Relaxed Currency Serializability for Middle-Tier Caching and Replication (extended version)

Philip A. Bernstein, Microsoft Corporation
Alan Fekete, University of Sydney
Hongfei Guo, Microsoft Corporation
Raghu Ramakrishnan, University of Wisconsin
Pradeep Tamma, University of Wisconsin

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Microsoft Corporation
One Microsoft Way
Redmond, WA 98052
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Philip A. Bernstein
Microsoft Corporation
philbe@microsoft.com

Alan Fekete
University of Sydney
fekete@it.usyd.edu.au

Hongfei Guo
Microsoft Corporation
hongfei@microsoft.com

Raghu Ramakrishnan
University of Wisconsin-Madison
{raghu,pradeep}@cs.wisc.edu

Pradeep Tamma

ABSTRACT

Many applications, such as e-commerce, routinely use copies of data that are not in sync with the database due to heuristic caching strategies used to enhance performance. We study concurrency control for a transactional model that allows update transactions to read out-of-date copies. Each read operation carries a “freshness constraint” that specifies how fresh a copy must be in order to be read. We offer a definition of correctness for this model and present algorithms to ensure several of the most interesting freshness constraints. We outline a serializability-theoretic correctness proof and present the results of a detailed performance study.

This is an extended version of a paper with the same title that appeared in SIGMOD 2006.¹

1 INTRODUCTION

The use of copies is widespread in e-commerce sites. For example, consider an auction site: when browsing auctions in a category, it is apparent that the data (e.g., item prices, number of bids) is a little out of date. However, most users understand and accept this, as long as they see completely current data when they click on an individual item. As another example, a purchase transaction may calculate and report to the user an estimated arrival date for the goods, based on reading a log of shipment timings at the cache. Reading slightly stale information might be acceptable given the business requirements, but it is essential to record this promised arrival date at the centralized backend server.

As a concrete example, the following query returns a summary of books that have the specified title:

```sql
SELECT I.ID, I.COST, I.SUBJECT, I.ISBN,
      I.INSTOCK, A.FNAME, A.LNAME
FROM ITEM I, AUTHOR A
WHERE I.AID = A.AID AND I.TITLE = "SQL"
```

Different users might have different freshness requirements for this query. For example, user $U_1$ is about to purchase all of these books and calls transaction $T_1$ to get a current price result. User $U_2$ is browsing and runs transaction $T_2$, which offers the associated freshness constraints. Our protocols accumulate constraints on the updater’s time, which can be checked at read-time or as part of commit processing.

We investigate concurrency control for a transactional model that allows an update transaction to read out-of-date copies. Each read operation carries a “freshness constraint” [12] that bounds how up-to-date a copy must be in order to be read. Up-to-date-ness may be relative to read-time, reader’s commit-time, or freshness of copies of other items that are read. Our main contributions are:

- An extended serializability model called Relaxed Currency (or RC-) Serializability that accounts for out-of-date reads that are justified by user-specified freshness constraints.
- Concurrency control protocols that allow update transactions to read out-of-date data safely, that is, read data that satisfies the associated freshness constraints. Our protocols accumulate constraints on the updater’s time, which can be checked at read-time or as part of commit processing.
- A proof sketch that the protocols produce RC-serializable executions.
- A detailed simulation study of the protocols. It shows that performance is similar to the weakest protocol used in practice, read-committed, which offers no freshness guarantee.

The paper is organized as follows. In Section 2 we introduce our model of correctness for interleaved executions of transactions that read cached and possibly out-of-date copies. We define our models of a replicated database and transactions that can read out-

¹ The extensions are Sections 7.7, 7.8 and Appendix A.
of-date copies in Section 3, and several classes of freshness constraints in Section 4. We present several concurrency control algorithms to ensure RC-serializability in Section 5 and prove their correctness in Section 6. We present a performance study in Section 7 and discuss implementation extensions in Section 8. Section 9 covers related work. Section 10 is the conclusion.

2 RC SERIALIZABILITY

We define a new user model of computation where transactions may update data yet read cached copies, subject to various freshness constraints. We also define implementations that realize it. To relate the user model and implementation, we use serializability theory [6]. However, all forms of correctness stated in that theory require each transaction to read values as in a serial one-copy execution, which does not hold if an update transaction reads slightly stale replicas. We need a weaker correctness property that captures our intuition of what a transaction does when it reads stale values.

As in traditional serializability theory, we define the correctness of an implemented physical system by saying that its behavior must be indistinguishable from that of a simpler, ideal system (i.e., the user’s model) where transactions execute serially. In the ideal system, we model a database as a collection of data items. Since the freshness constraints are part of the user’s model, the user is aware that transactions read out-of-date values of data items. Such out-of-date values are called versions of a data item. Thus, in the ideal system a data item is a sequence of versions and a computation is a multiversion execution. That is, the user must understand that (i) each time a transaction updates a data item a new version of that data item is (conceptually) created, and (ii) when a transaction reads a data item, it may read an old version of that item, not the most recent one. Each read operation’s freshness constraint tells whether a version is satisfactory.

We therefore define the ideal system to be a one-copy multiversion database. By one-copy, we mean there is a multiversioned master copy of each data item and no replicas (i.e., cached copies). We define a correct execution of the ideal system to be a serial multiversion execution in which each read operation satisfies its freshness constraint. A correct execution of a physical system is one that is “equivalent” to a correct execution of the ideal system. A correct execution of a physical system is called relaxed-currency (RC) serializable.

To our knowledge, this is the first time a multiversion database has been taken to be the user’s model of computation. By contrast, the well-known correctness definition for multiversion and replicated databases is one-copy serializability, which uses a one-copy single-version database as the ideal system.

3 THE PHYSICAL SYSTEM MODEL

One of the main questions we consider in this paper is how to extend the theory of transactions and serializability to account for freshness requirements. To do this, we need to model the physical system consisting of transactions, data managers, and databases. Since cached copies are replicas of a master copy, we can model the system as an ordinary non-versioned replicated database. This section defines our replicated database model (see Figure 1).

We assume that each data item has a unique identity. A transaction is modeled as a set of partially-ordered actions, which include reads and writes of data items, and, possibly, a final commit or abort. For simplicity, we do not allow inserts or deletes of items, since they would lead to many notational complexities. We consider them and the associated issue of phantoms as an extension in Section 8. We denote the set of items read by transaction $T$ as $\text{readset}(T)$ and the set written by $T$ as $\text{writeset}(T)$.

Every data item has a set of physical copies: one master and zero or more replicas. The collection consisting of masters of all of the data items is called the master database. A cache is a collection of replicas of one or more data items. There is only one version of each physical copy, which contains its latest value.

We assume that the master database has a master manager that controls the execution of reads and writes to the master. Similarly, each cache has a cache manager that controls the execution of reads and writes to the cache.

We distinguish between user-transactions (or simply transactions) that are issued by users, and copy-transactions (or simply copyers) that are issued by the master manager. Each transaction issues each read operation either to the master manager or to a particular cache manager. For now, we assume that it only modifies the master database, so it sends all of its write operations to the master manager, which is where they are processed.

![Figure 1: Replicated Database Model](image)

After a transaction $T$ commits, the master manager invokes a copier for each pair $[x, P]$ where $x$ is a data item updated by $T$ and $P$ is a cache that has a replica of $x$. A copier reads the value of the master $x^M$ and writes that value into the corresponding replica $x^C$ in cache $P$. Note that a copier runs outside of the transaction that caused the copier to be invoked. For now, we leave open whether the copiers from the master to a given cache are pipelined in commit order or grouped (e.g., one copier contains reads and corresponding writes for multiple $[x, P]$ pairs).

The techniques in this paper are applicable when the master database includes materialized views of base data and when replicas of those materialized views are stored in caches. In this case, we assume that each transaction maintains the consistency of materialized views with respect to base data. Whether this is done by user code or system code executing within the transaction is unimportant to our model. What is important is that we assume that the master and cache managers treat updates to materialized views the same as those on base data. Solutions that relax this assumption may be useful, but are outside the scope of this paper.
FRESHNESS CONSTRAINTS

To capture the concept of freshness introduced in Section 1, we require that each read request from a transaction includes a freshness constraint [12]. The cache manager or master manager that services a read must return a value of the requested item that satisfies the read’s freshness constraint. Freshness constraints are defined based on the time-drift or value-drift of an item from its correct value, or on the mutual consistency of item values. To define these precisely, we use the well-known concepts of timestamps and snapshots.

We assume a timestamp domain, which is a subset of the natural numbers. The master manager assigns to every committed transaction \( T \) a unique transaction timestamp \( t(T) \) greater than all timestamps assigned to previously committed transactions.

We assume that the master manager and cache managers can calculate a user-comprehensible wall-clock time from each timestamp and vice versa. Thus, the system can translate a freshness constraint into three classes of data items (since there is just one copy of each). We classify each data item (and hence no copiers), and all of its committed versions (from all past writes) are visible to user-transactions.

Thus, the semantics of freshness constraints can refer to versions (from all past writes) of one data item, and can be combined. For example, a transaction’s reads must see changes made by a committed transaction is called a freshness constraint.

Every copy \( x^t \) (i.e., a master or replica) has an associated last-modified timestamp, \( \text{last-modified}(x^t) \), which is the timestamp of the last committed transaction that wrote \( x^t \). The value of a copy written by a committed transaction is called a committed version.

Each committed version \( x_t \) of a copy \( x^t \) of data item \( x \) is said to be valid over the half-open interval \([t, t')\) where \( t = \text{last-modified}(x_t) \) and \( t' \) is either the next larger last-modified timestamp of another committed version of \( x^t \) or \( \infty \) if no version of \( x^t \) has a larger last-modified timestamp. We call \([t, t')\) the valid interval of \( x_t \).

A snapshot is a database state produced by a certain set of transactions—usually, the committed transactions in the prefix of an execution. A snapshot is associated with a timestamp \( t \) and maps each data item \( x \) to the version of \( x \) whose valid interval includes \( t \).

The ideal system, which is the user’s model, is just like the physical replicated database model except there is just one copy of each data item (and hence no copiers), and all of its committed versions (from all past writes) are visible to user-transactions. Thus, the semantics of freshness constraints can refer to versions of data items (since there is just one copy of each). We classify freshness constraints into three classes \( B, D, \) and \( M \) as follows.

**B. Time-Bound and Value-Bound Constraints**

**B1. Time-Bound Constraints.** We can limit the currency bound, which is the difference between the time a version becomes invalid and the time it is read (the read-time). For example, we can specify that it is acceptable to read a version of \( x \) that is up to \( 10 \) seconds out of date: \( \text{bound}(x; 10) \). If \( \text{bound}(x; 0) \), then the read must be given a valid version of \( x \). For simplicity, we omit the time unit and assume it is seconds.

**B2. Value-Bound Constraints.** We can specify that the value read for \( x \) is within a certain percent of the value of the valid version: \( \text{bound}(x; 10\%) \).

**D. “Drift” Constraints on multiple data items.** When a transaction \( T \) reads more than one data item, we can specify constraints over a subset \( S \) of readset(\( T \)).

