

STABILITY OF ANISOTROPIC-PRESSURE TOKAMAK EQUILIBRIA TO IDEAL BALLOONING MODES

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ABSTRACT. The effect of pressure anisotropy on the stability of ideal-MHD ballooning modes is studied for a large-aspect-ratio tokamak of circular cross-section. Significant increase of the perpendicular energy of electrons or ions can cause strong modifications of the stability boundary, especially at low shear.

1. INTRODUCTION

With the use of high-power auxiliary heating in recent tokamak experiments there is evidence for the generation of pressure anisotropy. In TOSCA, for example, operating at densities of order $5 \times 10^{18} \text{ m}^{-3}$ with ECR input power of 150 kW, poloidal asymmetry of the soft-X-ray signal was observed [1], suggesting significant poloidal modulation of density or temperature, and possibly an increased population of energetic trapped electrons.

Neutral-beam injection perpendicular to the field and ICRH may be expected to modify the pressures in a similar way. Under these circumstances the distortions of the particle distribution functions from Maxwellian have two characteristics: (i) higher energy in the perpendicular direction and hence pressure anisotropy with $p_{\perp} > p_{\parallel}$; (ii) the presence of a suprathermal tail.

Such distortions can modify MHD stability properties, particularly the β -limit for ballooning instability. It is therefore of interest to determine (i) whether such modifications are favourable or unfavourable, and (ii) at what power level of the auxiliary heating, if any, they become significant.

The ballooning stability of anisotropic-pressure tokamak equilibria has been investigated by Fielding and Haas [2, 3], Cooper et al. [4] and Mikhailovskii [5]. Fielding and Haas pointed out that stability could be improved relative to an isotropic plasma if nearly perpendicular neutral-beam injection is employed to generate anisotropy in which the perpendicular pressure

component is modulated in poloidal angle θ with maximum value on the *inside* of the tokamak minor cross-section. Thus, with $\theta = 0$ defining the outside, pressures of the form

$$p_{\perp} = p_0(1 + \eta \cos\theta) \quad (1)$$

with

$$\eta < 0$$

were found to enhance stability.

In Ref. [3] a model large-aspect-ratio, circular-cross-section equilibrium in which $B\theta$ is constant on a magnetic surface, was used. Cooper extended this work and confirmed the stabilizing effect by studying the stability of global equilibria obtained from a numerical solution of the anisotropic Grad-Shafranov equation. In contrast, Mikhailovskii found that excess perpendicular energy, $p_{\perp} > p_{\parallel}$ has a destabilizing effect on ballooning and interchange modes in a large-aspect-ratio tokamak.

In this paper, we extend these results and investigate the apparent conflict between Refs [3] and [5]. In Section 2, the validity of the ideal-MHD ballooning stability theory is briefly discussed. Section 3 is devoted to a description of the large-aspect-ratio equilibrium model, and the form of the stability equation for this model. In Section 4, the results of numerical and analytic calculations of the stability boundaries are presented, and these results are discussed and conclusions are drawn in Section 5.

2. STABILITY OF BALLOONING MODES

The ballooning stability of anisotropic tokamak equilibria is determined by the anisotropic ballooning equation [3, 5]:

$$\vec{B} \cdot \nabla \left\{ \frac{1}{R^2 B_p^2} (1 + G^2) \vec{B} \cdot \nabla F \right\} + \frac{F}{R B_p} \{ K_\psi - G K_s \} \left\{ \frac{\partial}{\partial \psi} (p_\parallel + p_\perp) - \frac{B_\phi}{R B_p^2} \frac{K_\psi}{K_s} \vec{B} \cdot \nabla (p_\parallel + p_\perp) \right\} = 0 \quad (2)$$

where the magnetic field is defined by

$$\vec{B} = I \nabla \psi - \nabla \psi \times \nabla \psi$$

and B_p and B_ϕ denote the poloidal and toroidal components, respectively. The distance to the axis of symmetry is R , the poloidal magnetic flux is ψ , and K_ψ and K_s are the principal and geodesic curvatures:

