable to ask for the period until a playout buffer, initially occupied to level L, either overflows (reaches a level $L + K$) or empties (reaches 0). Chernoff's approximation allows us to construct estimates of the associated probabilities. The logarithm of the probability that the buffer overflows after N cells is approximately

$$\log \mathsf{P}\left\{ \sum_{i=1}^{N}(X_i - d) > K \right\} \simeq \inf_s [N \log h_n(e^s) - (Nd + K)s].$$

The logarithm of the probability that the playout buffer empties after N cells is approximately

$$\log \mathsf{P}\left\{ \sum_{i=1}^{N}(X_i - d) < -L \right\} \simeq \inf_s [N \log h_n(e^{-s}) + (Nd - L)s].$$

3. Variants

3.1 Batch traffic

Suppose both the reference flow and cross traffic in the model of Figure 1 is Poisson batch traffic, where the batch sizes are independent and identically distributed, with probability generating function $f(z)$. Then

$$h_n(z) = \mathsf{E}[z^{D_n}] = f(z) h_{n-1}(e^{-\lambda(1-f(z))}).$$

In particular

$$\mathsf{E}D_{n+1} = \mu + \mu\lambda \mathsf{E}D_n \tag{11}$$

where μ is the mean batch size. For the traffic intensity to approach one we require $\mathsf{E}D_n = \mathsf{E}D_{n+1}$, and so

$$d = \mathsf{E}D = \frac{\mu}{1 - \lambda\mu}.$$

Also

$$\mathrm{Var}(D_{n+1}) = \sigma^2 + \lambda\sigma^2 \mathsf{E}D_n + \mu^2 \mathrm{Var}(W)$$

where σ^2 is the variance of the batch size and W is Poisson random variable with (random) mean λD_n. Thus

$$\begin{aligned} \mathrm{Var}(W) &= \mathsf{E}[\mathrm{Var}(W|D_n)] + \mathrm{Var}[\mathsf{E}(W|D_n)] \\ &= \mathsf{E}[\lambda D_n] + \mathrm{Var}[\lambda D_n] = \lambda \mathsf{E}D_n + \lambda^2 \mathrm{Var} D_n . \end{aligned}$$

Hence

$$\mathrm{Var}(D_{n+1}) = \sigma^2 + \lambda(\sigma^2 + \mu^2)\mathsf{E}D_n + \lambda^2\mu^2 \mathrm{Var}(D_n). \tag{12}$$

From (11) and (12) we see that the mean and variance of D_n converge geometrically as n increases to limit values of

$$\mathsf{E}D_\infty = d$$
$$\mathrm{Var}(D_\infty) = \frac{d^2(d(\sigma^2+\mu^2)-\mu^3)}{\mu^2(2d-\mu)},$$

agreeing with (5) and (6) when $(\mu, \sigma^2) = (1, 0)$.

3.2 Light cross traffic

In [1] and [5] interesting numerical results are given for cases where the queues are not heavily loaded. Asymptotic approaches are also available for various limiting regimes. For a simple example, suppose that the number of queues, n, is fixed, and that the period, d, of the reference stream increases. In the limit the sojourns in the system of different cells from the reference stream are independent.

As a further example, suppose that rate of the Poisson cross traffic is approaching zero and suppose that the queues are synchronized so that services start at integer time points. Then the reference flow is perturbed on passing through the sequence of queues in a manner exactly corresponding to an infinite sequence of a certain form of quasi-reversible queue [2]. The queue is a single server exponential queue, with no additional waiting room, and a customer arriving to find the server occupied moves straight through to the next queue in the sequence ([2], Fig 4.14, p. 119). The stationary distribution for such a sequence is of product form. Thus we can deduce that after an *infinite* series of queues lightly loaded with Poisson cross traffic the reference flow will converge to a Bernoulli flow of the same mean rate (d^{-1}, say). Thus in a window of size τ the number of arrivals will have the binomial distribution $B(\tau, d^{-1})$, approximately the normal distribution