**D1. Snapshot consistency.** We can require that data items in \( S \) must be read from the same snapshot, denoted \( \text{snapshot}(S) \).

**D2. Limited Time-Drift.** The constraint \( \text{drift}(S, b) \) says that for every two items \( x \) and \( y \) in \( S \), the versions of \( x \) and \( y \) that are read are within \( b \) seconds of each other. That is, if the read transaction reads version \( x_t \) of \( x \) and \( y_{t'} \) of \( y \), then there are timestamps \( t \) and \( t' \) such that \( x_t \)’s valid interval includes \( t \), \( y_{t'} \)’s valid interval includes \( t \), and \( |t - t'| \leq b \). Observe that snapshot consistency is the special case of \( b = 0 \).

**D3. Limited Aggregate Value Drift.** We can require that an aggregate computed over (a subset of) the readset of \( T \) be within a certain percentage of a “recent” value (even though individual data items may be further out of sync). We use the notation \( \text{drift}(S, \text{AGG}, b\%, w) \), where \( \text{AGG} \) is an aggregate operation, and \( w \) is a time window. It means that \( \text{AGG}(S) \) must be within \( b\% \) of \( \text{AGG}(S') \), where \( S' \) denotes the value(s) of committed versions of items in \( S \) at some instant less than \( w \) seconds prior to the current time.

Constraints of type \( B \) and \( D \) can be combined. For example, a transaction can set \( \text{bound}(x; 10) \) and \( \text{snapshot}(\{x, y\}) \) [12]. We can further classify these constraints using two orthogonal criteria:

- **Granularity:** Constraints over individual data items, sets of items, aggregates over sets of items, subsets of snapshots, and complete snapshots.
- **Unit of skew:** Specify the skew using timestamps, number of subsequently committed transactions, or value.

**M. Multi-statement Constraints**

We can specify “session level” constraints that refer to points in time external to the execution of the current statement. For example, a transaction’s reads must see changes made by preceding statements in this transaction or in this session.

When evaluating constraint types \( B1, D2, \) and \( M \), there are two possible notions of read-time: the time when the read executes, or when its transaction (later) commits. Even in the user’s model of computation (a serial execution of the ideal system), these read-times are not the same unless the transaction has negligible execution time. For most of this paper, we use a transaction’s commit time to be the read-time of all of the transaction’s reads. We consider the other semantics in Section 8.

5 ENFORCING RC SERIALIZABILITY

Enforcing RC-serializability requires ensuring that transactions are serializable and satisfy their freshness constraints. Enforcing RC-serializability is non-trivial because these requirements are not independent: freshness constraints are affected by factors related to transaction order, such as transaction timestamps. So the different transaction orders in two equivalent serial executions might affect whether certain freshness constraints are satisfied. The goal of RC-serializability is to ensure that each execution has at least one equivalent serial execution in which freshness constraints are satisfied.
The problem of proving RC-serializability is also non-trivial. As in one-copy serializability, the notion of equivalence in the definition of RC-serializability requires mapping between two different kinds of executions—multiversion executions of the ideal system and replicated data executions of the physical system—and proving that they have the same behavior. We will do this in Section 6. But first we introduce some algorithms that produce RC-serializable results for various kinds of freshness constraints. We describe how to deal with transactions that include both reads and writes; read-only transactions (queries) can use the techniques of [12].

5.1 Baseline Synchronization

We first present a general concurrency control algorithm, called the baseline synchronization algorithm (or BSA). It provides a common framework for the algorithms we discuss later that enforce freshness constraints, but does not by itself enforce such constraints. In summary, user-transactions read replicas using read-duration locks and write masters using commit-duration locks. Thus, user-transactions run at the read committed isolation level. Commit and abort are processed at the master site, which generates transaction timestamps for committed transactions. For copiers, the master pipelines writes to caches in timestamp order.

We describe BSA in terms of how it handles the following four actions: reads, writes and commits of user-transactions, and copiers. See the boxed algorithm descriptions below and their descriptions that immediately follow.

**BSA-Read(x)** // issued by a user-transaction

Choose a manager R from which to read x;
// R can be the master for x or a cache
read-lock x at R;
read x;
release read-lock;

**BSA-Write(x)** // issued by a user-transaction

write-lock x to the master for x; // lock held until commit
write x;

**BSA-Commit(T)** // issued by a user-transaction

// and processed at the master site

\( ts(T) \leftarrow \text{new transaction timestamp} \)
// ts(T) > all timestamps previously issued

for all items x in writeset(T)
\( \text{last-modified}(x^M) \leftarrow ts(T); \)
release all locks;

**BSA-Copy(x, P)** // a copier transaction

read-lock x at the master for x;
read x and last-modified(x^M);
release read-lock;
write-lock x at cache manager P;
\( x^P \leftarrow x^M; \)
\( \text{last-modified}(x^P) \leftarrow \text{last-modified}(x^M); \)
// assumes writes are applied in last-modified order
release write-lock

Writes set exclusive locks on master copies. The locks are released only after the transaction commits and its updates are applied at the master. Thus, for each unlocked master item \( x^M \), \( \text{last-modified}(x^M) \) is the timestamp of the last committed transaction that updated \( x \).

Reads set short-duration shared locks. This ensures that each read operation sees a committed value. It also ensures that the data and last-modified value are mutually consistent at the time the read occurs, since there may be a gap between a transaction updating a copy’s data value and updating its last-modified timestamp.

Copiers are generated by the system. They set short-duration shared locks to read the master copy of an item before propagating the value to a target cache. Updates from copiers are pipelined to each cache in timestamp order, which is the same as commit order. This is how primary-copy replication works in most of today’s commercial database products.

Since updates are propagated in timestamp order to each cache, successive reads of the same data item in a cache see time moving forward. But if a transaction reads the same item from different caches, this may not hold. To avoid this, assume all updates of each transaction are applied atomically at each cache, and define \( \text{last-modified}(A) \) for cache A to be the largest value of \( \text{last-modified}(x^A) \) of any copy \( x^A \) at A. Each transaction \( T_x \) remembers the maximum value \( m_x \) of \( \text{last-modified}(A) \) over all caches it reads from and attaches it to all reads. Before processing a read of \( x^A \) from \( T_x \), cache B checks \( \text{last-modified}(B) \geq m_x \). If not, it can wait till more updates arrive so the check is satisfied, or it can reject the read.

The rest of this paper is independent of whether this is done.

At this point, we make no assumptions about how and when each copier is generated. It might be generated on behalf of a particular transaction \( T \) to copy writeset(\( T \)), or periodically for each item or set of items, or using some other strategy.

5.2 Algorithms for Ensuring Freshness

5.2.1 Algorithm BSA-FC

We now propose tests to determine whether a given set of copies satisfies reads with various freshness constraints. The freshness tests we propose are implemented as additional steps in BSA-Read, BSA-Write and BSA-Commit. In BSA, every copy \( x^A \) has an associated timestamp \( \text{last-modified}(x^A) \). Our tests use another timestamp associated with each copy, called valid-till \( (x^A) \). Intuitively, valid-till \( (x^A) \) is the smallest timestamp that could be associated with the next version of \( x^A \). Regardless of how it is maintained, we require the following property to be preserved:

**Property VT** The valid interval of the value held in a copy \( x^A \) must include the closed interval \( [\text{last-modified}(x^A), \text{valid-till}(x^A)] \).

Recall that the valid interval of version \( x_T \) of \( x \) is the half-open interval \([t, \infty)\) where \( t = \text{last-modified}(x_T) \) and \( t \) is either the next larger last-modified timestamp associated with another committed version of \( x \) or \( \infty \) if no other version of \( x \) has a larger last-modified timestamp. For the master copy \( x^M \), we can take valid-till \( (x^M) \) to be the largest timestamp issued so far, say \( t^* \). This works because the next larger timestamp \( t^* \) of a version of \( x \) will surely be larger than \( t^* \), so Property VT holds. For a replica \( x^A \), we can take valid-till \( (x^A) \) to be last-modified \( (x^A) \), for which Property VT trivially holds. However, since updates are propagated to each cache in timestamp order, and assuming updates of each transaction are applied atomically at each cache, valid-till \( (x^A) \) can be taken as
last-modified(A). In this case, valid-till has the same value for all items in a cache and can be maintained at cache granularity.

When a read or group of reads is performed, the system uses the read’s freshness condition, values of last-modified and valid-till, and perhaps other information to deduce a constraint on the timestamp of the reader’s transaction. For each transaction \( T \), these timestamp constraints are remembered. When \( T \) is ready to commit, the master site assigns \( T \)’s timestamp and then checks if it satisfies all of \( T \)’s timestamp constraints. If any constraint is false, then \( T \) aborts. Alternatively, it could be backed out to a savestate preceding the read for which the system deduced the failed constraint, or if aborted it could be restarted. But these strategies are independent of the approach to RC-serializability, so to simplify the algorithm descriptions we ignore them in what follows.

The rest of this section describes variations of BSA that handle Time-Bound and Limited-Time-Drift freshness constraints (classes B1 and D2 in Section 4). Section 5.2.2 shows how to reduce commit-time work at the master. Section 5.2.3 briefly discusses other classes of constraints.

To test freshness conditions associated with read operations of transaction \( T \), the following checks are added to BSA-Read:

a) Time Bound \( \text{bound}(x; b) \): Add the constraint \( "\text{ts}(T) \leq vt+b" \) to transaction \( T \)’s constraint \( C_{T} \), where \( vt \) is the value of valid-till(\( x^b \)) associated with the value being read. The simplest implementation uses valid-till(\( x^b \)) at the moment the read is performed; we discuss alternative implementations below.

b) Limited Time-Drift Bound \( \text{drift}(S, b) \): Recall that \( S \) denotes a subset of \( T \)’s readset. Let \( \text{max}(\text{last-modified}(S)) \) denote the largest last-modified timestamp associated with any of the copies in \( S \) that are read by \( T \). Similarly, let \( \text{min}(\text{valid-till}(S)) \) denote the smallest valid-till timestamp of a copy in \( S \) that is read by \( T \). Add the constraint \( "\text{max}(\text{last-modified}(S)) < \text{min}(\text{valid-till}(S))+b" \) to the transaction’s constraint \( C_{T} \).

And the following change is made to BSA-Commit for \( T \):

- If possible, assign a timestamp to \( T \) that is greater than all previously issued timestamps and also satisfies all of its constraints in \( C_{T} \). Otherwise, abort \( T \).

We will refer to BSA with the above changes as BSA-FC (for “freshness checking”).

To understand Step (a), recall that \( \text{bound}(x; b) \) says that the value read by \( T \) can be at most \( b \) seconds out-of-date. Since the value of \( x^b \) that was read is valid until at least \( vt \), if \( ts(T) \leq vt+b \), then \( \text{bound}(x; b) \) is satisfied.

