Abstract

This paper investigates the connections between two state of the art classifiers: decision forests (DFs, including decision jungles) and convolutional neural networks (CNNs). Decision forests are computationally efficient thanks to their conditional computation property (computation is confined to only a small region of the tree, the nodes along a single branch). CNNs achieve state of the art accuracy, thanks to their representation learning capabilities. We present a systematic analysis of how to fuse conditional computation with representation learning and achieve a continuum of hybrid models with different ratios of accuracy vs. efficiency. We call this new family of hybrid models conditional networks. Conditional networks can be thought of as: i) decision trees augmented with data transformation operators, or ii) CNNs, with block-diagonal sparse weight matrices, and explicit data routing functions.

Experimental validation is performed on the common task of image classification on both the CIFAR and Imagenet datasets. Compared to state of the art CNNs, our hybrid models yield the same accuracy with a fraction of the compute cost and much smaller number of parameters.

1. Introduction

Machine learning has enjoyed much success in recent years for both academic and commercial scenarios. Two learning approaches have gained particular attention: (i) random forests [1, 4, 5, 22], as used e.g. in Microsoft Kinect [15]; and (ii) deep neural networks (DNNs) [13, 20], as used for speech recognition [31] and image classification [9], among other applications. Decision trees are characterized by a routed behavior: conditioned on some learned routing function, the data is sent either to one child or another. This conditional computation means that at test time only a small fraction of all the nodes are visited, thus achieving high efficiency. Convolutional neural networks repeatedly transform their input through several (learned) non-linear transformations. Typically, at each layer all units need to perform computation. CNNs achieve state-of-the-art accuracy in many tasks, but decision trees have the potential to be more efficient. This paper investigates the connections between these two popular models, highlighting differences and similarities in theory and practice.

Related work. Decision forests were introduced in [1, 4] as efficient models for classification and regression. Forests were extended to density estimation, manifold learning and semi-supervised learning in [5]. The decision jungle variant [22] replaces trees with DAGs (directed acyclical graphs) to reduce memory consumption.

Convolutional networks were introduced for the task of digit recognition in [6]. More recently they have been applied with great success to the task of image classification over 1,000 classes [2, 7, 8, 9, 13, 14, 20, 23, 27, 32].

In general, decision trees and neural networks are perceived to be very different models. However, the work in [21, 30] demonstrates how any decision tree or DAG can be represented as a two-layer perceptron with a special pattern of sparsity in the weight matrices. Some recent papers have addressed the issue of mixing properties of trees and convolutional networks together. For example, the two-routed CNN architecture in [13] is a stump (a tree with only two branches). GoogLeNet [27] is another example of a (imbalanced) tree-like CNN architecture.

The work in [26, 28] combines multiple “expert” CNNs into one, manually designed DAG architecture. Each component CNN is trained on a specific task (e.g. detecting an object part), using a part-specific loss. In contrast, here we investigate training a single, tree-shaped CNN model by minimizing one global training loss. In our model the various branches are not explicitly trained to recognize parts (though they may do so if this minimizes the overall loss).

The work in [33] is a cascade [29] of CNN classifiers, each trained at a different level of recognition difficulty. Their model does not consider tree-based architectures. Finally, the work in [11] achieves state of the art classification accuracy by replacing the fully-connected layers of a CNN with a forest. This model is at least as expensive as the orig-
In this section, we introduce the conditional networks model, in comparison to trees and CNNs, and discuss their efficiency and training. For clarity, we first introduce a compact graphical notation for representing both trees and CNNs.

**Representing CNNs.** Figure 2a shows the conventional way to represent an MLP, with its units (circles) connected via edges (for weights). Our new notation is shown in Fig. 2b, where the symbol $P_{ij}$ denotes the popular non-linear transformation $v_j = \sigma(Pv_i)$ between consecutive layers $i$ and $j$. The linear projection matrix is denoted ‘$P$’, and ‘$\odot$’ indicates a non-linear function $\sigma(.)$ (e.g. a sigmoid or ReLU). In the case of CNNs the function $\sigma$ could also incorporate e.g. max-pooling and drop-out. Deep CNNs are long concatenations of the structure in Fig. 2b.

**Representing trees and DAGs.** The same graphical language can also represent trees and DAGs (Fig. 3). Usually, in a tree the data is moved from one node to another transformed. In our notation this is achieved via the identity matrix $I$ (i.e. $v_j = Iv_i$). Additionally, Selecting a subset of features $v'$ from a longer vector $v$ is achieved as $v' = \Sigma v$, with $\Sigma$ non-square matrix with only one element per row equal to 1, and 0 everywhere else. Identity and selection transforms are special instances of linear projections.

