Influence diffusion dynamics and influence maximization in complex social networks

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(Social) networks are natural phenomena
Booming of online social networks

facebook

开心网

人人网

twitter

www.tianya.cn
Opportunities and challenges on the research of online social networks

- **Opportunities**
  - massive data set, real time, dynamic, open
  - help social scientists to understand social interactions in a large scale
  - help marketing people to target to the right audience
  - help economists to understand social economic networks

- **Challenges**
  - graph structure based large scale data analysis
  - scalable graph algorithm design
  - realistic modeling of network formation, evolution, and information/influence diffusion in networks
Our recent work on social network related research

- Social influence in social networks
  - scalable influence maximization
  - influence maximization with complex social interactions
- Game-theoretic based modeling of social interaction
  - bounded budget betweenness centrality game for network formation
  - Optimal pricing in social networks with networked effect
- Fundamental algorithms for large graphs
  - fast distance queries in power-law graphs
  - game-theoretic approach to community detection
Scalable Influence Maximization in Social Networks

[KDD’09, KDD’10, ICDM’10]

Collaborators:
Chi Wang, Yajun Wang, Siyu Yang,
Yifei Yuan, Li Zhang
Word-of-mouth (WoM) effect in social networks

- Word-of-mouth effect is believed to be a promising advertising strategy.
- Increasing popularity of online social networks may enable large scale WoM marketing

Harvard, Oct. 18, 2011
WoM (or Viral) Marketing

Level of trust on different types of ads *

- Family and friends: very effective

*Source from Forrester Research and Intelliseek

Harvard, Oct. 18, 2011
Two key components for studying WoM marketing

- **Modeling influence diffusion dynamics**, prior work includes:
  - independent cascade (IC) model
  - linear threshold (LT) model
  - voter model

- **Influence maximization**, prior work includes:
  - greedy approximation algorithm
  - centrality based heuristics
The Problem of Influence Maximization

- Social influence graph
  - vertices are individuals
  - links are social relationships
  - number $p(u,v)$ on a directed link from $u$ to $v$ is the probability that $v$ is activated by $u$ after $u$ is activated

- Independent cascade model
  - initially some seed nodes are activated
  - At each step, each newly activated node $u$ activates its neighbor $v$ with probability $p(u,v)$

- Influence maximization:
  - find $k$ seeds that generate the largest expected influence
Prior Work

- Influence maximization as a discrete optimization problem proposed by Kempe, Kleinberg, and Tardos, 2003
  - Introduce Independent Cascade (IC) and Linear Threshold (LT) models
  - Finding optimal solution is provably hard (NP-hard)
  - Greedy approximation algorithm, 63% approximation of the optimal solution
    - select k seeds in k iterations
    - in each iteration, select one seed that provides the largest marginal increase in influence spread

- Several subsequent studies improved the running time

- Serious drawback:
  - very slow, not scalable: > 3 hrs on a 30k node graph for 50 seeds
Our Work

- Exact influence computation is \#P hard, for both IC and LT models --- computation bottleneck

- Design new heuristics
  - MIA (maximum influence arborescence) heuristic [KDD’10]
    - for general independent cascade model (more realistic)
    - 10^3 speedup --- from hours to seconds
    - influence spread close to that of the greedy algorithm of [KKT’03]
  - Degree discount heuristic [KDD’09]
    - for uniform independent cascade model
    - 10^6 speedup --- from hours to milliseconds
  - LDAG (local directed acyclic graph) heuristic [ICDM’10]
    - for the linear threshold model
    - 10^3 speedup --- from hours to seconds

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Maximum Influence Arborescence (MIA) Heuristic

- For any pair of nodes u and v, find the maximum influence path (MIP) from u to v
- Ignore MIPs with too small probabilities (< parameter \( \theta \))
MIA Heuristic (cont’d)

Local influence regions

for every node v, all MIPs to v form its maximum influence in-arborescence (MIIA)
MIA Heuristic (cont’d)

- Local influence regions
  - for every node $v$, all MIPs to $v$ form its maximum influence in-arborescence (MIIA)
  - for every node $u$, all MIPs from $u$ form its maximum influence out-arborescence (MIOA)
  - computing MIAs and the influence through MIAs is fast
MIA Heuristic III: Computing Influence through the MIA structure