$$K_\psi = \frac{\nabla \psi \cdot \nabla (p_\perp + \frac{1}{2} B^2)}{B^2 (1 - \sigma) |\nabla \psi|} \quad (3)$$

$$K_s = \frac{\vec{B} \times \nabla \psi \cdot \nabla (p_\perp + \frac{1}{2} B^2)}{B^3 (1 - \sigma) |\nabla \psi|} \quad (4)$$

with

$$\sigma = (p_\parallel - p_\perp) / B^2$$

Finally, the quantity G is defined by

$$G = \frac{R^2 B_p^2}{B} \int_{\chi_0}^{\chi} \frac{\partial}{\partial \psi} \left(\frac{I J}{R^2} \right) d\chi \quad (5)$$

where χ is the poloidal co-ordinate with $\nabla \chi$ orthogonal to $\nabla \psi$ and $\nabla \psi$, and J is the Jacobian.

Equation (2) was derived in Refs [3, 4] by minimizing the Kinetic Energy Principle of Kruskal and Oberman [6]. For plasmas with $p_\parallel \geq p_\perp$, it can be shown [7] that the kinetic term of this energy principle yields a small additional contribution of order $\epsilon^{3/2}$, where ϵ is the inverse aspect ratio. For the case of strong perpendicular anisotropy considered in this paper, the kinetic term may not be negligible, but provided the distribu-

tion function is monotonic in energy it can be shown to be positive-definite. Its neglect, therefore, yields a sufficient condition for stability.

Another requirement must be satisfied in order that the use of Eq.(2) to study β limits be justified. This arises when some of the plasma pressure and pressure gradients are associated with a high-energy, non-Maxwellian tail in the particle distribution. When such high-energy particles are present, Eq.(2) still provides a valid stability test, provided that the mode frequency at marginal stability exceeds the toroidal-precession frequency of such energetic trapped particles. Finite-Larmor-radius effects determine the frequency at marginal stability to be

$$\omega = \frac{1}{2} \omega_{*p_i}$$

where

$$\omega_{*p_i} = \frac{\ell}{N e_i} \frac{d p_i}{d \psi}$$

with ℓ the toroidal mode number. Thus the condition becomes

$$\frac{1}{2} \omega_{*p_i} > \langle \omega_d \rangle$$

where $\langle \omega_d \rangle$ is the precession frequency of the energetic particles. This is effectively a constraint on the energy W of such suprathermals and is equivalent to

$$\frac{W}{kT} < \epsilon^{-1} \quad (6)$$

When this condition is violated, it has been shown [8] that the suprathermal particles have a stabilizing effect, and that higher β values can be attained than Eq.(2) would predict.

The calculations presented here assume that inequality (6) is satisfied so that all parts of the electron and ion distributions behave in a fluid-like manner.

3. EQUILIBRIUM MODEL

The equilibrium is calculated locally, following Mercier and Luc [9], by expansion around a magnetic surface. For an isotropic equilibrium this method requires the specification of the shape of the magnetic surface and the variation of the poloidal magnetic field, B_p , around the surface. We shall consider a large-aspect-ratio equilibrium with strong perpendicular

anisotropy, i.e. $C/B^2 = O(1)$ in the aspect ratio expansion, where

$$C = 4\pi \sum_j m_j \int \frac{B}{|v_{\parallel}|} (\mu B)^2 \frac{\partial F_j}{\partial K} d\mu dK \quad (7)$$

where μ and K are the magnetic moment and particle energy per unit mass, respectively. In this case it is necessary to specify, in addition, the variation of both p_{\perp} and $\partial p_{\perp}/\partial\psi$ around the surface.

The equilibrium is determined by the following conditions:

- (a) Large aspect ratio: $\epsilon \ll 1$.
- (b) $\beta_{\parallel} \sim \beta_{\perp} \sim \epsilon$.
- (c) Circular magnetic surface.
- (d) Variation of B_p round the surface specified.
- (e) $C/B^2 \sim O(1)$ and therefore finite variation of p_{\perp} and $\partial p_{\perp}/\partial\psi$ round the magnetic surface.

