LDA 证明

2014 年 12 月 15 日

Gamma函数的定义为:

$$\Gamma(x) = \int_0^{\infty} u^{x-1} e^{-u} du$$

Gama函数的性质：$\Gamma(1) = 1; \Gamma(x+1) = x\Gamma(x); \Gamma(x+1) = x!;$

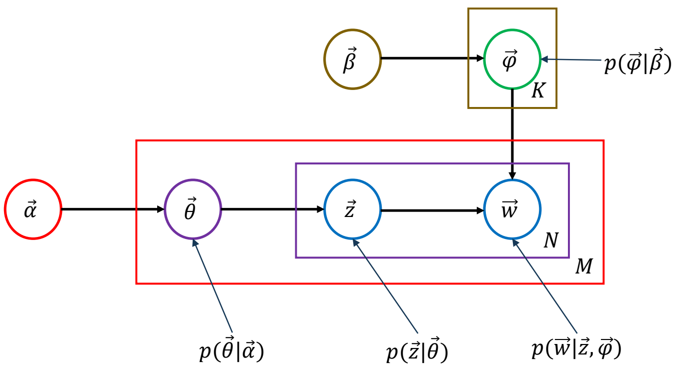
参数为$\vec{A} = (a_1, a_2 ... a_K)$的Dirichlet分布的定义如下：

$$Dir(\mu|\vec{A}) = \frac{\Gamma(\sum_{k=1}^{K} a_k)}{\prod_{k=1}^{K} \Gamma(a_k)} \prod_{k=1}^{K} \mu_k^{a_k - 1}$$

给定数据集D，K个不同类型出现的次数向量$\vec{M}$，当Dirichlet分布作为先验分布式时：

$$p(\mu|\vec{A}, \vec{M}) = Dir(\mu|\vec{A} + \vec{M}) = \frac{\Gamma(\sum_{k=1}^{K} (a_k + m_k))}{\prod_{k=1}^{K} \Gamma(a_k + m_k)} \prod_{k=1}^{K} \mu_k^{a_k + m_k - 1}$$

即：给定一个Dirichlet分布的先验和一个多项式分布的条件概率，可以得到一个Dirichlet分布的一个后验。



整个训练语料库为$\vec{w}$，文档数为M，文档m中词的个数为N_m，则第m个文档第n词为$w_{m,n}$。$\vec{w}$的似然函数为：

$$p(\vec{w}|\vec{\alpha}, \vec{\beta}) = \sum_{\vec{z}} p(\vec{w}, \vec{z}|\vec{\alpha}, \vec{\beta}) = \sum_{\vec{z}} p(\vec{w}|\vec{z}, \vec{\beta}) p(\vec{z}|\vec{\alpha})$$

主题数为K，词表大小V，则主题产生词的概率表$\vec{\phi}$为一个$K \times V$的一个矩阵，且每个主题（k）的概率表$\vec{\phi}_k$服从参数为$\vec{\beta}$的Dirichlet 分布：

$$p(\vec{\phi}|\vec{\beta}) = \prod_{k=1}^{K} p(\vec{\varphi}_k|\vec{\beta}) = \prod_{k=1}^{K} \frac{\Gamma(\sum_{t=1}^{V} \beta_t)}{\prod_{t=1}^{V} \Gamma(\beta_t)} \prod_{t=1}^{V} \varphi_{k,t}^{\beta_t - 1}$$

给定主题产生词的概率表$\vec{\phi}$以及M个文档中所有词的主题编号向量矩阵$\vec{z}$（文档数为M，文档m中词的个数为N_m，则第m个文档第n词的主题编号为$z_{m,n}$）。矩阵$\vec{z}$跟文档$\vec{w}$是等大小一一对应的一个矩阵。$p(\vec{w}|\vec{z},\vec{\phi})$的定义为:

$$p(\vec{w}|\vec{z},\vec{\phi}) = \prod_{d=1}^{M} p(\vec{w}_d|\vec{z}_d,\vec{\phi}) = \prod_{d=1}^{M}\prod_{n=1}^{N_d} \varphi_{z_{d,n},w_{d,n}} = \prod_{k=1}^{K}\prod_{t=1}^{V} \varphi_{k,t}^{n(k,t)}$$

其中，$\varphi_{z_{d,n},w_{d,n}}$表示第$d$个文档的第$n$个位置的主题$z_{d,n}$产生词$w_{d,n}$的概率。$n_{k,t}$指的是在所有文档中主题$k$产生词$t$的次数。

给定$p(\vec{\phi}|\vec{\beta})$和$p(\vec{w}|\vec{z},\vec{\phi})$，则$p(\vec{w}|\vec{z},\vec{\beta})$的定义为：