Recursive computation of activation probability \( ap(u) \) of a node \( u \) in its in-arborescence, given a seed set \( S \)

```
Algorithm 2 \( ap(u, S, MIA(v, \theta)) \)
1: if \( u \in S \) then
2: \( ap(u) = 1 \)
3: else if \( Ch(u) = \emptyset \) then
4: \( ap(u) = 0 \)
5: else
6: \( ap(u) = 1 - \prod_{w \in Ch(u)} (1 - ap(w) \cdot pp(w, u)) \)
7: end if
```

Can be used in the greedy algorithm for selecting \( k \) seeds, but not efficient enough
MIA Heuristic IV: Efficient updates on incremental activation probabilities

- $u$ is the new seed in $MIIA(v)$
- Naive update: for each candidate $w$, redo the computation in the previous page to compute $w$’s incremental influence to $v$
  \[ O(|MIIA(v)|^2) \]
- Fast update: based on linear relationship of activation probabilities between any node $w$ and root $v$, update incremental influence of all $w$’s to $v$ in two passes
  \[ O(|MIIA(v)|) \]
MIA Heuristic (cont’d)

- Iteration between two steps
  - Selecting the node \( v \) giving the largest marginal influence
  - Update MIAs after selecting \( v \) as the seed

- Key features:
  - updates are local
  - local updates are linear to the local tree structure
Experiment Results on MIA heuristic

Experiment setup:
- 35k nodes from coauthorship graph in physics archive
- influence probability to a node $v = 1 / (\# \text{ of neighbors of } v)$
- running time is for selecting 50 seeds

Influence spread vs. seed set size

- Greedy
- MIA
- DegreeDiscount
- Degree
- Random

Running time

- 10^3 times speed up

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Scalability of MIA heuristic

Experiment setup:
- synthesized graphs of different sizes generated from power-law graph model
- influence probability to a node $v = 1 / \text{(number of neighbors of } v)$
- running time is for selecting 50 seeds
Summary

Scalable influence maximization algorithms
- MixedGreedy and DegreeDiscount [KDD’09]
- PMIA for the IC model [KDD’10]
- LDAG for the LT model [ICDM’10]

PMIA/LDAG have become state-of-the-art benchmark algorithms for Inf. Max.

Collective citation count above 110 in less than 2 years
Handling Complex Social Interactions

[SDM’11, others under submissions]
Alex Collins, Rachel Cummings, Te Ke, Zhenming Liu, David Rincon, Xiaorui Sun, Yajun Wang, Wei Wei, Yifei Yuan, Xinran He, Guojie Song, Yanhua Li, Katie Everett, Zhi-Li Zhang
Handling complex social interactions

- people may dislike a product after usage and spread bad words about it
- a competing product may compete for social influence in the social network
- social relationships may be friends or foes
Our solutions

- people may dislike a product after usage and spread bad words about it
  - IC-N model and MIA-N algorithm
- a competing product may compete for social influence in the social network
  - CLT model and CLDAG algorithm for influence blocking maximization
- social relationships may be friends or foes
  - voter model in signed networks with exact inf. max. algorithm
IC-N model and MIA-N algorithm for the emergence and propagation of negative opinions
Negative opinions may emerge and propagate

- Negative opinions originates from poor product/service quality
- Negative opinions may be more contagious --- *negativity bias*

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Negative opinion model

- Extention of the independent cascade model
- The quality of the product to be advertised is characterized by the quality factor (QF) \( q \in [0,1] \).
- Each node could be in 3 states
  - Inactive, positive, and negative.
- When node \( v \) becomes active,
  - If the influencer is negative, the activated influencee is also negative (negative node generates negative opinions).
  - If the influencer is positive, the activated influencee
    - is positive with prob. \( q \).
    - is negative with prob. \( 1 - q \).
- If multiple activations of a node occur at the same step, randomly pick one
- Asymmetric --- negativity bias
Independent Cascading Process (without considering QF)
Independent Cascading Process (when considering QF)
## Our results (1)