A consequence of conditions (a) and (b) above follows from the equilibrium relation

$$\vec{B} \cdot \nabla p_{\parallel} + \frac{p_{\perp} - p_{\parallel}}{B} \vec{B} \cdot \nabla B = 0 \quad (8)$$

i.e. the variation of p_{\parallel} and $\partial p_{\parallel}/\partial\psi$ around the surface is of order ϵ . The $O(\epsilon)$ modulation of $(\partial p_{\parallel}/\partial\psi)_{\chi}$ round the magnetic surface is nevertheless required in the stability calculation and is obtained explicitly from Eq.(8).

To simplify the ballooning equation, a convenient variable is the poloidal angle, θ , defined in terms of χ by:

$$\theta = \int^{\chi} \frac{JB_p}{r} d\chi \quad (9)$$

where r is the radius of curvature of the magnetic surface under consideration. Since we assume that this surface is circular, r is independent of θ .

Using the large-aspect-ratio expansion and $\beta \sim \epsilon$ ordering, the curvatures K_{ψ} and K_s can be evaluated to leading order in ϵ to give

$$K_{\psi} = - \frac{\cos\theta}{R_0} \quad (10)$$

$$K_s = \frac{\sin\theta}{R_0} \quad (11)$$

where

$$R = R_0 + r \cos\theta$$

The ballooning equation (2) now takes the form

$$\begin{aligned} B_p \frac{\partial}{\partial\theta} \left\{ \frac{1}{B_p} (1 + G^2) \frac{\partial F}{\partial\theta} \right\} \\ - \frac{r^2}{B_p} F \{ \cos\theta + G \sin\theta \} \{ p'_{\parallel} + p'_{\perp} \\ + \frac{\cos\theta}{\sin\theta} \frac{\partial p_{\perp}}{\partial\theta} \frac{1}{R_0 r B_p} \} = 0 \end{aligned} \quad (12)$$

where

$$p'_{\parallel} = \frac{\partial p_{\parallel}}{\partial\psi}$$

and

$$p'_{\perp} = \frac{\partial p_{\perp}}{\partial\psi}$$

Making use of the aspect ratio expansion, the expression for G may be simplified to the form

$$\begin{aligned} G = - R r B_p^2 \int_{\theta_0}^{\theta} \frac{d\theta}{B_p^3} \left\{ \frac{\partial B^2}{\partial\psi} \right. \\ \left. + (1 - \sigma) \frac{\partial p_{\parallel}}{\partial\psi} + \frac{L L'}{R^2} \right\} \end{aligned} \quad (13)$$

where $L(\psi) = I(1 - \sigma)$, and $\partial B_p^2/\partial\psi$ may be obtained from a local solution of the Grad-Shafranov equation with the result

$$\frac{\partial B_p^2}{\partial\psi} = - 2 \left\{ \frac{B_p}{R r} + (1 - \sigma) \frac{\partial p_{\parallel}}{\partial\psi} + \frac{L L'}{R^2} \right\} \quad (14)$$

In Eqs (13) and (14) the small terms

$$\sigma \frac{\partial p_{\parallel}}{\partial\psi}$$

and B_p/rR are retained because of the lowest-order cancellation of

$$\frac{\partial p_{\parallel}}{\partial\psi} + \frac{L L'}{R^2}$$

The order- ϵ correction to $\partial p_{\parallel}/\partial\psi$ is also required, and to determine this we introduce explicit forms for $p_{\perp}(\theta)$, $\partial p_{\perp}(\theta)/\partial\psi$, $B_p(\theta)$, etc. Thus we take

$$p_{\perp} = p_{\perp 0} (1 + \eta \cos\theta)$$

$$\frac{\partial p_{\perp}}{\partial\psi} = p'_{\perp 0} (1 + \eta \cos\theta)$$

$$B_p = B_{p0}/g(\theta); \quad g(\theta) = 1 - \Lambda \cos\theta$$

$$p_{\parallel} = p_{\parallel 0} + O(\epsilon)$$

$$\frac{\partial p_{\parallel}}{\partial\psi} = p'_{\parallel 0} + O(\epsilon)$$

The parameter Λ represents the effect of the outward shift of the magnetic surfaces in toroidal equilibrium – the Shafranov shift. The degree of anisotropy is controlled by the parameters $(p_{\perp 0} - p_{\parallel 0})$, $(p'_{\perp 0} - p'_{\parallel 0})$ and η .