To enforce \( \text{drift}(S, b) \), Step (b) must ensure that there is a timestamp within \( b \) seconds of the valid interval of every version read by \( T \). To see how it accomplishes this, consider any two data items \( x \in S \) and \( y \in S \). Let the valid intervals of the versions read by \( T \) be \( [t_x', t_x"], [t_y', t_y"] \) and \( [t'_y, t_y"] \). Without loss of generality, suppose \( t_x' \leq t_y' \). As shown in Figure 2, either (i) \( t_x" \leq t_y" \) and the intervals overlap or (ii) \( t_x" < t_y' \) and the intervals are disjoint. In case (i) \( \text{drift}(x,y), b \) is satisfied. In case (ii) we need \( t_x"-t_y' \leq b \), or \( t_x" \leq t_x'+b \), to satisfy \( \text{drift}(x,y), b \). But this follows immediately from the check in Step (b), since \( t_x' \leq \text{max}(\text{last-modified}(S)) \leq \text{min}(\text{valid-till}(S))+b \) and \( \text{min}(\text{valid-till}(S)) \leq t_x" \). Since we chose \( x \) and \( y \) arbitrarily, \( \text{drift}(S, b) \) holds for all \( x \) and \( y \) in \( S \).

Figure 2: Valid Intervals for Checking \( \text{drift}(S, b) \)

The tests described above cover freshness constraints of types B1 and D2. Type D1 is a special case of D2 where \( b = 0 \) and is therefore covered as well.

5.2.2 Variants of BSA-FC

This section describes several optimizations or variant implementations of BSA-FC. One optimization is that when a constraint \( C \) is generated, if the system knows that \( C \) cannot be satisfied, then it can reject the operation immediately. For example, suppose the check for \( \text{bound}(x;b) \) takes \( vt \) as the value of valid-till(\( x^b \)) at the time the read occurs. The cache manager might know that the most recently committed transaction’s timestamp is already greater than \( vt+b \), because it saw an update with timestamp greater than \( vt+b \). Since the constraint will not be satisfied when the commit is eventually attempted, the read should be rejected. However, by using more expensive implementations, one can get larger correct values of \( vt \), which reduce the number of unnecessary aborts in four scenarios where \( vt \) may increase over time:

- If the master is a multiversion database, then it knows the exact value of valid-till of every version of every data item, which is at least \( vt \). So the constraint \( vt+b \) can be replaced by \( [x^l, \text{lm}, b] \) where \( \text{lm} \) is last-modified(\( x^b \)) of the value read. Using this, the master can calculate \( vt+b \) at commit time, where \( vt \) is valid-till of the version of \( x^b \) identified by \( \text{lm} \).

- If the transaction includes last-modified(\( x^b \)) with its update, and if the master finds last-modified(\( x^b \)) to be the same, then it can use its local value valid-till(\( x^b \)) as \( vt \) when checking the constraint.

- The cache could be made to participate in two-phase commit (which it may do anyway if the transaction updated the cache). The phase-one processing can re-read last-modified(\( x^b \)) and valid-till(\( x^b \)). If last-modified(\( x^b \)) is unchanged from the time the transaction read \( x^b \), but valid-till(\( x^b \)) has increased (which is likely) then the new value of valid-till(\( x^b \)) can be returned in the phase-one reply and used by the master in the constraint check.

- The cache could remember the identity of every replica read by every active transaction. When a replica \( x^b \) is updated by a copier, the value of valid-till(\( x^b \)) is recorded and frozen for all active transactions that read it. This is more accurate than the previous method, but adds bookkeeping cost. Again, phase-one processing obtains the latest values of valid-till.

If a time-bound constraint \( \text{bound}(x, b) \) is applied to each \( x \) read by transaction \( T \), then the processing of Step (a) can be optimized for the common case when a transaction’s read operations are issued in sequence. The system maintains a counter \( \text{high} \) on behalf of \( T \), initialized to \( \infty \) and updated to \( \text{min}(\text{high}, \text{valid-till}(x^b)+b) \) after each read, where \( x^b \) is the copy that was read. At commit time, it ensures that \( ts(T) < \text{high} \), a single constraint that implies \( ts(T) \leq \text{valid-till}(x^b)+b \) for every data item \( x^b \) that it accessed.
A similar optimization applies to drift(S, b). The system maintains two counters on behalf of Ti: high (as above) and low. It initializes low to -\infty. For each read of (say) x^i by Ti, if max(low, last-modified(x^i)) > min(high, valid-till(x^i)+b), then it rejects the read. Otherwise, it performs the read, sets low = max(low, last-modified(x^i)) and sets high as above. At commit time, it knows that low < high, so drift(S, b) is surely satisfied.

Insofar as last-modified is used for drift constraints, these timestamps can be maintained at coarse granularity, reducing the space required in the cache for storing these values. The downside is that in place of the accurate last-modified(x^i) we use the largest last-modified(y^i) across all y^i in x^i's granule. Thus, max(low, last-modified(x^i)) may be larger than it needs to be, increasing the chance that max(low, last-modified(x^i)) > min(high, valid-till(x^i) + b), causing the read of x^i to be rejected. That is, it may increase the number of aborts, but does not lead to erroneous results.

5.2.3 Handling Other Classes of Constraints
Value-Bound conditions can be processed if there is a known bound on the rate at which the value of an item can change. For example, the location of a vehicle will not change faster than the vehicle’s speed allows. Given such a rate bound, we can deduce a bound on timestamp drift from one on value drift and enforce the deduced time-drift bound. Limited aggregate-drift bounds can be handled similarly, given bounds on the rate of change of values.

Multi-statement constraints were discussed in Section 4. Such a constraint might say that a transaction sees all updates made by previous transactions in the same session. To implement this, the system can remember the maximum timestamp session-max-ts of any committed transaction in the session. When it executes a read of x^i, it checks that valid-till(x^i) \geq session-max-ts.

A more challenging multi-statement constraint is that a transaction Ti’s reads must see the results of its preceding writes. To enforce this, we can maintain a list Li of all writes executed by Ti. Every read from Ti is checked against Li. If Ti previously wrote the item to be read, then the read must be executed at the master so that it sees the prior write. We discuss this further in Section 8.

6 CORRECTNESS
Intuitively, BSA-FC is correct because it serializes transactions in timestamp order based on write-write conflicts and checks that each read satisfies its freshness constraints relative to that order. To prove this, we need to formally define the physical and ideal systems and prove that every execution of the former is equivalent to a correct execution of the latter. In this section, we give a short proof sketch. A complete proof appears in Appendix A.

The physical system is modeled by a physical history, which is a replicated data history. The ideal system is modeled by an ideal history, which is a multiversion history. To define equivalence, we extend the usual reads-from relation to include the semantics of copiers: Ti takes-x-from Tj if either Ti reads-x-from Tj, or there is a copier Cj such that Ti reads-x-from Cj and Cj reads-x-from Tj. A physical history is equivalent to an ideal history if they have the same takes-from relation and final writes.

We first prove that a history H is bare-RCSR, meaning that H is equivalent to some serial ideal history H'. To do this, we define an RC serialization graph (RCSG) over H, whose nodes are transac-

tions and whose edges capture takes-from relationships and write-write conflicts. We then characterize BSA’s physical histories and prove every such history has an acyclic RCSG.

Theorem 1 For any primary-copy physical history H, if RCSG(H) is acyclic then H is bare-RCSR. □

Proposition 1: A physical history H produced by BSA has the following properties, where \( <_H \) denotes the order of operations in H:
A. Every committed transaction Ti has a unique timestamp \( ts(T_i) \).
B. \( ts(T_i) < ts(T_j) \) iff \( c_j <_H c_i \).
C. If Ti reads-x-from Tj then \( ts(T_i) < ts(T_j) \).
D. If \( r_i(x^i) \in H \) for copier \( C_i \), master \( x^M \), and some copy \( x^i \), then \( r_i(x^i) <_H w_i(x^i) \).
E. If Ti takes-x-from Tj then \( ts(T_i) < ts(T_j) \).
F. If \( w_i(x^i) <_H w_j(x^j) \) then \( ts(T_i) < ts(T_j) \). □

Theorem 2 For every physical history H produced by BSA, RCSG(H) is acyclic. □

By Theorems 1 and 2, every physical history H produced by BSA is bare RCSR. In fact, from Prop. 1B, 1E and 1F, H is strictly bare RCSR, meaning there is an equivalent serial ideal history H' such that commits in H appear in the same order as in H'.

Corollary 1 Every physical history H produced by BSA is strictly bare RCSR. □

BSA-FC extends BSA with commit-time checks of time-bound and limited time-drift bound constraints. To prove these constraints are satisfied, we need to refer to the timestamps of transactions in the ideal history. We say that a timestamp assignment for ideal history H’ is valid if, for all Ti, Tj, \( ts(T_i) < ts(T_j) \) iff \( c_j <_H c_i \). The following proposition follows from Corollary 1.

Proposition 2: If a history H is strictly bare-RCSR, then a valid timestamp assignment for H is also a valid timestamp assignment for an ideal history equivalent to H. □

Theorem 3: BSA-FC is RCSR.
Proof: By Prop. 2, every physical history H produced by BSA-FC is equivalent to an ideal history H’ with the same timestamp assignment. We need to show freshness constraints hold in H’.

First, consider a time-bound constraint \( bound(x, b) \) on read operation \( r_i(x^i) \). Let \( x_j \) be the version of \( x^i \) read by \( r_i(x^i) \). If \( x_j \) is the last version of \( x^i \), then \( x_j \) never becomes invalid so \( bound(x, b) \) holds. Else let \( x_k \) be the next version of \( x^i \) —the one whose transaction has next larger timestamp among those that updated \( x^i \). By definition, \( bound(x, b) \) holds if \( ts(T_i) < ts(x_k) + b \). By Property VT, \( vt \leq ts(x_k) \). BSA-FC ensures the constraint \( ts(T_i) < vt + b \) is satisfied when \( ts(T_i) \) is allocated, thus \( ts(T_i) < ts(x_k) + b \) as desired.

Next, consider a time-drift constraint \( drift(S, b) \) and two reads \( r_i[x^i] \) and \( r_j[y^j] \), where \( x \in S \) and \( y \in S \). To satisfy \( drift(S, b) \), if \( r_i \) \( [x^i] \) and \( r_j \) \( [y^j] \) read versions \( x_i \) of \( x \) and \( y_j \) or \( y_i \) respectively, then there must be timestamps \( t_i \) and \( t_j \), such that \( x_i \)’s valid interval includes \( t_i \), \( y_j \)’s valid interval includes \( t_j \), and \( |t_j - t_i| \leq b \). This interval was proved at the end of Section 5.2.1, so we are done. □
7 PERFORMANCE

This section evaluates the performance of the BSA-FC protocol with time-bound constraints. It shows the potential gains when stale data is acceptable, and the cost of guaranteeing currency bounds. With currency bound infinity, BSA-FC behaves like BSA, allowing the most concurrency. At the other extreme, with currency bound 0, BSA-FC guarantees one-copy serializability. We address the following issues:

- How does BSA-FC perform with increased data contention and resource contention in the system? (Section 7.3)
- What is gained by executing reads of update transactions, in addition to read-only transactions, at caches? How does refresh interval impact throughput? (Section 7.4)
- How does performance scale as we add resources and caches? (Section 7.5)
- How does BSA-FC perform with different cache configurations and compare with a no-cache setting? (Section 7.6)

Our results show that superior performance and good scalability can be achieved with relatively tight currency bounds. We begin by describing our performance model in Section 7.1 and the workloads and metrics we consider in Section 7.2.

