Ballooning Δ' in the second stable region

C. M. Bishop, R. J. Hastie, A. Sykes, and H. R. Wilson
Culham Laboratory (UKAEA/Euratom Fusion Association), Abingdon, Oxfordshire OX14 3DB, England

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The stability of resistive ballooning modes in the second ideal stability region is an important issue that has recently received much attention. In this paper it is shown that the ballooning Δ' is negative, and therefore that the Δ'-driven modes are stable, throughout most of the second region. These results are compared with those of Kim and Choi [Phys. Fluids B **1**, 1444 (1989)].

I. INTRODUCTION

When the radial pressure gradient in a tokamak exceeds a critical value the equilibrium becomes unstable to the ideal ballooning mode. At still higher values of the pressure gradient, however, the plasma is once again stable. This phenomenon is known as the second stability regime, and it offers the possibility of plasmas with very high β (where β is the ratio of plasma pressure to magnetic pressure). An important issue is whether plasmas in the second regime are stable or unstable to resistive ballooning modes.

Sykes, Bishop, and Hastie[1] studied the resistive ballooning stability of the well-known s-α equilibrium by evaluating the quantity Δ', the ratio of coefficients of the small and large solutions of the ballooning equation in the asymptotic region. The resistive ballooning mode is unstable when Δ' exceeds a critical value Δ'_c. In the incompressible limit $\Delta'_c = 0$, while the inclusion of compressibility leads to $\Delta'_c > 0$.[2,3] From now on we shall, for simplicity, consider $\Delta' = 0$ to be the condition for marginal stability. Sykes, Bishop, and Hastie[1] showed that although the first stable region is unstable to resistive ballooning modes, throughout most of the second region we have $\Delta' < 0$, and so the plasma is then stable. A recent paper by Kim and Choi,[4] however, claims that Δ' is only negative over a small part of the second region. One purpose of the present paper is to point out an important error in Ref. 5 and to show that the results presented in Ref. 1 are indeed correct. The behavior of Δ' in the second region is discussed in some detail in Sec. II.

In addition to the Δ'-driven modes there exist modes driven from within the resonant layer,[5,6] and these may be unstable even when $\Delta' < 0$, as discussed in Ref. 4. Such modes are considered in Sec. III, and conclusions are presented in Sec. IV.

II. BEHAVIOR OF Δ' IN THE SECOND REGION

It was claimed by Kim and Choi[4] that Δ' for the s-α model is negative only over a small part of the second region. We shall show that this is not the case, and that Δ' is negative almost everywhere in the second region except very close to the ideal stability boundary. The error in Ref. 5 was in setting the ballooning mode parameter θ_0 equal to zero. As was shown by Sykes, Bishop, and Hastie[1] the value of θ_0 must be chosen so as to maximize the unstable zone, and in the second region this leads to values of $\theta_0 \neq 0$. By choosing the correct value for θ_0 we obtain results in agreement with those of Ref. 1 as we now show.

Figure 1 shows a plot of Δ' vs α for $s = 0.5$ and with $\theta_0 = 0$. The asymptotes where $\Delta' \to \infty$ correspond to the ideal stability boundaries. This diagram agrees with Fig. 1 of Kim and Choi,[4] although the latter was only plotted as far as $\alpha = 2.5$, and therefore did not show the final change of sign of Δ' to negative values at large α. The corresponding s-α plot is shown in Fig. 2, again with $\theta_0 = 0$. The dashed curves show the ideal stability boundaries, while the solid curves correspond to $\Delta' = 0$. A resistively unstable band with $\Delta' > 0$ is clearly seen in the second region.

We now replot these diagrams using the correct values of θ_0, i.e., those that maximize the extent of the unstable regions. As was shown in Ref. 1 the optimum value of θ_0 for resistive modes is $\theta_0 \simeq 0$, and thus the $\Delta' = 0$ curve in Fig. 2 (which was plotted for $\theta_0 = 0$) is essentially unchanged in the optimization over θ_0. For ideal modes in the second region, however, we have $\theta_0 \simeq \pi$ and this produces a significant shift in the second ideal stability boundary to larger values of α so as to engulf most of the resistively unstable region of Fig. 2. Figure 3 shows Δ' vs α, again for $s = 0.5$. The ideal second stability boundary has moved to a larger value of α, and Δ' is only positive very close to the ideal boundary. This can also be seen in Fig. 4, which shows the corresponding s-α plot, and it is clear that Δ' is negative throughout most of the second region, as is claimed in Ref. 1.

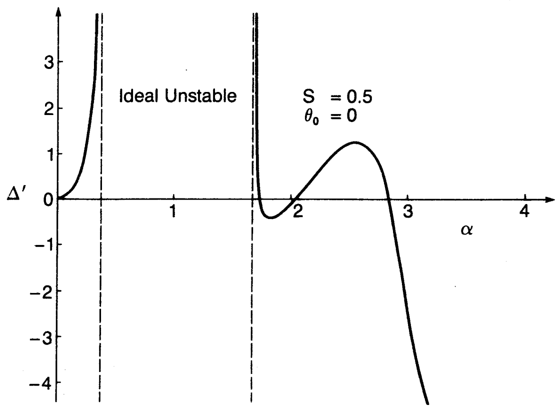

FIG. 1. Plot of Δ' vs α for $s = 0.5$ and $\theta_0 = 0$.