---

1. The correlation matrix $A_{12}$ is not the same as the weight matrix $P_{12}$.

2. This is in contrast to representation learning approaches which estimate optimal data transformation processes. Exceptions are [16, 17].
A key operator of trees which is not present in CNNs is data routing. Routers send the incoming data to a selected sub-branch and enable conditional computation. Routers (red nodes in Fig. 3) are represented here as perceptrons, though other choices are possible. In general, a router outputs real-valued weights, which may be used to select a single best route, multiple routes (multi-way routing), or send the data fractionally to all children (soft routing).

A conditional network exhibits both data routing and non-linear data transformation within a highly branched architecture (Fig. 4).

3.1. Computational Efficiency

Efficiency through explicit data routing. Split nodes can have explicit routers where data is conditionally sent to the children according to the output of a routing function (e.g., node 2 in Fig. 4), or have implicit routers where the data is unconditionally but selectively sent to the children using selection matrices $S$ (e.g., node 1). If the routing is explicit and hard (like in trees), then successive operations will be applied to ever smaller subsets of incoming data, with the associated compute savings. Next we show how implicit conditional networks can also yield efficiency.

Efficiency of implicit routed networks. Figures 5 compares a standard CNN with a 2-routed architecture. The total numbers of filters at each layer is fixed for both to $c_1$, $c_2$, and $c_3$. The number of multiplications necessary in the first convolution is $c_2 \times c_1 k_x k_y W H$, with $W, H$ the size of the feature map and $k_x, k_y$ the kernel size (for simplicity here we ignore max-pooling operations). This is the same for both architectures. However, due to routing, the depth of the second set of filters is different between the two architectures. Therefore, for the conventional CNN the cost of the second convolution is $c_3 \times c_2 k_x k_y W H$, while for the branched architecture the cost is $c_3 \times \left(\frac{c_2}{2}\right)^2 k_x k_y W H$, i.e. half the cost of the standard CNN. The increased efficiency is due only to the fact that shallower kernels are convolved with shallower feature maps. Simultaneous processing of parallel routes may yield additional time savings.\footnotemark

\footnotetext[3]{Feature not yet implemented in Caffe [28].}

3.2. Back-propagation Training

Implicitly-routed conditional networks can be trained with the standard back-propagation algorithm [13, 27]. The selection functions $S$ become extra parameters to optimize over, and their gradients can be derived straightforwardly. Now we show that explicitly-routed networks can also be trained using back-propagation. To do so we need to compute partial derivatives with respect to the router’s parameters (all other differentiation operations are as in conventional CNNs). We illustrate this using the small network in Fig. 6. Here subscripts index layers and superscripts index
the routing weights with \( V \) of the second set of kernels (in yellow) changes between the two architectures. The reduction in kernel size yields fewer computations and thus higher efficiency in the branched network.

4. Experiments and Comparisons

Conditional networks generalize decision trees, DAGs and CNNs, and thus could be used in all tasks where those are successful. Here we compare those models with one another on the popular task of image-level classification. We explore the effect of different “branched” architectures on a joint measure of: i) classification accuracy, ii) test-time compute cost and iii) model size.

4.1. Conditional Sparsification of a Perceptron

We begin with a toy experiment, designed to illustrate potential advantages of using explicit routes within a neural network. We take a perceptron (the last layer of “VGG11” [23]) and train it on the 1,000 Imagenet classes, with no scale or relighting augmentation [10]. Then we turn the perceptron into a small tree, with \( R \) routes and an additional, compact perceptron as a router (see fig. 7a). The router \( P^R \) and the projection matrices \( P^i \) are trained to minimize the overall classification loss (Sec. 3.2).

Interpolating between trees and CNNs. Given a test image we apply the convolutional layers until the beginning of the tree. Then we apply the router, and its \( R \) outputs are soft-max normalized and treated as probabilities for deciding which route/s to send the image to. We can send the image only to the highest probability route only (as done in trees) or we could send it to multiple routes, e.g. the \( \tau \) most probable ones. For \( \tau = 1 \) we reproduce the behaviour of a tree. This corresponds to the left-most point in the curves in fig. 7b (lowest cost and highest error). Setting \( \tau = R \) corresponds to sending the image to all routes. The latter reproduces the same behaviour as the CNN, with nearly the same cost (lowest error and highest compute cost point in the curves). Different values of \( \tau \in \{1, \ldots, R\} \) correspond to different points along the error-cost curves.
Figure 7. Conditional sparsification of a single-layer perceptron. (a) We take the deep CNN model in [23] (‘VGG11’) and turn
the last fully connected layer (size $4095 \times 1000$) into a tree with $R$ routes ($R = 4$ shown in figure). (b) The top-5-error vs. test-time-
cost curves for six conditional networks trained with different val-
ues of $R \in \{2, 4, 6, 8, 16, 24, 32\}$. Test-time cost is computed as
number of floating point operations per image, and is hardware-