7.1 Performance Model

Our performance model covers two scenarios, single-site and master-cache. We implemented it in about 8000 lines of code using CSIM [8], a process-oriented discrete event simulation package. The performance model captures the relevant characteristics of the database system, users, and transactions. The database system model includes the physical resources (CPUs and disks), their associated schedulers, the mechanism for controlling the number of active transactions, and the concurrency control algorithm. The user model captures the arrival rate of transactions. The transaction model captures the reference behavior and processing requirements in the workload.

Our single-site model is the closed queuing model of Agrawal, Carey, & Livny [2] and is outlined in Section 7.1.1. (See [2] for details.) We extend it to the master-cache setting in Section 7.1.2.

7.1.1 Single-Site Setting

As shown in Figure 3, which is reproduced from [2], transactions originate from a fixed number of terminals. The number of active transactions in the system is bounded by the multiprogramming level (MPL). A transaction is active if it is either receiving or waiting for service. When a new transaction arrives, if the system is full, it enters the ready queue and waits for an active transaction to commit or abort. It then enters the CC queue and requests its first lock. Upon success, it enters the object queue, accesses the object, and re-enters the CC queue for its next lock. If the lock request is blocked, the transaction enters the blocked queue and waits. If the lock request causes a deadlock, the transaction aborts and enters the ready queue after a random restart delay. Once all the accesses are done, the concurrency control algorithm may choose to commit the transaction. If the transaction is read-only, it is finished. Otherwise, it enters the update queue and writes its deferred updates into the database. For BSA-FC, the system notes the timestamps needed for checking transaction timestamp constraints, as in Section 5.2. In all experiments BSA-FC uses time bound constraints with the same currency bound for all reads.

Underlying this logical model is a physical model consisting of CPU and IO Resources, where a transaction accesses CPU and IO resources a certain number of times. For details, see [2].

7.1.2 Master-Cache Setting

This setting comprises a single master and multiple caches. We make the simplifying assumption that the master and all caches have the same set of objects. The logical model is shown in Figure 4. Each terminal is associated with a specific site and submits user transactions to that site. Each site processes transactions as described in the single-site setting, with the changes described below.

Each cache executes two types of transactions: user transactions and copiers. Further, the cache distinguishes read-only from read-write user transactions. As in [12], read-only transactions that execute entirely at a cache are validated there, instead of at the master, and are sent to the master only if the validation fails. For a read-write user transaction, the cache executes the read part and notes the timestamps needed for transaction timestamp constraints, as in Section 5.2. At commit time, it sends the writes and timestamp constraints to the master, where either the transaction receives a timestamp and commits or it aborts. The transaction then returns to the cache, which returns the result to the terminal.

The master executes three types of transactions: user transactions submitted from local terminals, read-only transactions submitted from caches (due to failed freshness constraint checks), and (possibly parts of) read-write transactions submitted from caches. All three types of transactions are handled as in the single-site case.

Copiers are implemented using the standard log-sniffing approach. The master has a distribution agent per cache. It wakes
up after every refresh-interval and sends the tail of the log to the cache. When a cache receives the log, for each transaction in the log it runs a copier to refresh its local database. Each copier updates the valid-till timestamp of the cache and the last-modified time of each object written by the logged transaction. As for the single-site setting, the master-cache logical model has a corresponding physical model (similar to the one for single-site).

Increasing the number of resources at the master increases the parallelism, leading to more read-write transactions being committed. After transactions are committed, their writes should be replayed at each cache in commit order. This sequential execution leads to poor performance of copiers at caches. We address this issue with two optimizations. First we increase the priority of copiers executing at a cache. This reduces the disk waiting time for copiers. Second, we execute a batch of copiers in parallel. That is, we consolidate multiple copiers into one multi-threaded transaction; locks are released together at commit. If an object is written by multiple copiers, we ensure the value and timestamp of the last copier in the batch is reflected. We set the number of consolidated copiers to twice the number of disks in the system.

7.2 Workload Parameters and Metrics

Major parameter settings, shown in Table 1, were adopted from [2]. Although they do not reflect current technology, the ratios are still sensible. Using them makes our measurements comparable to those in [2] for validation, and avoids long simulation times without affecting the relative performance of the methods. For example, substituting speeds of today's disks, throughputs increase by a factor of 10 but the shapes of the curves remain the same. We added more parameters to capture system characteristics specific to our context, such as currency bounds and distribution costs.

![Figure 5: 1 Resource Unit](image5.png)

![Figure 6: Infinite Resource Units](image6.png)

![Figure 7: Multiple Resource Units](image7.png)

Table 1: Parameter Settings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>db_size</td>
<td>1000 Pages</td>
<td>restart_delay</td>
<td>adaptive</td>
</tr>
<tr>
<td>tran_size</td>
<td>8-page readset</td>
<td>num_terminals</td>
<td>200</td>
</tr>
<tr>
<td>max_size</td>
<td>12-page readset</td>
<td>ext_think_time</td>
<td>1 second</td>
</tr>
<tr>
<td>min_size</td>
<td>4-page readset</td>
<td>obj_io</td>
<td>35 ms</td>
</tr>
<tr>
<td>write_prob</td>
<td>0.25</td>
<td>obj_cpu</td>
<td>15 ms</td>
</tr>
</tbody>
</table>

We briefly describe the main parameters. The number of objects in the database is \( db_{size} \). Each transaction’s reads are uniformly distributed between \( min_{size} \) and \( max_{size} \). Each object read by a transaction is written with probability \( write_{prob} \). The number of user terminals in the system is \( num_{terminals} \). \( Obj_{cpu} \) and \( obj_{io} \) give the service times of CPU and IO resources respectively. If a transaction restarts due to deadlock, it is delayed by an amount controlled by an exponential distribution whose mean is average response time times MPL/num_terminals, to ensure that transactions are not delayed for long at lower MPLs. After each transaction, a terminal waits for a time that is exponentially distributed with mean \( ext_{think}_{time} \). For BSA-FC, we varied the currency bound from 0 to 100 seconds. Distribution cost involves reading the log. We assume that the log is sequential and has an IO cost of 5 milliseconds (ms) per page plus 30 ms seek time. Further, we assume that about 100 updates can be stored in a page. We assume network cost similar to IO cost, but we only delay the corresponding action instead of charging the IO resources.

Performance Metrics: We only present system throughput. But to verify that our results are consistent, we measured other performance metrics too, including response time, resource utilization, conflict, and restart ratios. We ran each experiment 10 times, with sufficiently long simulation time to produce 90 percent confidence intervals, which we omit from the graphs for clarity.

7.3 Single-Site Experiments

The goal of our single-site experiments is to compare different protocols with respect to data and resource contention. We compare BSA-FC (with three different currency bounds), Strict 2PL (two-phase locking with write locks held till after commit) and Read Committed (which is BSA without copiers). We control data contention by varying the MPL. We control resource contention by varying the number of resources. Our 1st experiment examines high resource and data contention. The 2nd and 3rd look at data contention and resource contention in isolation. We found similar differences between Strict 2PL and Read Committed as in [7].

7.3.1 Expt 1: Limited Resources

A resource unit is 1 CPU and 2 disks. We fixed the number of resource units to 1 and varied the MPL. The throughput is shown in Figure 5. Resource contention increases significantly as the MPL increases. This causes long response times (40~60s), and hence more constraint failures for BSA-FC with tight currency bounds (<=10), leading to Strict 2PL outperforming BSA-FC. On the other hand, BSA-FC with a large enough currency bound (100s) performs as well as Read Committed, i.e., the guarantees on freshness come “for free”. Here, the difference between the time of invalidation and the commit-time of the reading transaction could be the length of the transaction. (On average, this is 40-60s, but there is no limit on outlier cases.)
7.3.2 Expt 2: Infinite Resources
We set the number of resources in the system to a large number (“infinity”) to eliminate resource contention. Although this is not realistic, it serves as an upper bound. As shown in Figure 6, BSA-FC outperforms Strict 2PL even with currency bound 0. This is because 2PL’s throughput is limited at low MPLs due to its blocking behavior, and at higher levels due to lock thrashing. In contrast, BSA-FC achieves increasing throughput until it reaches a performance plateau. The conflict ratio (number of times a transaction is blocked) is 2.7 at MPL 200 for 2PL but less than 0.1 for BSA-FC. Response time at this point was observed to be about 0.7 sec. With 1 sec think time between transactions and 200 terminals, increasing MPL above this level does not increase throughput as the remaining terminals are in the think state. In a single-site setting, the only way constraint checks can fail in BSA-FC is if a transaction reads a value and takes longer to complete than the associated currency bound. Since the response time in this experiment is 1–2 sec, BSA-FC with currency bound 10 does as well as Read Committed.

7.3.3 Expt 3: Multiple Resource Units
This experiment fills the gap between the previous two by varying the number of resource units from 5 to 50. Another important parameter here is the MPL. Setting it to lower values results in the underutilization of resources. Setting it to higher values results in thrashing. We fixed its values for each protocol to maximize its throughput, based on the previous experiments and results in [2]. Strict 2PL’s performance peaks at a MPL of 50. BSA-FC reaches a plateau above 100. The throughput results are shown in Figure 7. Increasing the resources in the system increases the throughput thanks to increased parallelism. But the increase is more for BSA-FC, because there is less blocking compared to Strict 2PL (conflict ratio 0.33 vs. 0.05 with 50 resource units).

In summary, in the single-site scenario BSA-FC outperforms Strict 2PL if the system has sufficient resources. BSA-FC also gives much higher throughput (a factor of 3) with currency bounds of a few times the response time, which we believe is appropriate for many practical situations. As currency bounds are relaxed, BSA-FC achieves the throughput of Read Committed, which is the expected ideal, performance-wise.

7.4 Master-Cache Experiments
In the approach proposed in [12], only read-only transactions can execute at a cache. The RCSR theory developed in this paper allows us to improve on this by doing reads at the cache even for transactions that also do updates (which, of course, are still sent to the master). In this section, we evaluate the performance gains due to this improvement. We also study how the value of the refresh interval relative to currency bounds affects performance.

7.4.1 Expt 4: Reads of RW Transactions at Caches
In [12], the authors proposed extending SQL with currency and consistency constraints for caches that only handle read-only transactions. What do we gain by executing the reads of read-write transactions at the caches? To simulate their scenario, we run Strict 2PL at the master and BSA-FC at the cache. In scenario RO-2PL, the cache only handles read-only transactions and forwards all read-write transactions to the master. In scenario RW-2PL, the cache also handles reads from read-write transactions (master checks their freshness). In another setting, we modify these two scenarios by using BSA-FC at the master, and refer to them as RO-BSAFC and RW-BSAFC respectively.