$$N\left(\frac{\tau}{d}, \tau\frac{d-1}{d^2}\right).$$

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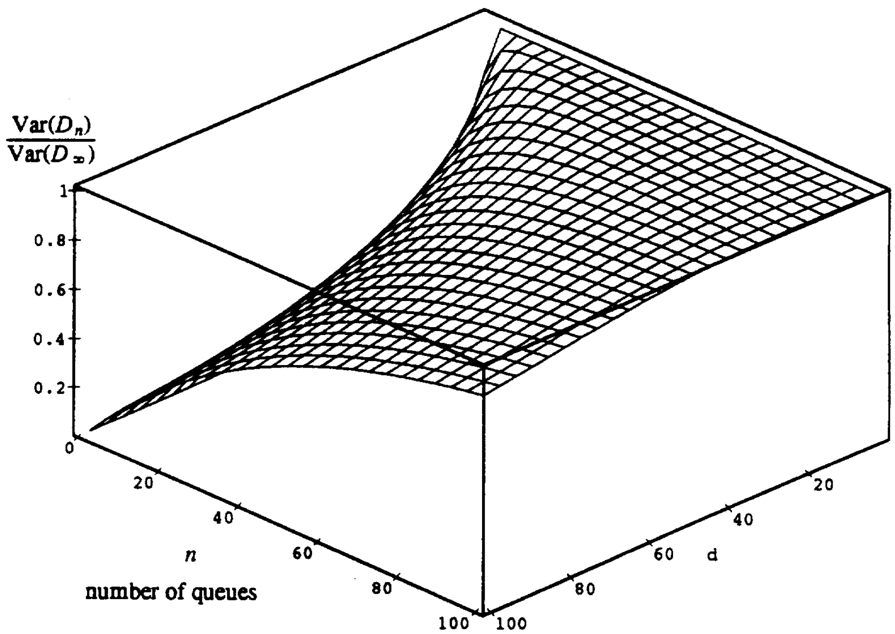

Figure 2: Convergence of Variance of Interexit Times

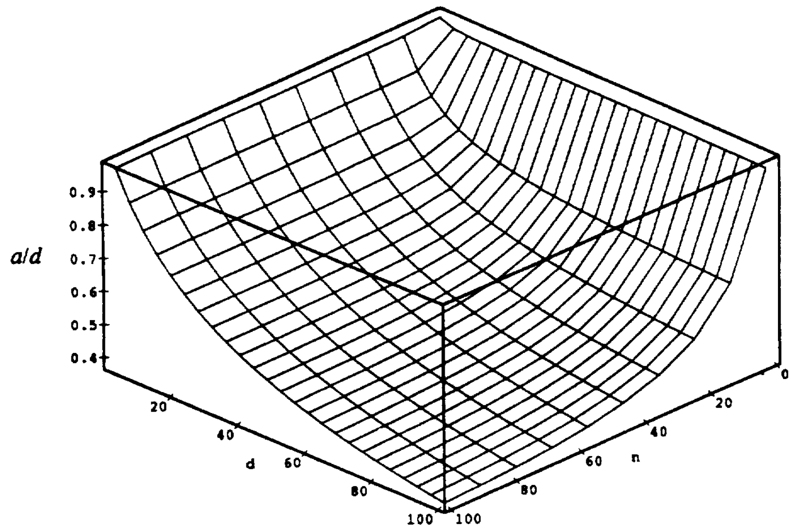

Figure 3. Efficiency of leaky bucket (a/d) when γ=20, w=200.