 Dynamic accuracy-efficiency trade-off. The ability to se-
lect the desired accuracy-efficiency operating point at run-
time allows e.g. better battery management in mobile appli-
cations. In contrast, a CNN corresponds to a single point in
the accuracy-efficiency space (see the black point in fig. 7b).
The pronounced sub-linear behaviour of the curves in fig. 7b
suggests that we can increase the efficiency considerably
with little accuracy reduction (in the figure a 4-fold effi-
ciency increase yields an increase in error of less than 1%).

 Why care about the amount of computation? Modern
parallel architectures (such as GPUs) yield high classifica-
tion accuracy in little time. But parallelism is not the only
way of increasing efficiency. Here we focus on reducing
the total amount of computations while maintaining high
accuracy. Computation affects power consumption, which
is of huge practical importance in mobile applications (to
increase battery life on a smartphone) as well as in cloud
services (the biggest costs in data centres are due to their
cooling). Next we extend conditional processing also to the
expensive convolutional layers of a deep CNN.

4.2. Comparing Various Architectures on Imagenet

Here we validate the use of conditional networks for im-
age classification in the ILSVRC2012 dataset [18]. The
dataset consists of $\sim$1.2M training images for 1000 classes,
and 50K validation images. We base our experiments on
the VGG network [23] on which the current best models
are also based [9]. Specifically, we focus on the VGG11
model as it is deep (11 layers) and relatively memory ef-
cient (trains with Caffe [10] on a single Nvidia K40 GPU).

 Global max-pooling. We found that using global max-
pooling, after the last convolutional layer is effective in
reducing the number of parameters while maintaining the
same accuracy. We trained a new network (‘VGG11-GMP’)
with such pooling, and achieved lower top-5 error than the
baseline VGG11 network (13.3% vs. 13.8%), with a de-
crease in the number of parameters of over 72%.

 Designing an efficient conditional architecture. Then we
designed the conditional network in Fig. 8 by starting with
the unrouted VGG11-GMP and splitting the convolutional
layers (the most computationally expensive layers) into a
DAG-like, routed architecture. The hypothesis is that each
filter should only need to be applied to a small number of
channels in the input feature map. Data routing is imple-
mented via filter groups [13]. Thus, at the $n$-th convolu-
tional level (with $n = 3 \ldots 5$) the filters of VGG11-GMP
are divided into $2^{(n-2)}$ groups. Each group depends only on
the results of 128 previous filters. The feature maps of the
last convolutional layer are concatenated together, and glo-
ally max-pooled before the single-routed, fully-connected
layers, which remain the same as those in VGG11-GMP.

 Training. We trained the architecture in Fig. 8 from scratch,
with the same parameters as in [23], except for using
the initialization of [9], and a learning schedule of $\gamma_t = 
\gamma_0(1 + \gamma_0 \lambda t)^{-1}$, where $\gamma_0$, $\gamma_t$ and $\lambda$ are the initial learning
rate, learning rate at iteration $t$, and weight decay, respec-
tively [3]. When the validation accuracy levelled out, the
learning rate was decreased by a factor 10, twice. Our archi-
tecture took twice as many epochs to train than VGG11, but
Figure 9. Comparing different network architectures on Imagenet. Top-5 error as a function of test-time compute and model size, for various networks, validated on the Imagenet dataset. (left) A 3D view. (middle) Error vs. compute cost. (right) Error vs. model size. Our VGG11-GMP net (dark green) reduces model size significantly. Conditional networks (denoted with circles) yield points closest to the origin, corresponding to the best accuracy-efficiency trade-off. The conditional architecture of fig.8 is the second closest to the origin. Thanks to higher efficiency it took roughly the same time.