In this set of experiments, there is one master and one cache. The master has 10 resource units and 200 terminals. The cache has 5 resource units and 100 terminals. We add a new parameter, read-only transaction percentage; decreasing it increases data contention. The refresh interval is set to 0, i.e., we continuously refresh the cache. Each site’s MPL is set to the optimum value for its read-only transaction percentage, as in Section 7.3.3.

The results are shown in Figure 8, Figure 9 and Figure 10, corresponding to currency bound 0, 5 and 10 seconds, respectively. In each figure, we show the throughput of scenarios RO-2PL to RW-BSAFC as the read-only percentage increases.

As the read-only percentage increases, throughput increases. Also as the currency bound increases, throughput increases except for the 100% read-only case. Similar to the single-site experiments, BSA-FC dominates Strict 2PL. Even for small currency bounds, BSA-FC gains 20–35% throughput over Strict 2PL. When the currency bound is zero (Figure 8), all the scenarios behave similarly as most transactions fail the freshness constraint check. As the currency bound increases, BSA-FC improves over Strict 2PL (Figure 9). Further, with currency bound 10 (Figure 10), the scenarios with read-write transactions reading at the cache dominate those executing only read-only transactions at the cache.

7.4.2 Expt 5: Refresh Interval vs. Currency Bound
Clearly, the performance of BSA-FC depends greatly on the time between cache refreshes, relative to currency bounds. In this experiment, we study the effect of varying the refresh interval from 0 to 100 seconds, while also varying currency bounds from 0 to 100 seconds. We consider the same settings as the previous experiment except that we fix the read-only percentage to 70. The
results are shown in Figure 11. Once the refresh interval exceeds the currency bound, throughput drops drastically; at this point, the throughput is solely due to the master. Thus, it is important to refresh more often than typical currency bounds in transactions.

7.5 Scalability Experiments
Earlier sections showed the benefit of BSA-FC in single-site and master-cache scenarios. Next, we study scalability. For scale-out, we increase the number of caches. For scale-up, we increase the number of resource units at a single site. Our results show that caching with BSA-FC achieves good scale-up and scale-out. Since caching is not very useful in a write-dominated workload, we target read-dominated ones, where reads frequently occur in transactions that contain some update operations.

7.5.1 Experiment 6: Scale-Out
We increased the number of caches in the system from 0 to 5. We associated 100 terminals with each additional cache. The master has 15 resource units and 100 terminals. We fix the read-only transaction percentage to 70. The throughput results are shown in Figure 12. They show a linear scale-out when the currency bounds are more than 10 seconds. With a currency bound of 0 seconds, freshness constraint checks often fail at caches as it takes time to propagate changes. With a currency bound of 10 seconds or higher, the updates are propagated and the freshness requirements are satisfied. Since we start with no caches, this experiment also sheds light on the gains due to caching.

7.5.2 Experiment 7: Scale-Up
The set-up for this experiment is very similar to our single-site ones, except that we varied the number of terminals in the system. We also varied the number of resources from 15 to 40. Throughput results are shown in Figure 13. The corresponding points in Figure 12’s x-axis have the same number of resources in the system; the master with 0 caches has 15 resources and each cache has 5 more resources. As in the master-cache scenario, BSA-FC is scalable. The throughput increases with increased workload and resources. However, when the system has the same number of resources, single-site always performs better for the given workload, mainly because of replication overhead (copier, distribution and network) and concentrating resources at one site smooths bursts in the workload. If resources are divided across cache sites, some sites have resource contention even while others have idle resources.

7.6 Master-Cache vs. Single Site
In this experiment we compare three caching configurations with a single-site configuration, to study how each deployment of the same resources performs on throughput and throughput-per-dollar. All configurations use 64 resource units (64 CPUs and 128 disks). The first configuration has 48 caches with 1 unit each, the second has 12 caches with 4 units each, and the third has 3 caches with 16 units each; the remaining units in each case are at the master.

The throughput results with 70 percent read-only transactions are shown in Figure 14. The 48-cache configuration performs poorly because the caches are spending all their resources executing copiers from the master. Observe that with fewer caches, and therefore more resources per cache, throughput steadily increases. To validate our results we repeated the experiment with 90% read-only transactions, and the throughput is uniformly higher (Figure 15) because of the lower copier overhead. In particular, as the percentage of read-only transactions increases, throughput for the cache scenarios approaches that of the single-site configuration. Although a single-site system with the same number of resources achieves better performance, as expected, the cost of a single-site system grows much faster than the cost of a loosely-coupled master-cache system with the same number of resources. Thus, the price-performance achievable with caching and BSA-FC is significantly better than for a single-site system. To make this point, we show the throughput per dollar for each of these settings at a 90% read-only level (Figure 16), using system costs from the TPC website (www.tpc.org).

7.7 Partial Caching
We believe the benefits of caching can be considerably increased by caching only portions of the database, rather than the entire database at each cache. In this section, we present experiments that demonstrate the effects of partial caching. The settings are similar to those of in Section 7.6 except for the following changes.

In the case with 48 caches we divide the database into 10 parts. Each cache caches a tenth of the database. Furthermore, the workload at a cache consists only of data objects present in that cache. We divide the database into 10 parts in the case of 12 caches and 3 parts in the case of 3 caches. The results are shown in Figure 17, Figure 18, and Figure 19. As can seen from the figures with 70% read-only transactions we gain substantially with just 10 second currency bound. The performance/price graph shown demonstrates the low end machines offer a major benefit with partial caching.

7.8 Effect of Buffer Hits
In all of our previous experiments, we ignored the effect of the buffer pool size and hence the hit ratio. But today’s large main memory sizes may sometimes fit the whole database. So in this experiment, we vary the buffer hit ratio and repeat the experi-
ments in Section 7.6. As we increase the percentage of buffer hits, low end caches perform as well as high end ones. This is easiest to see by comparing the throughput for the case of 48 machines with one cache unit each in Figure 20 to Figure 21. The throughput also increases for the other three configurations. But the effect is less pronounced because these cases were not I/O bound.

The resources at caches are used to execute both normal and copier transactions. In low end caches, they are mostly used to run copiers as they have higher priority. Note that each cache with one resource unit has to execute copiers generated by a master that has 16 resource units.

8 OTHER IMPLEMENTATION ISSUES

As in any concurrency control algorithm, many variations and optimizations of BSA-FC are possible. We mentioned many of them throughout the paper, and briefly describe a few others here.

Reflexive Reads: A read is reflexive if its transaction previously wrote the same item. We briefly introduced them in Section 5.2.3, suggesting they be executed at the master. This requires that the system identify them. One way is to tag a transaction that might perform a reflexive read and run it entirely at the master. Another is to have the master tell the cache which items the transaction writes. When the cache receives a read, it checks whether the transaction previously wrote it. To limit the overhead, an item could be coarse-grained, such as a table. It could even be the entire database, which means all of a transaction’s reads that follow its first write execute at the master. This simplifies the bookkeeping but forces some reads to execute at the master unnecessarily.

Insertions and Deletions: Our model can easily be extended to insertions and deletions. Since inserts and deletes do not return data to their caller, and do not carry freshness constraints they can be executed at the master like any other update. This can include setting index-range locks or other locks used to avoid phantoms. Predicate-based queries, such as a SQL Select, use indices and other auxiliary structures for deciding what to access and for avoiding phantoms (e.g., using index-range locking), not just in a master but also in a cache. Copiers need to include a last-modified timestamp on these structures, since these timestamps are used when generating timestamp constraints for time-drift constraints.
For deletes, copiers need to leave a tombstone (i.e., empty object) that includes the deleted item’s last-modified and valid-till timestamps. To see why, consider a transaction \( T \) that issues a query that reads all objects satisfying a predicate \( P \) and suppose a deleted object satisfies \( P \). Then the deleted object’s timestamps must be considered when generating constraints on \( T \)’s timestamp.

**Read time currency check:** Suppose we evaluate constraints relative to the timestamp when the read executes or when the transaction starts executing, rather than transaction commit time. Thus, constraints can be evaluated during the execution of the read, rather than being delayed till the commit time. As this allows cache managers to assign timestamps, some clock synchronization between the master and caches is needed to avoid timing anomalies.

**Thomas Write Rule:** Thomas’ Write Rule (TWR) can be used to avoid requiring updates to be pipelined from the master to caches [26]. This is commonly used in multi-master replication. In TWR a write \( w(x^i) \) is applied to \( x^i \) only if \( ts(T_i) \geq ts(x^i) \). In our case, \( w(x^i) \) is issued by a copier so \( T_i \) is the timestamp of the user-transaction \( T_i \) that induced the copier execution.

The key property of TWR is that given a set of writes into \( x^i \), the final value of \( x^i \) is the value written by the copier with largest timestamp, which is independent of the order in which the writes are applied. Thus, TWR is ordinarily used to avoid requiring that writes be applied in timestamp order. If we no longer require that copiers apply updates to each cache in timestamp order, then we can no longer use the timestamp of the latest update to the cache as the value of valid-till. However, the four alternative techniques for determining valid-till in Section 5.2.2 still apply.

**9 RELATED WORK**

Tradeoffs between data freshness and availability, concurrency, and maintenance costs have been explored extensively. We discuss the most closely related work in this section.

Guo et al. proposed that applications be allowed to specify relaxed currency and consistency requirements [12]. They introduced freshness constraints and corresponding extensions to SQL syntax, and showed how to modify query optimization and execution to handle queries with such constraints. In [11] this was extended to deal with row-level caching and currency/consistency constraints rather than table-level. However, they only considered read-only queries. So they did not address the semantics of interleaved executions or concurrency control issues that arise when update transactions are allowed to see out-of-date copies. Their algorithms check whether a copy is suitable at compile time or when the read occurs, without commit time checks as in our work.

FAS [24] also allows a query to specify a currency bound. It only supports database-granularity currency control, using a freshness index that is loosely-correlated to the delay of cache refresh. The implementation uses middleware that routes each query to a database whose freshness is appropriate. In [3] this is extended to allow table-granularity decisions, and to allow a query to access multiple databases. In both papers, queries need commit-duration locks, unlike the short-duration locks we take. Because this work demands exact consistency (in our terminology, zero drift) the correctness criterion is one-copy serializability, not RCSR.

Techniques to improve concurrency by relaxing data freshness requirements were introduced in [10][13][18]. They allow use of out-of-date copies for hot-spots, which are frequently-updated items that can be incremented or decremented, and where reads use the range of an item’s value rather than its exact value. In [1], this approach was generalized to arbitrary objects through consistency assertions on the database. The main idea is that in an equivalent serial execution, transactions are only required to see the same consistency assertions, rather than the same database states.

In [20] the authors aim to increase concurrency while reducing abort rate in a multiversion system. They explicitly require full serializability for update transactions, i.e., they do not allow divergence for reads in update transactions. They measure database-level inconsistency (fuzziness), while we measure divergence in terms of user-specified currency limits on individual objects or groups of objects read by a transaction.