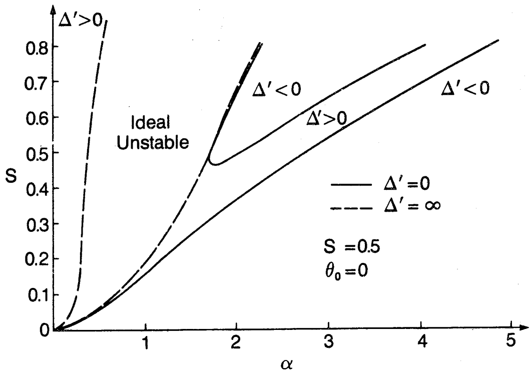

FIG. 2. A s-α diagram with $\theta_0 = 0$. Dashed curves show the ideal stability boundaries, and solid curves correspond to $\Delta' = 0$.

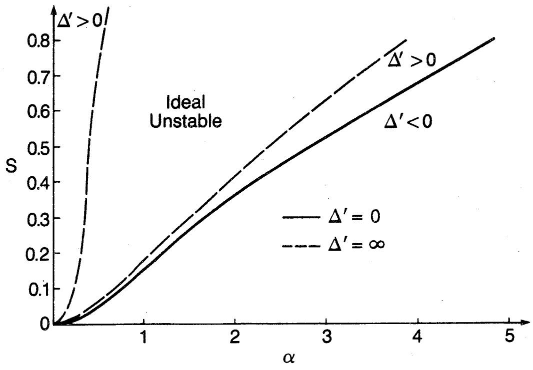

FIG. 4. A s-α diagram using the optimum value of θ_0 at each point. Dashed curves show the ideal stability boundaries, and solid curves correspond to $\Delta' = 0$.

III. ADDITIONAL CONSIDERATIONS

Kim and Choi[4] have stated that the results of Sykes, Bishop, and Hastie[1] were limited to low values of the toroidal mode number n as a result of the use of the two length-scale assumption, and the consequent introduction of Δ' as the quantity determining stability. From Drake and Antonsen[3] it follows that the two length-scale assumption is valid provided

$$n \ll \Delta' S^{1/2}/s, \tag{1}$$

where S is the magnetic Reynolds number. Strictly, this is violated at marginal stability ($\Delta' = 0$), but if we take a small value, say $\Delta' = \frac{1}{10}$, as representative of marginality, and insert typical values for a tokamak such as JET: $S \sim 10^8$, $s \sim 1$, we have $n \ll 10^3$. This is a very large number and poses no serious limitation to the theory. Indeed, the use of resistive magnetohydrodynamic (MHD) to describe ballooning stability will itself break down at lower values of n than this. For instance, ion kinetic effects will be important when $k_\perp \rho_i \sim O(1)$, where $k_\perp \sim m/r \sim nq/r$ is the perpendicular wave vector, ρ_i is the ion Lamor radius, q is the safety factor, and r is the minor radius. Again, typical values for JET would be $\rho_i \sim 2 \times 10^{-3}$ m, $r \sim 0.8$ m, and $q \sim 2$, giving $n \ll 200$. Again, this is not a serious limitation on the theory, although it does indicate that little is to be gained from avoiding the two length-scale assumption.

As discussed by Kadomtsev *et al.*[5] and Carreras *et al.*[6] there also exists a different class of resistive ballooning mode driven by the resistive layer, whose stability properties are independent of Δ'. Such modes may be unstable in the second region even when $\Delta' < 0$, as was found by Kim and Choi.[4] These results are in agreement with those found by Hender *et al.*,[7] who considered not only the s-α model but who also investigated the effects of a separatrix geometry. Drake and Antonsen[3] have shown that compressibility effects, which were absent in Ref. 4, have a significant stabilizing effect.

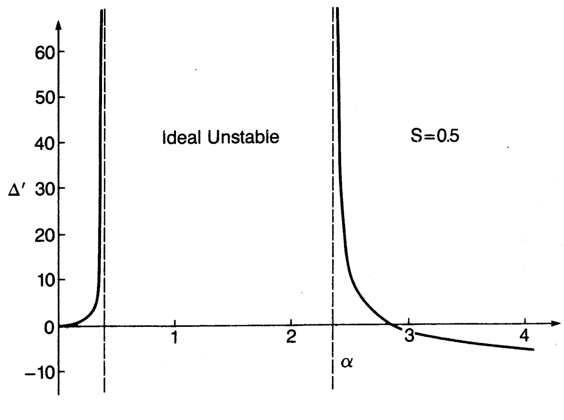

FIG. 3. Plot of Δ' vs α for $s = 0.5$, using the optimum value of θ_0 for each value of α.

IV. CONCLUSIONS

We have shown that in the ideal second stability region, $\Delta' < 0$ almost everywhere, and so equilibria in the second region will be stable to Δ'-driven resistive ballooning modes. These results differ from those of Kim and Choi,[4] and we have discussed the source of their error. The second region, may, however, be unstable to a layer-driven ballooning mode, although finite compressibility has a significant stabilizing effect.

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