Results: accuracy vs. compute vs. size. In order to compare different network architectures as fairly as possible, here we did not use any training augmentation aside from that supported by Caffe (mirroring and random crops). Similarly, we report test-time accuracy based only on centre-cropped images, without potentially expensive data over-sampling. This reduces the overall accuracy (w.r.t. to state of the art), but constitutes a fairer test bed for teasing out the effects of different architectures. Applying the same over-sampling to all networks produced a similar accuracy improvement in all models, without changing their ranking.

Figure 9 shows top-5 error as a function of test-time compute cost and model size, for various architectures. Compute cost is measured as the number of multiply-accumulate operations. We chose this measure of efficiency because it is directly related to the theoretical complexity on the testing (run-time) algorithm, and it is machine/implementation independent. Later we will also show how in our parallel implementation this measure of efficiency correlates well with measured timings on both CPU and GPU. Model size is defined here as the total number of parameters (network weights) and it relates to memory efficiency. Larger model sizes tend to yield overfitting (for fixed accuracy). Architectures closest to the axes origin are both more accurate and more efficient.

The conditional network of Fig. 8 corresponds to the bright green circle in Fig. 9. It achieves a top-5 error of ∼13.8%, identical to that of the VGG11 network (yellow diamond) it is based upon. However, our conditional network requires less than half the compute (45%), and almost one-fifth (21%) of the parameters. Our conditional architecture is the second closest to the origin after GoogLeNet [27] (in purple). Both [27] and [13] obtain efficiency by sending data to different branches of the network. Although they do not use “highly branched” tree structures they can still be thought as special instances of (implicit) conditional networks. GoogLeNet achieves the best results in our joint three-way metric, probably thanks to their use of: i) multiple intermediate training losses, ii) learnt low-dimensional embeddings, and iii) better tuning of the architecture to the specific image dataset. Finally, the best accuracy is achieved by [9], but even their most efficient model uses 1.9E+10 flops, and thus falls outside the plot.

Do fewer operations correspond to faster execution? Figure 10 reports a layer-wise comparison between the predicted test-time compute cost (measured as number of multiply-accumulate operations in the model) and the actual measured timings (both on CPU and GPU) for the network architecture in Fig. 8. The three histograms have been “max normalized” to aid comparison.
from the fact that in branched architectures successive layers need to run convolutions with smaller shorter kernels, on ever smaller feature maps. All architectures tested in our experiments are implemented in the same Caffe framework and enjoy the same two levels of parallelism: i) parallel matrix multiplications (thanks to BLAS\(^4\)), and ii) data parallelism, thanks to the use of mini-batches. Although highly-branched conditional networks could in theory benefit from model parallelism (computing different branches on different GPUs, simultaneously), this feature is not yet implemented in Caffe [28].

### 4.3. Comparing Various Architectures on CIFAR

We further validate our hybrid model on the task of classifying images in the CIFAR10 [12] dataset. The dataset contains 60K images of 10 classes, typically divided into 50K training images and 10K test images. We take the state-of-the-art network in Network (NiN) model as a reference [14], and we build a conditional version of it. This time the optimal conditional architecture (in Fig. 11) is constructed automatically, by using Bayesian search [24] on a parametrized family of architectures.

**Designing a family of conditional networks.** The NiN model has a large number (192) of filters in the first convolutional layer, representing a sizable amount of the overall compute.\(^5\) We build a variant (‘NiN-64’) that prepends a layer of 64 filters to the NiN model. While this variant is more complex than NiN, when routed (as described later) it allows us to split the larger layers into many routes and increase the efficiency. By changing the number of routes at each level of the NiN-64 model (from conv2) we can generate a whole family of possible conditional architectures.

**Learning the optimal network architecture.** Next we search this parametrized space of routed architectures by using Bayesian optimization [24]. In the optimization we maximized the size-normalized accuracy \(\alpha = \frac{\text{classification accuracy}}{\text{model size}}\) with respect to the parameters \(R_l = 2^i, \{i \in \mathbb{N} : 0 \leq i \leq 5\}\), where \(R_l\) is the number of nodes at layer \(l\) in the conditional network. Fig. 11 shows the resulting architecture. It turns out to be a DAG with 10 layers.

For a fair comparison, we use Bayesian optimization on the NiN architecture too. We reduce the complexity of the unrouted NiN-64 network by learning a reduction in the number of per-layer filters. *i.e.* we maximize \(\alpha\) over \(F_i = F_{\text{orig}}/2^i, \{i \in \mathbb{N} : 0 \leq i \leq 4\}\), where \(F_{\text{orig}}\) is the number of filters in layer \(l\). All networks were trained with the same parameters as [14], except for using the initialization of [9], and a learning schedule of \(\gamma_t = \gamma_0 (1 + \gamma_0 \lambda t)^{-1}\), where \(\gamma_0, \gamma_t\) and \(\lambda\) are the initial learning rate, learning rate at iteration \(t\), and weight decay, respectively [3]. Training was run for 400 epochs (max), or until the validation accuracy had not changed in 10K iterations. We split the original training set into 40K training images and 10K validation images. The remaining 10K images are used for testing.