Our use of error bounds is related to epsilon-serializability [19], which allows queries to read concurrently updated data while bounding the resulting inconsistency. However, their notion of inconsistency is limited to changes made by concurrently executing transactions. Thus, a query that does not read objects modified by concurrent transactions has no inconsistency in their sense. But it could still read an out-of-date copy created by a transaction that committed prior to the reader. In our setting, this read may or may not be allowed, depending on its freshness constraint. Further, epsilon-serializability has a quantitative model of inconsistency. It assumes there is a measure of “distance” between two database states and transactions can accumulate inconsistency up to a bound; there is no such notion in our approach. The only formal accounting of epsilon-serializability is [21], which only considers histories where updaters are not allowed to see any inconsistency.

Of the approaches that use integrity constraints on databases for improved concurrency, the closest to our work is probably the predicate-wise serializability (PWSR) model [15][16][23] which includes features needed by long transactions in design applications. The key idea is that often only projections of histories that access subsets of the database need to be serialized, and these data subsets can be specified by integrity constraints. Korth and Speegle [16] proposed extensions of PWSR to handle versions and nested transactions. One of their correctness classes, multiversion serializability, is similar to RCSR, in that it allows each read to be given any version that satisfies the read’s pre-condition and allows different reads in a transaction to be given versions of the same item. In this respect, our model specializes theirs. Their predicates can refer only to data items and constants, not to versions or timestamps, and thus cannot express freshness constraints. Their model allows a broad class of predicates over database values as pre- and post-conditions for transactions; in contrast, ours does not allow post-conditions, and only reads have associated pre-conditions. Moreover, their model does not consider replication or distinguish the user’s execution on a multiversion database from the system’s execution.

The following papers study availability and autonomy of distributed databases when local data is allowed to diverge from the master copy: [9][5][27][4][28]. They differ from each other in divergence metrics, the concrete update protocols, and the guaranteed divergence bound; none of these approaches support a transaction containing both updates and divergence-bounded queries. Divergence caching [14] and Olston’s work [17] consider how to set optimal bounds for approximate values given queries with
precision bounds and an update stream. For real-time data updated by sensors, [22] discusses the frequency of updates needed to keep items acceptably fresh. [25] considers how to ensure that reads see fresh enough data in a content-distribution network by optimizing the allocation of many reads with value-bounds among copies whose value-divergence is known. These papers allow precision or freshness bounds only on the queries.

10 CONCLUSION
We presented a new transaction model that allows update transactions to read stale data satisfying their freshness constraints. Our main results are: (i) a formal correctness criterion for this model, called RC-serializability, which requires an execution to be equivalent to a serial execution on a multiversion database where the freshness constraints of reads are satisfied; (ii) new algorithms that implement a variety of constraints; (iii) a proof that these algorithms are correct; and (iv) a simulation study that shows that the new algorithms perform like Read Committed for reasonable freshness levels, while also guaranteeing freshness constraints. We also discussed some optimization and implementation issues.

11 REFERENCES

12 Appendix A: CORRECTNESS PROOFS
12.1 Formal Model
Our goal is to prove that each execution of the physical system that is permitted by the synchronization algorithm is RC-serializable. That is, it is computationally equivalent to a serial execution in the ideal system such that every read operation satisfies its freshness constraint. Our approach is to first define a general correctness condition which applies to transactions with no freshness conditions, in a system with deferred updates of cached copies. We then add constraints that correspond to each variety of freshness condition. One popular approach to proving the correctness of transaction synchronization algorithms is serializability theory [6]. In this theory, the physical execution of a set of transactions is modeled as a history, which is a partial-order, denoted by $\prec$, of the transactions’ read, write, commit, and abort operations, reflecting the order in which the operations executed. A read or write operation by transaction $T_i$ on a data item $x$ is represented by $r_i[x]$ or $w_i[x]$ respectively. The commit operation of $T_i$ is represented by $c_i$ and its abort operation by $a_i$.

We need to consider three kinds of history. First, there are the user-supplied transactions, each of which is expressed as a partial-order of operations that refer to data items. User-supplied transactions do not mention copies or versions. Second, there are executions of the ideal system. In these histories, transactions read
and write versions of data items. But there are no replicas in the picture. Third, there are executions of the physical system. In these histories, there are copiers as well as transactions. Read and write operations access copies of data items, but there are no versions in the picture. In this sub-section, we define this formal model. In later sub-sections we use it to prove RC-serializability.

**User-Transactions:** A user-transaction $T_i$ is a partial order $\prec_i$ over operations on data items such that

1. $T_i \subseteq \{r_i(x), w_i(x) \mid x$ is a data item $\} \cup \{c_i\}$
2. $c_j \in T_i$
3. for all read and write operations $p \in T_p, p \prec_i c_i$
4. if $r_i(x), w_i(x) \in T_p$, then $r_i(x) \prec_i w_i(x)$

To keep the notation simple, we assume that each transaction performs at most one read and at most one write on any item (U1) above. This allows us to talk later about “the version of $x$ read by $T_p$” without mentioning the particular read operation that read $x$. (U2) says that we only deal with committed transactions. In effect, this assumes that a rollback mechanism removes all effects of aborted transactions on the database or on other transactions, so we do not include them in our formal model. (U3) says that all of a transaction’s reads and writes precede its commit in the partial order, which reflects the fact that all of its reads and writes execute before it terminates. In (U4), we assume that a transaction does not read a value that it previously wrote. This is not practical in many cases but it simplifies much of the formalism. We discuss how to weaken it in Section 8.1.

**Histories in the Ideal System:** A history of the ideal system with respect to a set of user-transactions $T = \{T_1, \ldots, T_n\}$ is a partial order over the operations of transactions in $T$ where data items are replaced by versions of data items. We formalize this using the approach in [6], page 147. We formalize the replacement of data items by versions by a version translation function $h$ that maps each $w_i(x)$ into $w_i[x]$, each $r_i(x)$ into $r_i[x]$ for some $k$, and each $c_i$ into $c_i$, where $x_i$ denotes the version of $x$ written by $T_i$. Given a version translation function $h$, an ideal history $(H, \prec_i)$ with respect to $T$ is defined as follows:

1. $H = h(\bigcup_{i=1}^n T_i)$, where $h$ is total.
2. for each $T_i$ and all ops $p, q \in T_i$, if $p \prec_i q$ then $h(p) \prec_i h(q)$
3. if $r_i[x] \in H$, then $w_i[x] \prec_i r_i[x]$
4. if $h(r_i[x]) = r_i[x]$, then $c_i \prec_i c_i$
5. if $w_i[x], w_i[x] \in H$, then $c_i \prec_i c_i$ or $c_i \prec_i c_i$

Condition (11) says that each operation on a data item is translated into an operation on a version. (12) says that the history preserves each transaction’s required orderings. (13) says that a transaction can read a version only after the version was produced. (14) says that before a transaction $T_i$ commits, every transaction $T_j$ that produced a version it read has committed. In (13) and (14), note that $i \neq k$ due to property (U4) of user-transactions. (15) says that the commit operations of transactions that write each data item are totally ordered. We will see why momentarily.

In our correctness proofs we will need to argue that two histories are “equivalent,” in the sense that each transaction that appears in both histories performs the same computation and that the histories leave the database in the same final state. We formalize this using the concepts of reads-from and final-write [6]. If transactions are determinate, we can argue that they perform the same computation by saying that the histories are defined over the same transactions and each transaction reads the same input in both histories. We formalize the latter as the reads-from relation between transactions: In an ideal history $(H, \prec_i)$, transaction $T_i$ reads-x-from $T_j$ if $r_i[x] \in H$.

If each transaction reads the same input in both histories, it performs the same computation in both histories, and hence writes the same values in both histories. Thus, we can argue that the histories leave the database in the same final state by showing that the last transaction to write into each data item is the same in both histories. We therefore define the final write of $x$ in $H$ to be the write on $x$ whose transaction committed last among all writers of $x$. Formally, in an ideal history $(H, \prec_i)$, $w_i[x] \in H$ is the final-write of $x$ if, for all $w_i[x] \in H$, $i \neq k$ implies $c_i < c_k$. Property (15) ensures this notion is well-defined for any item on which writes occur.

We are particularly interested in serial ideal histories, which are the user’s model of computation. We say that an ideal history $(H, \prec_i)$ is serial if for all transactions $T_i, T_j \in H$ either all operations of $T_i$ precede those of $T_j$ or vice-versa. That is $(\forall p \in T_i)((\forall q \in T_j)(p \prec_i q))$.

**Histories in the Physical System:** A history of the physical system with respect to a set of user-transactions $T = \{T_1, \ldots, T_n\}$ and copiers $C = \{C_1, \ldots, C_m\}$ is a partial order over (i) the operations of transactions in $T$ where data items are replaced by copies of data items, and (ii) the operations of copiers. We formalize this as in [6] page 271, by mapping each $r_i[x]$ into $r_i[x^p]$ and each $w_i[x]$ into $w_i[x^p]$ for some copies $x^p$ and $x^q$ of $x$ (possibly $A=B$), and each $c_i$ into $c_i$. A copier $C_i$ is a partial order $\prec$ over operations on copies such that

1. $C_i \subseteq \{r[x^p], w[x^p] \mid x^p$ is $x$’s master and $x^q$ is a replica}$
2. if $w[x^p] \in C_i$, then $r[x^q] \in C_i$ and $r[x^q] \prec w[x^p]$

Given a copy translation function $h$, a physical history $(H, \prec_p)$ with respect to transactions $T$ and copiers $C$ is defined as follows:

1. $H = h(\bigcup_{i=1}^n T_i)$, the union of $T_i$'s
2. for each $T_i$ and all ops $p, q \in T_i$, if $p \prec_p q$ then $h(p) \prec_i h(q)$
3. if $r_i[x] \in H$, then $w_i[x] \prec_i r_i[x]$
4. if $h(r_i[x]) = r_i[x]$, then $c_i \prec_i c_i$
5. if $w_i[x], w_i[x] \in H$, then $c_i \prec_i c_i$ or $c_i \prec_i c_i$

Condition (P1) says that each operation on a data item is translated into an operation on a copy. (P2) says that the history preserves each transaction’s required orderings. (P3) says that a transaction can read a copy only if the copy was previously written at least once. (P4) ensures that the value read by each read operation and the final value of each copy are well-defined.
A physical history models the execution of a particular system. The behavior of that system affects the definitions of reads-from and final-write. For generality, we define reads-from and final-write here assuming ordinary read and write semantics. We modify them to include timestamp considerations in the next section.

Let \( D_1, D_2, \) and \( D_3 \) each be either a user-transaction or a copier. That is, \( \{ D_1, D_2, D_3 \} \subseteq (T \cup C) \). In a physical history \( (H, \prec_H) \), \( D_i \) reads-x-from \( D_j \) if there exists a copy \( x^i \) such that \( w_i(x^i) \prec_H r_j(x^i) \) and there is no \( w_i(x^i) \prec_H w_j(x^i) \prec_H r_j(x^i) \). We also need an extended reads-from relationship that incorporates the special semantics of a copier, namely, that it writes the same values that it reads. We therefore define the takes-from relationship: if \( T_i \) and \( T_j \) are user-transactions, then \( T_i \) takes-x-from \( T_j \) if either \( T_i \) reads-x-from \( T_j \) or there exists a copier \( C_j \) such that \( T_i \) reads-x-from \( C_j \) and \( C_j \) reads-x-from \( T_j \).