Since we are particularly interested in the effect of poloidal variation of the perpendicular pressure on stability ($\eta \neq 0$), we simplify the equilibrium model by choosing $p_{\parallel 0} = p_{\perp 0} \equiv p_0$, $p'_{\parallel 0} = p'_{\perp 0} \equiv p'_0$.

The small modulation in θ of $\partial p_{\parallel}/\partial\psi$ is now obtained from Eq.(8) and found to be

$$\left(\frac{\partial p_{\parallel}}{\partial\psi}\right)_1 = \frac{\eta\epsilon}{4} \cos 2\theta \left\{ p'_0 \left(1 + \frac{\eta\beta}{\epsilon}\right) + \frac{p_0}{R_0 r_B p_0} \right\} - \frac{\eta\epsilon p_0 \Lambda}{2R_0 r_B p_0} \left\{ \cos\theta + \frac{1}{3} \cos 3\theta \right\} \quad (15)$$

The parameter $p'_0 + LL'/R_0^2$ is then eliminated from the expression for G (Eqs (13) and (14)) in favour of the global shear, s , defined by

$$s = \frac{R_0 r_B p_0}{q} \frac{dq}{d\psi} \quad (16)$$

with q the safety factor.

The ballooning equation then takes the form

$$\frac{\partial}{\partial\theta} \{g(\theta)(1 + G^2) \frac{\partial F}{\partial\theta}\} + g^2 F \{ \cos\theta + G \sin\theta \} \left\{ \alpha + \frac{\alpha\eta}{2} \cos\theta \right\}$$

$$+ \frac{\eta\beta q^2}{2\epsilon} g \cos\theta \} = 0 \quad (17)$$

where

$$\alpha = -2R_0^2 \frac{B_{p0} p'_0 q^2}{B^2}; \quad \beta = \frac{2p_0}{B^2}$$

and

$$g^2 G = s(\theta - \theta_0)$$

$$+ 2 \int_{\theta_0}^{\theta} [g^2] d\theta - \alpha \left(1 + \frac{\eta\beta}{4\epsilon}\right) \int_{\theta_0}^{\theta} [g^3 \cos\theta] d\theta - \frac{\eta}{8} \left\{ \alpha \left(1 + \frac{\eta\beta}{\epsilon}\right) - \frac{\beta q^2}{\epsilon} \right\} \int_{\theta_0}^{\theta} [g^3 \cos 2\theta] d\theta - \frac{\eta\beta q^2 \Lambda}{3\epsilon} \int_{\theta_0}^{\theta} [g^3 \cos^3\theta] d\theta + \frac{Q}{\langle g^3 \rangle} \int_{\theta_0}^{\theta} [g^3] d\theta \quad (18)$$

where

$$Q = s - 2\langle g^2 \rangle + \alpha \left(1 + \frac{\eta\beta}{4\epsilon}\right) \langle g^3 \cos\theta \rangle + \frac{\eta}{8} \left\{ \alpha \left(1 + \frac{\eta\beta}{\epsilon}\right) - \frac{\beta q^2}{\epsilon} \right\} \langle g^3 \cos 2\theta \rangle + \frac{\eta\beta q^2 \Lambda}{3\epsilon} \langle g^3 \cos^3\theta \rangle \quad (19)$$

and the bracket notations in Eqs (18) and (19) are defined by:

$$\langle x \rangle = \frac{1}{2\pi} \oint x d\theta, \quad [x] = x - \langle x \rangle$$

If the poloidal magnetic field is constant round a magnetic surface (i.e. $\Lambda = 0$), then in Eq.(17)

$$g \rightarrow 1 \\ G \rightarrow s(\theta - \theta_0) - \alpha \left(1 + \frac{\eta\beta}{4\epsilon}\right) (\sin\theta - \sin\theta_0) - \frac{\eta}{16} \left\{ \alpha \left(1 + \frac{\eta\beta}{\epsilon}\right) - \frac{\beta q^2}{\epsilon} \right\} (\sin 2\theta - \sin 2\theta_0) \quad (20)$$

If the further limit $\beta/\epsilon \rightarrow 0$ is taken, Eq.(17) reduces to the ballooning equation solved in Ref.[3], while if the isotropic limit, $\eta \rightarrow 0$, is taken, Eq.(17) reduces to the $(s-\alpha)$ equation of Connor et al. [10].