**Results: accuracy vs. compute vs. size.** Fig. 12 shows test errors with respect to test-time cost and model size for multiple architectures. Diamonds denote unrouted networks and circles denote conditional networks. The original NiN is shown in red, and samples of unrouted, filter-reduced versions explored during the Bayesian optimization are shown in pink. A sample of 300 conditional variants are shown as grey circles. The green circle denotes one such conditional architecture close to the origin of the 3D space \((\text{test} \, - \, \text{error}, \text{test} \, - \, \text{cost}, \text{model} \, - \, \text{size})\). Most of the conditional networks proposed by the optimization are distributed near a 3D surface with either low error, low size, low compute cost, or all of them. The conditional samples are in average closer to the origin than the unrouted counterparts. The accuracy of the best conditional network is almost identical to that of the NiN model, but it is about 5 times faster and 6 times smaller.

### 4.4. Conditional Ensembles of CNNs

A key difference between CNNs and conditional networks is that the latter may include (trainable) data routers. Here we use an explicitly-routed architecture to create an ensemble of CNNs where the data traverses only selected, component CNNs (and not necessarily all of them), thus saving computation.

As an example, the branched network in Fig. 13 is applied to the ILSVRC2012 image classification task. The network has \(R = 2\) routes, each of which is itself a deep CNN. Here, we use GoogLeNet [27] as the basis of each component route, although other architectures may be used. Generalizing to \(R > 2\) is straightforward. The routes have different compute cost (denoted by different-sized rectangles), arising from differing degrees of test-time oversampling. We use no oversampling for the first route and 10X oversampling for the second route. The router determines

\(^4\)http://www.netlib.org/blas/

\(^5\)Most Imagenet networks typically use 64 – 96 conv1 filters.
Figure 12. **Comparing network architectures on CIFAR10.** Classification error as a function of test-time compute and model size, for various networks, validated on the CIFAR10 dataset. *(left)* A 3D view. *(middle)* Error vs. compute cost. *(right)* Error vs. model size. Our automatically-optimized conditional architecture (green circle) is \( \sim 5 \) times faster and \( \sim 6 \) times smaller than NiN, with same accuracy.

Figure 13. **Explicit data routing for conditional ensembles.** An explicitly-routed conditional network that mixes existing deep CNNs in a learned, data-dependent fashion.

Figure 14 shows the resulting error-cost curve. All costs, including the cost of applying the router are taken into consideration here. Given our trained conditional network, we use dynamic, multi-way data routing (Section 4.1) to generate a curve in the error-compute space. Each point on the curve shows the top-5 error on the validation set at a given compute cost, which is an amortized average over the validation set. The dashed line corresponds to the trivial error vs. compute trade-off that could be made by selecting one or other base network at random, with a probability chosen so as to achieve a required average compute cost. The fact that the green curve lies significantly below this straight line confirms the much improved trade-off achieved by the conditional network. In the operating point indicated by the green circle we achieve nearly the same accuracy as the most accurate GoogLeNet with less than half its compute cost. A conventional CNN ensemble would incur a higher cost since all routes are used for all images.

Figure 14. **Error-accuracy results for conditional ensembles of CNNs.** Error-accuracy results for the two GoogLeNet base networks are shown in purple. The dynamic error-cost curve for our conditional ensemble is in green. In the green circle we achieve same accuracy as the most accurate GoogLeNet with half its cost.

5. **Discussion and Conclusion**

This paper has investigated similarities and differences between decision trees/forests and convolutional networks. This has led us to introduce a hybrid model (namely conditional network) which can be thought both as: i) trees which have been augmented with representation learning capabilities, and ii) CNNs which have been augmented with explicit data routers and a rich, branched architecture.

Experiments on image classification have shown that highly branched architectures yield improved accuracy-efficiency trade-off as compared to trees or CNNs. The desired accuracy-efficiency ratio can be selected at run time, without the need to train a new network. Finally, we have shown how explicit routers can improve the efficiency of ensembles of CNNs, without loss of accuracy. We hope these findings will help pave the way to a more systematic exploration of efficient architectures for deep learning at scale.
References