In a physical history \( (H, \prec_H) \), \( w_i(x^i) \in H \) is the final-write of copy \( x^i \) if, for all \( w_j(x^i) \in H, \ i \neq k \) implies \( w_j(x^i) < w_i(x^i) \). Property (P4) of physical histories ensures this notion is well-defined.

Correctness Criteria: Recall that our goal is to show that each physical history is computationally equivalent to a serial ideal history in which every read operation satisfies its freshness constraint. To formalize this, we say that a physical history \( (H, \prec_H) \) is **RC-serializable (RCSR)** if there exists a serial ideal history \( (H', \prec_{H'}) \) such that

1. \( H' \) is defined over the user-transactions of \( H \);
2. for every data item \( x \), and for every pair of user-transactions \( T_i, T_j \in H, T_i \) takes-x-from \( T_j \) in \( H' \) if and only if \( T_i \) reads-x-from \( T_j \) in \( H' \); and
3. for every data item \( x \), there is a copy \( x^i \) of \( x \) such that \( w_i(x^i) \) is the final write of \( x^i \) in \( H' \) if and only if \( w_i(x^i) \) is the final write of \( x^i \) in \( H' \). (R4)
4. every read operation in \( H' \) satisfies its freshness constraints. As explained earlier, conditions (R1) – (R3) show that \( H \) is equivalent to \( H' \) [6].

Condition (R3) says that \( H \) and \( H' \) have the same final-writes. It is stated to avoid referring to a master copy, since that concept is not in our formal definition of physical history. It basically says that for each \( x \), there is some copy of \( x \), that counts as the “master copy,” that is, the one whose value represents the value of \( x \) for the physical history.

We say \( H \) is strictly RCSR if there exists a serial ideal history \( (H', \prec_{H'}) \) such that conditions (R1) – (R4) hold and commit operations in \( H \) appear in the same order as in \( H' \). If (R1) – (R3) are satisfied, we say that \( H \) is **bare-RCSR**. That is, \( H \) is RCSR, discounting freshness constraints. \( H \) is strictly bare-RCSR if it is bare-RCSR and commits in \( H \) appear in the same order as in \( H' \). Unlike traditional definitions of Read Committed, bare-RCSR provides hooks to associate times to versions, which are needed to be precise about freshness constraints. We note that bare-RCSR is a strict subset of the class of all MV histories, and it includes as a strict subset the one-copy serializable histories (ISR), which in turn contain the schedules generated by most published multiversion concurrency control algorithms.

All of the algorithms we described generate histories that are strictly RCSR. We will use strictness to prove condition (R4) when freshness constraints are checked at commit time, as in BSA-FC. Strictness also implies that transaction handshakes are preserved. That is, if an external observer sees \( T_i \) before \( T_k \) in \( H \), then this will also hold in \( H' \).

12.2 Characterizing Bare-RCSR

In serializability theory, the usual approach to proving correctness is to define a directed graph over histories which is acyclic only if the history satisfies the relevant variation of serializability. We follow this approach by defining a **relaxed-currency serialization graph (RCSG)** for a physical history \( (H, \prec_H) \), denoted RCSG\((H)\), to be a directed graph whose nodes are the committed user-transactions in \( H \) and whose edges are defined as follows:

i. Takes-from edges - For each read operation \( r_i(x^i) \) if \( T_i \) takes-x-from \( T_k \) in \( H \) then \( T_k \rightarrow T_i \).

ii. Overwrites edges - For each pair of writes \( w_i(x^i), w_j(x^j) \) by user-transactions \( T_i \) and \( T_j \), if \( w_j(x^i) \prec_H w_i(x^i) \) then \( T_j \rightarrow T_i \).

As we will show in Corollary 1, if RCSG\((H)\) is acyclic then \( H \) is bare-RCSR. The intuition is that if RCSG\((H)\) is acyclic then there is a serial ideal history \( H' \) consistent with the edges of RCSG\((H)\). Conditions (i) and (ii) ensure that \( H \) and \( H' \) have the same takes-from relationships and final writes, that is, properties (R2) and (R3) in the definition of RCSR.

There is one complication with condition (ii): it fails to identify final-writes if \( T_i \) and \( T_k \) do not write into the same copy of \( x \). This does not arise in our algorithms, because all user-transactions write into the master. To capture this behavior formally we define the following property of physical histories: A physical history is **primary-copy** if for each data item \( x \) there is a copy \( x^i \) of \( x \) such that if a user-transaction writes into \( x \) then it writes into \( x^i \) and only into \( x^i \).

Our proof of Corollary 1 proceeds through a lemma and theorem that offer an alternative set of conditions to identify the equivalent serial ideal history needed to show a physical history is bare RCSR.

**Lemma 1** For each read operation \( r_i(x^i) \) in a physical history \( (H, \prec_H) \), \( H \) has exists \( T_i \) such that \( T_i \) takes-x-from \( T_k \) in \( H \).

**Proof** By (P3), there exists \( w_i(x^i) \in H \) such that \( w_i(x^i) \prec_H r_i(x^i) \). So there must be some \( T_k \) such that \( T_k \) reads-x-from \( T_k \). If \( T_k \) is a user-transaction, then we are done. If not, then \( T_k \) is a copier and by the same argument, there is a user-transaction \( T_k \) such that \( T_k \) reads-x-from \( T_k \). Thus, \( T_k \) takes-x-from \( T_k \) as desired.

**Theorem 1** Let \( (H, \prec_H) \) be a primary-copy physical history. Then any sequence \( (H', \prec_{H'}) \) and translation function \( h' \) with the following properties is an ideal history satisfying conditions (R1) - (R3).

a. \( H' \) is defined over the user-transactions in \( H \).

b. The translation function \( h' \) is defined as follows: For each write operation \( w_i(x^i) \in H \), let \( h'(w_i(x^i)) = w_i(x^i) \). For each read operation \( r_i(x^i) \) in \( H \), let \( h'(r_i(x^i)) = r_i(x^i) \) such that \( T_k \) takes-x-from \( T_k \) in \( H \).

c. \( (H', \prec_{H'}) \) is serial.
d. The order of transactions in $H'$ is consistent with RCSG($H$).
That is, if $T_i \rightarrow T_j$ is in RCSG($H$), then for every operation $p \in T_i$ and $q \in T_j$, $h(p) <_{H'} h(q)$.

e. The order of operations of each transaction in $H'$ is consistent with their order in $H$. That is, if $p, q \in T$ and $h(p) <_{H} h(q)$ then $h'(p) <_{H'} h'(q)$.

**Proof:** We first show that any $H'$ with properties (a) – (e) satisfies the conditions that define an ideal history, namely (I1) – (I5). We then show that it satisfies (R1) – (R3).

By Lemma 1 and property (b), $H'$ is total. This plus property (a) implies $H'$ satisfies (I1). To show (I2), suppose that $p$ and $q$ are operations of $T_i$ with $p < q$. Since $H$ is a physical history, by (P2) we have $h(p) <_{H} h(q)$. Hence, by (e), $h'(p) <_{H'} h'(q)$, which is (I2).

To prove (I3) if $r_{[H]} x \in H'$, then by (b) there exists $r_{[x]}^x$ in $H$ such that $h'(r_{[x]}^x) = r_{[x]} x$ and $T_i$ takes-x-from $T_i$ in $H$. Hence $w_i[x]^x$ is in $H$ for some $B$, so by (b), $H'$ contains $w_i[x]^x$. Since $T_i$ takes-x-from $T_i$ in $H$, $T_i \rightarrow T_i$ is in RCSG($H$). So by (d), $T_i$ precedes $T_i$ in $H'$. Thus, $w_i[x]^x <_{H'} r_{[x]}^x$, as desired to show (I3). From this and (c) we get $w_i[x] <_{H'} c_i$, $w_i[x] <_{H'} r_{[x]}^x$, as desired to show (I4). (I5) holds because $H'$ is serial.

$H'$ satisfies (R1) in the definition of bare-RCSR, as this is exactly (a). (R2) holds since $T_i$ reads-x-from $T_i$ in $H'$ if and only if $r_{[H]} x \in H'$, which by (b) is exactly when $T_i$ takes-x-from $T_i$ in $H$.

Condition (R3) requires that for every data item $x$, there is a copy $x'$ of $x$ such that $w_i[x'] = r_{[x']} x'$ in $H'$ if and only if $w_i[x]$ is the final write of $x$ in $H'$. $H'$ is a primary-copy history, so choose $x'$ to be the primary copy of $x$ in $H$. By definition of primary copy, the set of transactions that write $x$ is the same as the set that set write copy $x'$ in $H$. By (P4), writes on $x'$ in $H'$ are totally ordered. By (e), the corresponding writes of versions of $x$ in $H'$ are also totally ordered, and in the same order. Thus (R3) holds.

**Corollary 1** For any primary-copy physical history ($H, <_{H}$), if RCSG($H$) is acyclic then $H$ is bare-RCSR.

**Proof:** We construct a sequence ($H', <_{H'}$) and translation function $h'$ with properties (a) – (e) in the hypothesis of Theorem 1. By Theorem 1 $H'$ is a serial ideal history satisfying (R1) – (R3), and therefore $H$ is bare-RCSR.

Construct the set $H'$ and translation function $h'$ as defined by (a) and (b). By Lemma 1, $h'$ is well-defined. To define $<_{H'}$, totally order the operations of $H'$ so that $H'$ is serial and consistent with RCSG($H$) (which is possible because RCSG($H$) is acyclic) and operations within each user-transaction ($T_i, <_{i}$) are consistent with $<_{i}$ (which is consistent with $H$ because $H$ is a physical history and hence satisfies (P3)). Thus, $<_{H'}$ satisfies (c) to (e), as desired.

To characterize strictly bare-RCSR histories, we define an augmented RCSG (ARCSG) to be an RCSG with the following commit edges added: If $c_i < c_i$ in $H$, then $T_i \rightarrow T_i$.

**Corollary 2** For any primary-copy physical history ($H, <_{H}$), if ARCSG($H$) is acyclic then $H$ is strictly bare-RCSR.

**Proof:** Since ARCSG($H$) is acyclic, we can construct a sequence ($H', <_{H'}$) and translation function $h'$ exactly as in the proof of Corollary 1, but using the augmented graph ARCSG($H$) instead of RCSG($H$). Since ARCSG($H$) contains RCSG($H$) as a subgraph, we can still apply Theorem 1, to show that $H'$ is a serial ideal history satisfying (R1) – (R3). The commit-edges in ARCSG($H$), ensure commit operations in $H$ appear in the same order in $H'$ as in $H$. Therefore, $H$ is strictly bare-RCSR.

**12.3 Bare-RC-Serializability of BSA**

We defer considering freshness constraints for now and first prove that BSA is strictly bare-RC-RSR. This factors out the common parts to prove all the algorithms for various freshness constraints.