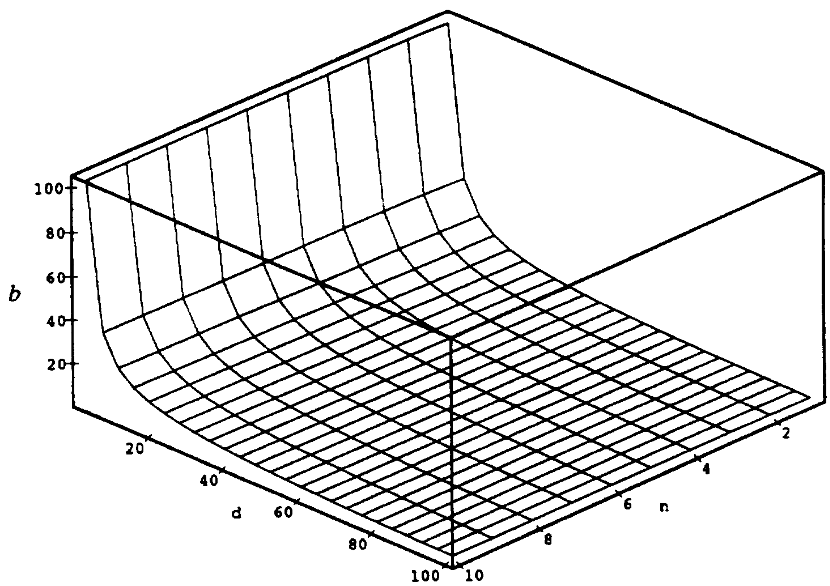

Figure 4. Bucket depth b when γ=20, w=200.

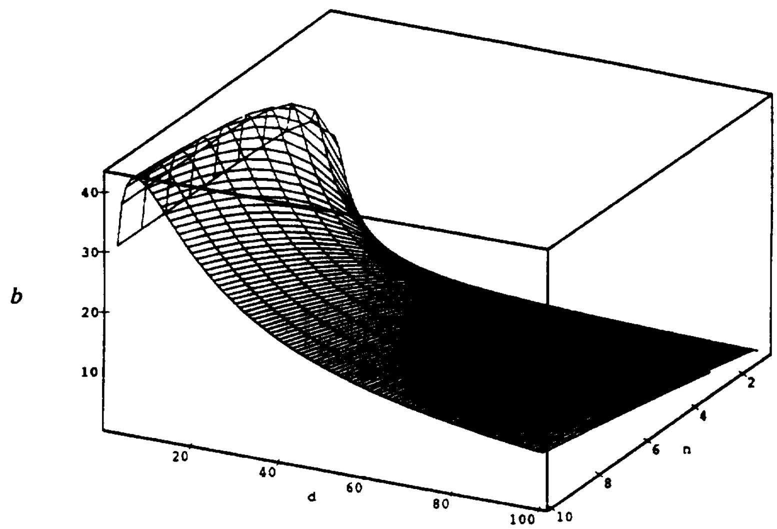

Figure 5. Bucket depth b for fixed speed-up when a=0.9d, γ=20.

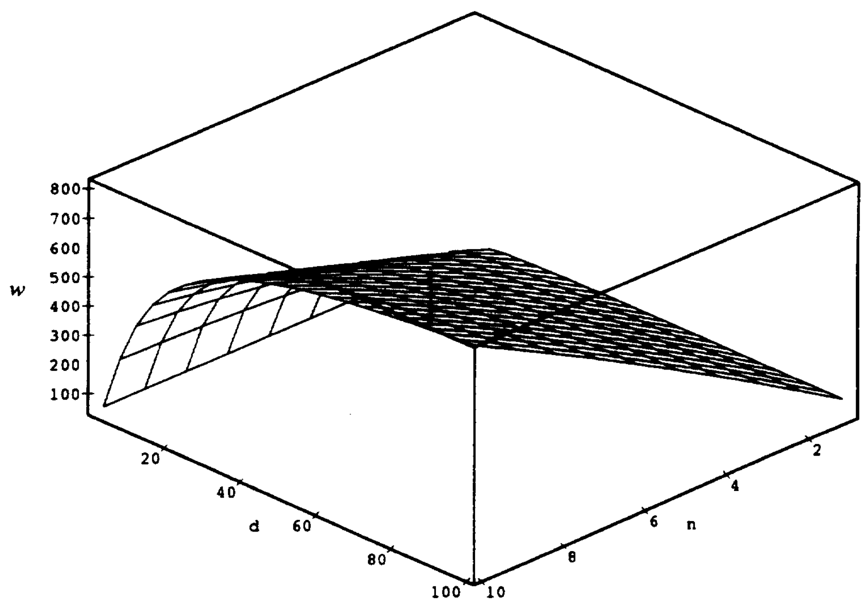

Figure 6. Cell Delay Variation w when a=0.9d, γ=20.