4. STABILITY BOUNDARIES FOR INTERCHANGE AND BALLOONING MODES

We first note that the criterion for mirror stability,

$$\{\vec{B} \cdot \nabla(p_{\perp} + \frac{1}{2} B^2)\} \{\vec{B} \cdot \nabla B\} > 0 \quad (21)$$

takes the form

$$1 + \frac{\eta\beta}{2\epsilon} > 0 \quad (22)$$

for the equilibria considered in the previous section. Instability is, therefore, only possible for rather large values of β/ϵ and for negative values of η (p_{\perp} larger on the inside of the tokamak cross-section).

Stability to interchange modes is determined by the Mercier criterion [11], which is obtained from the asymptotic behaviour of the ballooning equation. When $\Lambda \neq 0$, the resulting criterion is complicated and has been evaluated numerically for the stability diagrams presented below. When $\Lambda = 0$, however, it takes the relatively simple form:

$$s^2 > \eta \left\{ \alpha + \frac{\beta q^2}{\epsilon} - \frac{1}{2} \frac{\alpha^2 \beta}{\epsilon} + \frac{\eta}{16} \left(\alpha + \frac{\beta q^2}{\epsilon} \right) \frac{\beta q^2}{\epsilon} \left(1 - \frac{\eta \alpha}{2q^2} \right) \right\} \quad (23)$$

For small values of α and $\beta q^2/\epsilon$, positive values of η are seen to be destabilizing. The term $\eta\alpha$ on the right-hand side of the inequality arises because of the weighting of the radial pressure gradient towards the unfavourable curvature region. This effect is, however, reinforced by the longitudinal pressure gradient appearing as $\eta\beta q^2/\epsilon$. For negative η values, these effects are stabilizing so that interchange modes were not unstable for the inward-shifted p_{\perp} surfaces of Fielding and Haas.

The destabilizing effect of pressure weighting towards the unfavourable curvature also applies to ideal ballooning modes. These modes are now required to 'balloon' less and hence cause less field line bending with its consequent stabilizing effects.

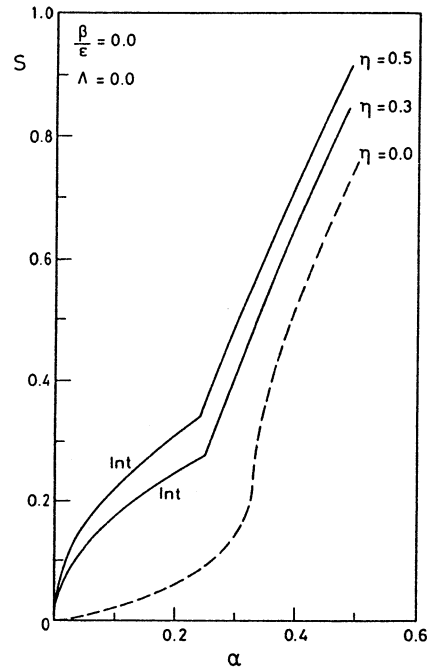


FIG.1. Effect of pressure anisotropy on the $s-\alpha$ stability boundary. In this and subsequent figures "Int" denotes the part of the stability boundary due to interchange modes.

In Fig.1 the stability boundaries in an $s-\alpha$ diagram are shown for $\beta/\epsilon = 0$ and several values of η . Figure 2 shows the destabilizing effect of longitudinal pressure gradients, $\beta/\epsilon \neq 0$, on the stability boundaries when $\eta = 0.5$. In both figures the interchange stability boundaries are included, and are seen to be important for small shear.

In Fig.3 we show the largely stabilizing effect of the outward toroidal equilibrium shift, $\Lambda = 0.25$, for an isotropic plasma, while Fig.4 shows the result of the competing influence of the equilibrium shift and unfavourable pressure weighting (positive η).