So we need to model physical histories produced by BSA. To do this, we refer to the timestamp of a write, $w_i[x]^x$, denoted $ts(w_i[x]^x)$. If $w_i[x]^x$ was issued by a user-transaction, $ts(w_i[x]^x) = ts(T_i)$. If it was issued by a copier, $ts(w_i[x]^x) = ts(w_i[x]^x)$, where $C_j$ reads-x-from $T_i$, which reflects the fact that a copier takes $T_i$’s timestamp when applying $w_i[x]^x$ to $x$. We say $D_j$, reads-x-from $D_i$ in BSA if there exists a copy $x^i$ such that $w_i[x]^x <_{H} r_{[x]}^x$ and there is no $w_i[x]^x$ such that $w_i[x]^x <_{H} w_i[x]^x <_{H} r_{[x]}^x$. We say $w_i[x]^x \in H$ is a final write of copy $x^i$ in BSA if, for all $w_i[x^i] \in H$, $i \neq j$ implies $ts(w_i[x^i]) < ts(w_i[x^i])$.

Every history produced by BSA is a physical history, provided that every copy is initialized by a write operation before being read. We characterize other properties of such histories in the following proposition.

**Proposition 1:** A physical history $H$ produced by BSA has the following properties:

A. Every committed transaction $T_i$ has a unique timestamp $ts(T_i)$
B. $ts(T_i) < ts(T_j)$ iff $c_j <_{H} c_i$
C. If $T_i$ reads-x-from $T_i$ then $ts(T_j) < ts(T_i)$
D. If $r_{[i]} x^i \in H$ for copier $C_i$, master $x^i$, and some copy $x^i$, then $r_{[i]} x^i <_{H} w_i[x^i]$
E. If $T_i$ takes-x-from $T_i$ then $ts(T_j) < ts(T_i)$
F. If $w_i[x^i] \leq w_i[x^i]$, then $ts(T_j) < ts(T_i)$

**Proof sketch:** We explain how each part of Proposition 1 follows from BSA behavior. (A) holds because the commit operation assigns a unique timestamp to every transaction. (B) holds because of (A) and that BSA commits one transaction at a time and assigns a timestamp that is larger than any previously committed transaction. (C) holds because the locking rules require the following event order: $T_i$ sets a write lock on $x^i$; $T_i$ commits and is assigned a timestamp; $T_i$ releases its write lock on $x^i$; $T_i$ sets its read lock on $x^i$; $T_i$ commits and is assigned a timestamp. (D) holds because a copier must read a data item’s master before writing into a copy.

For (E), if $T_i$ reads-x-from $T_i$ then the result follows from (C). If not, there exists a copier $C_j$ such that $T_i$ reads-x-from $C_j$ and $C_j$ reads-x-from $T_i$. The locking rules require the following event order: (i) $T_i$ sets a write lock on $x^i$; (ii) $T_i$ commits and is assigned a timestamp; (iii) $T_i$ releases its write lock on $x^i$; (iv) $C_j$ sets its read lock on $x^i$ and reads $x^i$; (v) $C_j$ writes into $x^i$ (D) implies (v) follows (iv); (vi) $T_i$ reads $x^i$; (vii) $T_i$ commits and is assigned a timestamp. Since (ii) precedes (viii), $ts(T_i) < ts(T_i)$ as desired.

For (F), the locking rules require the following event order: (i) $T_i$ sets a write lock on $x^i$; (ii) $T_i$ commits; (iii) $T_i$ releases its write
lock on \( x^M \); (iv) \( T_i \) sets a write lock on \( x^M \); (v) \( T_i \) commits. Since (ii) precedes (v), \( c_i \prec c_j \). So by (B), \( ts(T_j) < ts(T_i) \) as desired. □

**Theorem 2:** BSA is strictly bare-RCSR.

**Proof:** Let \( (H, \prec_B) \) be a physical history with respect to a copy translation function \( h \) produced by an execution of BSA. We claim that for each edge \( T_k \to T_i \) in \( \text{ARCSG}(H) \), \( ts(T_k) < ts(T_i) \). This follows immediately for takes-from edges by Proposition 1E, for overwrite edges by Proposition 1F, and for commit edges by Proposition 1B. Thus, \( \text{ARCSG}(H) \) is acyclic, so by Corollary 2, \( H \) is strictly bare-RCSR. □

12.4 RCSR with Freshness Conditions

BSA-FC extends BSA with commit-time checks of time-bound constraints, as described in Section 5.2. To prove that time-based constraints are satisfied, we need to refer to the timestamps of transactions in the ideal history. Given a timestamp domain \( tsd \), we say that a function \( ts: \bigcup_{i=1}^n T_i \to tsd \) is a valid timestamp assignment for history \( H \) if, for all \( T_i, T_j \), \( ts(T_i) < ts(T_j) \) if \( c_i < c_j \). Since a strictly bare-RCSR history \( H \) has an equivalent ideal history \( H' \) whose transactions commit in the same order as \( H \), we have the following proposition.

**Proposition 2:** If a history \( H \) is strictly bare-RCSR, then a valid timestamp assignment for \( H \) is also a valid timestamp assignment for an ideal history equivalent to \( H \). □

**Theorem 3:** BSA-FC is strictly RCSR.

**Proof** Let \( (H, \prec_B) \) be a physical history with respect to a copy translation function \( h \) produced by an execution of BSA-FC. Since BSA-FC extends BSA, it satisfies all of the properties of BSA. Thus, by Theorem 2, it is strictly bare-RCSR. Let \( H' \) be the ideal history that satisfies conditions (R1) – (R3) of RCSR and whose commits are in the same order as \( H \). By Proposition 2, the timestamp assignment for \( H \) is valid for \( H' \). All that remains is to prove condition (R4) that every read in \( H \) satisfies its freshness constraints. There are two types of freshness constraints to consider: time-bound constraints and limited-drift constraints.

First, consider a time-bound constraint \( \text{bound}(x, b) \) on read operation \( r[x^t] \). Let \( x_i \) be the version of \( x^t \) read by \( r[x^t] \). If \( x_i \) is the last version of \( x \), then \( x_j \) never becomes invalid so \( \text{bound}(x, b) \) holds. Else let \( x_j \) be the next version of \( x \), i.e. the one whose transaction has the next larger timestamp among those that updated \( x \). By the definition of \( \text{bound} \), \( \text{bound}(x, b) \) is true if \( ts(T_i) < ts(x_j)+b \). By Property VT, \( vt \leq ts(x_j) \). The BSA-FC algorithm ensures that \( ts(T_i) \leq vt+b \) is among the constraints satisfied when \( ts(T_i) \) is allocated, thus \( ts(T_i) < ts(x_j)+b \) as desired.

Next, consider a time-drift constraint \( \text{drift}(S, b) \) and two reads \( r[x^t] \) and \( r[y^t] \), where \( x \in S \) and \( y \in S \). To satisfy the definition of \( \text{drift}(S, b) \), if \( r[x^t] \) and \( r[y^t] \) read versions \( x_i \) and \( y_j \) of \( y \) respectively, then there must be timestamps \( t_i \) and \( t_j \) such that \( x_i \)'s valid interval includes \( t_i \), \( y_j \)'s valid interval includes \( t_j \), and \([t_i - t_j] < b \). This is exactly what was proved in the second-to-last paragraph of Section 5.2.1, so we are done. □

The same style of argument can be used to prove a session constraint that a transaction sees all updates made by previous transactions within the same session. Recall this is implemented by ensuring that at the time of a read \( r[x^t] \), \( \text{session-max-ts} \leq \text{valid-till}(x^t) \), where \( \text{session-max-ts} \) is the maximum timestamp of any previously committed transaction in the session. Let us abbreviate this by \( \text{sm} \leq vt(x^t) \).

To prove the session constraint formally, suppose \( r[x^t] \) and \( w[y^t] \) are in the same session, \( c_i < r[x^t] \), and \( T_i \) takes-x-from \( T_k \). We need to show either \( j = k \) or \( c_j < c_i \). If \( j = k \) we are done. If not, there are two cases. If \( smts \leq ts(T_k) \), then \( ts(T_k) \leq smts \) implies \( c_i < c_j \) and we are done. So suppose \( ts(T_k) < smts \). If \( c_i < c_j \), then we have \( ts(T_k) < ts(T_i) \), \( ts(T_i) \leq smts \) (by definition of \( smts \)), and \( smts \leq vt(x^t) \) (ensured by the implementation). But this implies \( ts(T_k) < ts(T_i) < vt(x^t) \), which contradicts Property VT, since \([ts(T_k), vt(x^t)] \) is within the valid interval of the version read by \( r[x^t] \). Thus, it must be that \( c_i < c_j \) as desired.

12.5 Final-Writes Revisited

Recall property (R3) in the definition of RCSR:

R3. For every data item \( x \), there is a copy \( x^t \) of \( x \) such that \( w[x^t] \) is the final write of \( x^t \) in \( H \) if and only if \( w[x] \) is the final write of \( x \) in \( H' \).

This requires that there be some copy \( x^t \) whose value represents the value of \( x \) for the physical history. To ensure this is the case in Theorem 1, we restricted our attention to primary-copy physical histories. That is, for each data item \( x \), there is a copy \( x^t \) of \( x \) such that if a user-transaction writes into \( x \) then it writes into \( x^t \) and only into \( x^t \). While this holds for all of the algorithms in this paper, there are practical cases where it does not hold, such as in a multi-master algorithm such as quorum consensus.

In multi-master algorithms, a weaker condition usually holds: The user-transactions that write into \( x \) can be totally ordered such that for each adjacent pair of transactions in the sequence, there is some copy they both write. For example, consider the following history: \( w_1[x^t] \ w_2[x^t] \ w_3[x^t] \ w_4[x^t] \ w_5[x^t] \ w_6[x^t] \). There is no copy that all transactions write. However, \( T_1 \) and \( T_2 \) both write \( x^t \), \( T_3 \) and \( T_4 \) both write \( x^t \), and \( T_5 \) and \( T_6 \) both write \( x^t \). Thus, the writes into \( x \) are effectively totally ordered, and the final write is \( w_6[x^t] \).

To cope with such situations, we need to extend our formalism. First, we need to allow a transaction to write into multiple copies of a data item. Second, we need to define the final-write of a data item (not a copy) in a physical history. For example, we could formalize the intuition of the previous paragraph as follows: In a physical history \( (H, \prec_B) \) where \( w_1[x^t] < w_2[x^t] \) implies \( c_i < w_2[x^t] \), \( w_1[x^t] \in H \) is a final-write of \( x \) if there is no write \( w_i[x^t] \in H, i \neq k \) such that \( w_1[x^t] < w_i[x^t] \). Third, we need to adjust property (R3) to be consistent with that definition. For example, we could say \( R3' \). For every data item \( x \), \( w_1[x^t] \) is the final write of \( x^t \) in \( H \) if and only if \( w_1[x] \) is the final write of \( x \) in \( H' \).

The exact definitions that are required depend on the kind of algorithms to be modelled. We leave the development of such algorithms and the associated serializability theory as a topic for further research.