- Complexity and approximation algorithm results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Objective function</th>
<th>Algorithm result</th>
<th>Negative result</th>
</tr>
</thead>
<tbody>
<tr>
<td>General directed graphs</td>
<td>Maximize expected positive nodes</td>
<td>$(1 - \frac{1}{e} - \varepsilon)$-approx alg, due to submodularity</td>
<td>Exact sol. is NP hard.</td>
</tr>
<tr>
<td>General directed graphs</td>
<td>Maximize expected (positive – negative) nodes.</td>
<td>Exists an $(1 - \frac{1}{e} - \varepsilon)$-approx alg. Only when $q$ is sufficiently large</td>
<td>Same as above</td>
</tr>
<tr>
<td>Directed graphs with different $q$ for different people</td>
<td>Maximize expected positive nodes</td>
<td>NA</td>
<td>Objective is non-submodular</td>
</tr>
</tbody>
</table>
Our results (2)

- Q: is the knowledge of quality factor important?
- guess a “universally good” value $q$ so that regardless of the actual quality factor, the seeds are good?
- No: $\exists$ social networks s.t. a wrong guess of $q$ could lead to a much worse result than the optimal one. ($\Theta(\sqrt{n/k})$)
- Intuition: which one seed to select in the following graph?

![Graph diagram with $\sqrt{n}$ nodes and $(n - \sqrt{n})$ nodes]
Our results (3)

- Q: what is the bottleneck of the approx. alg.
  - Given a specific seed set $S$, can we evaluate the expected number of positive nodes?
    - In general, #P-hard; can use Monte Carlo to approximate.
    - But exists efficient exact algorithm for arborescence (trees).
  - Developed scalable heuristic MIA-N based on influence calculation alg. for arborescences.
Computation in directed trees (in-arborescences)

- Without negative opinions, a simple recursion computes the activation probability of $u$:
  
  $$ap(u) = 1 - \prod_{w \in N_{in}(u)} (1 - ap(w)p(w, u))$$

- Difficulty with negative opinions: needs to know whether the neighbors of $u$ is positive or negative --- because of negativity bias
Solutions for in-arborescences

- Step 1: compute activation probability of $u$ at step $t$ (via dynamic programming):

$$ap(u, t) =
\begin{cases}
1 & t = 0 \land u \in S, \\
0 & t = 0 \land u \notin S, \\
0 & t > 0 \land u \in S, \\
\prod_{w \in N^{in}(u)} [1 - \sum_{i=0}^{t-2} ap(w, i)p(w, u)] - \prod_{w \in N^{in}(u)} [1 - \sum_{i=0}^{t-1} ap(w, i)p(w, u)] & t > 0 \land u \notin S.
\end{cases}$$

- Step 2: compute positive activation probability of $u$ at step $t$:

$$pap(u, t) = ap(u, t) \cdot q^{t+1}.$$
Influence spread and QF

- Results on a collaboration network with 15K nodes.
- Convex function because of negativity bias
Performance of the heuristic

- **MIA-N heuristic** performs nearly as good as the original greedy algorithm.
Scalability

- MIA-N heuristic is 3 orders of magnitude faster than Greedy
CLT model for competitive influence diffusion and CLDAG algorithm for the influence blocking maximization problem
The problem

- Consider two competing influence diffusion process, one positive and one negative
- Inf. Blocking Max.: selecting positive seeds to block the negative influence diffusion as much as possible
  - e.g. stop rumors on a company, on a political candidate, on public safety events, etc.
Our solution

- Competitive linear threshold model
  - positive influence and negative influence diffuse concurrently in the network
  - negative influence dominates in direct competition
- Prove that the objective function is submodular
- Design scalable algorithm CLDAG to achieve fast blocking effect
Influence diffusion on networks with friends and foes
The problem

- You would positively influence your friends, but influence your foes in the reverse direction
- How to model such influence?
- How to design influence maximization algorithm?
Our solution

- Voter model in signed networks
  - suitable for opinion changes from positive to negative or reverse
  - individual takes the opposite opinion from his foe
- Provide complete characterization of short term dynamics and long-term steady state behavior
- Provide exact solutions to the influence maximization problem
On going and future directions

- Model validation and influence analysis from real data
- Even faster heuristic algorithms
- Fast approximate algorithms
- Online and adaptive algorithms
Questions?