In Fig.5 we show the marked effect of unfavourable pressure weighting on the second stability region and, for comparison, the favourable effect of inward-shifted pressure surfaces. This shows the sensitivity of the ballooning instability to modulation of the perpendicular pressure round a magnetic surface.

5. DISCUSSION OF RESULTS

From the foregoing results, it is clear that auxiliary heating methods which generate 'strong perpendicular anisotropy', i.e. finite poloidal variation of $p_{\perp}(\theta)$, will have a significant effect on the critical pressure and

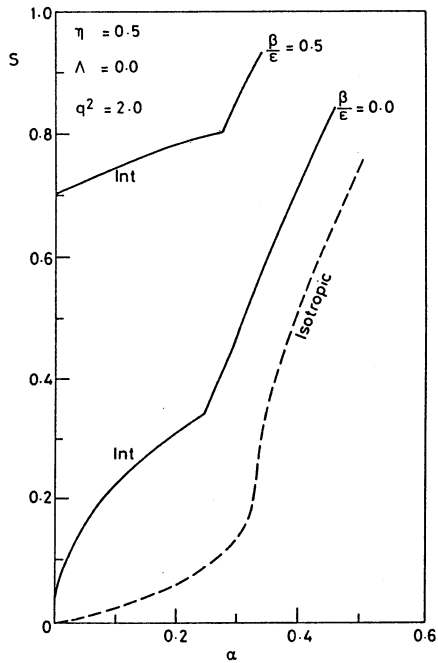


FIG.2. Destabilizing influence of longitudinal pressure gradients.

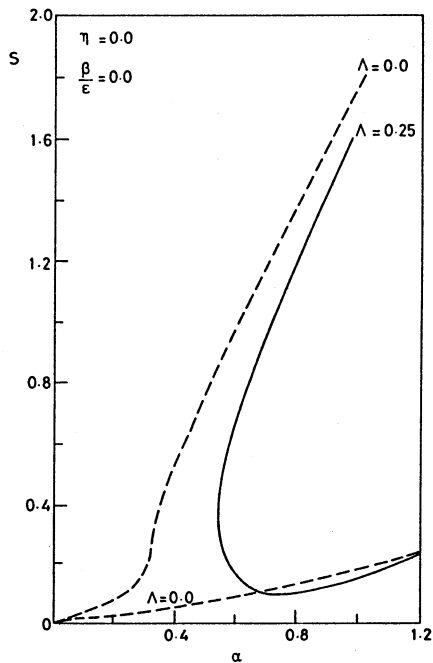


FIG.3. Effect of an outward toroidal shift in an isotropic plasma.

pressure gradient for ballooning instability. This is especially true in regions of low magnetic shear s . Fielding and Haas found that for an inward shift of the $p_{\perp} = \text{const.}$ surfaces ($\eta < 0$ in the previous section) the first and second stability regions coalesce for low

shear. The result of the present investigations is that for $\eta > 0$ interchange modes become unstable at low shear. In addition, we find that the effect of the longitudinal pressure gradient coupled to the geodesic curvature reinforces the stabilizing or destabilizing effects associated with the poloidal variations of the radial pressure gradient.

In regions of stronger shear the destabilizing effect of positive η persists, but is weaker.

These effects are quite distinct from the destabilization found by Mikhailovskii. The anisotropic equilibria studied by Mikhailovskii differ from those studied here in the ordering in the aspect ratio parameter $\epsilon = a/R$, of the pressure-like moment C defined by Eq.(7). The finite modulation of $p_{\perp}(\theta)$, in Eq.(1), implies that $C/B^2 \sim O(1)$. In Ref.[5], the ordering

$$\frac{p_{\parallel}}{B^2} \sim \frac{p_{\perp}}{B^2} \sim \frac{C}{B^2} \lesssim O(\epsilon)$$

is taken. For such equilibria both components of the pressure are constant round the magnetic surface in leading order in ϵ , so that $\eta = 0$. The degree of anisotropy in this case is measured by the parameters $(p_{\perp 0} - p_{\parallel 0})$, and $(p'_{\perp 0} - p'_{\parallel 0})$.

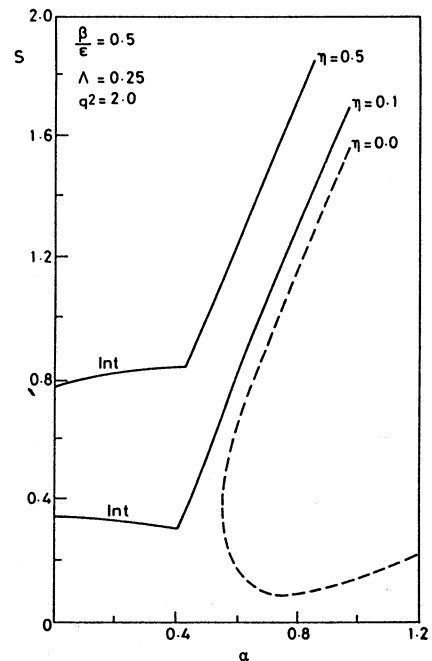


FIG.4. Competing influence of equilibrium shift and unfavourable pressure modulation.

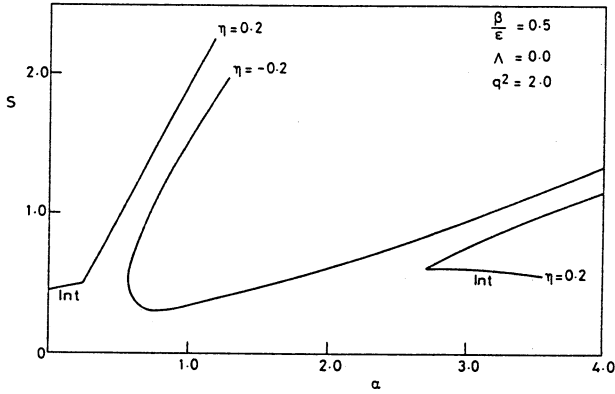


FIG.5. Effects of favourable and unfavourable pressure modulations on the second stability region.

If in Section 4 we had taken $p_{\parallel 0} \neq p_{\perp 0}$, $p'_{\parallel 0} \neq p'_{\perp 0}$ the interchange stability criterion would have become (when $\Lambda = 0$):

$$s^2 > \bar{\alpha} \left(\frac{\beta_{\perp} - \beta_{\parallel}}{\epsilon} \right) q^2 + \eta \left\{ \alpha_{\perp} + \beta_{\perp} q^2 / \epsilon - \bar{\alpha} \alpha_{\perp} \left(\beta_{\perp} - \frac{1}{2} \beta_{\parallel} \right) / \epsilon \right. \\ \left. + \frac{\eta}{64} \frac{\beta q^2}{\epsilon} \left(\alpha_{\perp} + \frac{\beta_{\perp} q^2}{\epsilon} \right) \left(1 - \frac{\eta \alpha_{\perp}}{2 q^2} \right) \right\}$$

where

$$\bar{\alpha} = - R_0^2 B_p^2 (p'_{\parallel 0} + p'_{\perp 0}) q^2 / B^2$$

$$\alpha_{\perp} = - 2 R_0^2 B_p^2 p'_{\perp 0} q^2 / B^2$$

$$\beta_{\perp} = 2 p_{\perp 0} / B^2$$

$$\beta_{\parallel} = 2 p_{\parallel 0} / B^2$$

Here the first term on the right-hand side shows the destabilizing effect of $\beta_{\perp} > \beta_{\parallel}$ found by Mikhailovskii.

We conclude that two parameters characterizing the anisotropy of a tokamak with auxiliary heating are of importance in assessing the effect of anisotropy on ballooning stability and, therefore, the β limits. These are $(p_{\perp} - p_{\parallel})/p_{\parallel}$, and $\{p_{\perp}(\theta = 0) - p_{\perp}(\theta = \pi)\}/p_{\perp}(\theta = \pi)$, which is a measure of the parameter η of the foregoing study. In optimizing the parameters of any heating scheme, one desirable feature ought to be the achievement of small or negative η values.

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