$$\begin{aligned}
p(\vec{w}|\vec{z},\vec{\beta}) &= \int p(\vec{w}|\vec{z},\vec{\phi})p(\vec{\phi}|\vec{\beta})d\vec{\phi} \\
&= \int \prod_{k=1}^{K} \frac{\Gamma(\sum_{t=1}^{V}\beta_t)}{\prod_{t=1}^{V}\Gamma(\beta_t)} \prod_{t=1}^{V} \varphi_{k,t}^{\beta_t-1} \cdot \prod_{k=1}^{K}\prod_{t=1}^{V} \varphi_{k,t}^{n(k,t)} d\varphi_{k,t} \\
&= \int \prod_{k=1}^{K} \frac{\Gamma(\sum_{t=1}^{V}\beta_t)}{\prod_{t=1}^{V}\Gamma(\beta_t)} \prod_{t=1}^{V} \varphi_{k,t}^{n(k,t)+\beta_t-1} d\varphi_{k,t} \\
&= \int \prod_{k=1}^{K} \frac{\triangle(\vec{n}_k+\vec{\beta})}{\triangle(\vec{\beta})} \frac{1}{\triangle(\vec{n}_k+\vec{\beta})} \prod_{t=1}^{V} \varphi_{k,t}^{n(k,t)+\beta_t-1} d\varphi_{k,t} \Longleftarrow \frac{1}{\triangle(\vec{\beta})} = \frac{\Gamma(\sum_{t=1}^{V}\beta_k)}{\prod_{t=1}^{V}\Gamma(\beta_t)} \\
&= \prod_{k=1}^{K} \frac{\triangle(\vec{n}_k+\vec{\beta})}{\triangle(\vec{\beta})} \underbrace{\int \frac{1}{\triangle(\vec{n}_k+\vec{\beta})} \prod_{t=1}^{V} \varphi_{k,t}^{n(k,t)+\beta_t-1} d\varphi_{k,t}}_{\int Dir(\vec{\varphi}|\vec{n}_k+\vec{\beta})d\vec{\varphi}=1} \\
&= \prod_{k=1}^{K} \frac{\triangle(\vec{n}_k+\vec{\beta})}{\triangle(\vec{\beta})}
\end{aligned}$$

文档数为M，主题数为K，则文档产生主题的概率表$\vec{\theta}$为一个$M\times K$的矩阵，且第d个文档的概率表$\vec{\theta}_d$服从参数为$\vec{\alpha}$的Dirichlet 分布：

$$p(\vec{\theta}|\vec{\alpha}) = \prod_{d=1}^{M} p(\vec{\theta}_d|\vec{\alpha}) = \prod_{d=1}^{M} \frac{\Gamma(\sum_{k=1}^{K}\alpha_k)}{\prod_{k=1}^{K}\Gamma(\alpha_k)} \prod_{k=1}^{K} \theta_{d,k}^{\alpha_k-1}$$

给定文档产生主题的概率表$\vec{\theta}$, $p(\vec{z}|\vec{\theta})$的定义为:

$$p(\vec{z}|\vec{\theta}) = \prod_{d=1}^{M} p(\vec{z}_d|\vec{\theta}_d) = \prod_{d=1}^{M}\prod_{n=1}^{N_d} \theta_{d,z_{d,n}} = \prod_{d=1}^{M}\prod_{k=1}^{K} \theta_{d,k}^{n(d,k)}$$

其中，$z_{d,n}$是第d个文档第n个单词的主题。$\vec{\theta}$中存放了d个文档产生主题$z_{d,n}$的概率$\theta_{d,z_{d,n}}$。$n(d,k)$表示第d个文档中词的主题是k的个数。

给定$\vec{\theta}$和$p(\vec{z}|\vec{\theta})$，则$p(\vec{z}|\vec{\alpha})$的定义为：

$$
\begin{aligned}
p(\vec{z}|\vec{\alpha}) &= \int p(\vec{z}|\vec{\theta})p(\vec{\theta}|\vec{\alpha})d\vec{\theta} \\
&= \int \prod_{d=1}^{M} \frac{\Gamma(\sum_{k=1}^{K}\alpha_k)}{\prod_{k=1}^{K}\Gamma(\alpha_k)} \prod_{k=1}^{K} \theta_{d,k}^{\alpha_k-1} \cdot \prod_{d=1}^{M}\prod_{k=1}^{K} \theta_{d,k}^{n(d,k)} d\vec{\theta}_{d,k} \\
&= \int \prod_{d=1}^{M} \frac{\Gamma(\sum_{k=1}^{K}\alpha_k)}{\prod_{k=1}^{K}\Gamma(\alpha_k)} \prod_{k=1}^{K} \theta_{d,k}^{n(d,k)+\alpha_k-1} d\vec{\theta}_{d,k} \\
&= \int \prod_{d=1}^{M} \frac{\triangle(\vec{n}_d+\vec{\alpha})}{\triangle(\vec{\alpha})} \frac{1}{\triangle(\vec{n}_d+\vec{\alpha})} \prod_{k=1}^{K} \theta_{d,k}^{n(d,k)+\alpha_k-1} d\vec{\theta}_{d,k} \\
&= \prod_{d=1}^{M} \frac{\triangle(\vec{n}_d+\vec{\alpha})}{\triangle(\vec{\alpha})} \underbrace{\int \frac{1}{\triangle(\vec{n}_d+\vec{\alpha})} \prod_{k=1}^{K} \theta_{d,k}^{n(d,k)+\alpha_k-1} d\vec{\theta}_{d,k}}_{\int Dir(\vec{\theta}|\vec{n}_d+\vec{\alpha})d\vec{\theta}=1} \\
&= \prod_{d=1}^{M} \frac{\triangle(\vec{n}_d+\vec{\alpha})}{\triangle(\vec{\alpha})}
\end{aligned}
$$

$\vec{w}$的似然函数为：

$$
\begin{aligned}
p(\vec{w}|\vec{\alpha},\vec{\beta}) &= \sum_{\vec{z}} p(\vec{w}|\vec{z},\vec{\beta})p(\vec{z}|\vec{\alpha}) \\
&= \sum_{\vec{z}} (\prod_{k=1}^{K} \frac{\triangle(\vec{n}_k+\vec{\beta})}{\triangle(\vec{\beta})} \cdot \prod_{d=1}^{M} \frac{\triangle(\vec{n}_d+\vec{\alpha})}{\triangle(\vec{\alpha})}) \\
&= \sum_{\vec{z}} (\prod_{k=1}^{K} \frac{\prod_{t=1}^{V}\Gamma((\vec{n}_k+\vec{\beta})_t)}{\Gamma(\sum_{t=1}^{V}(\vec{n}_k+\vec{\beta})_t)} \cdot \frac{\Gamma(\sum_{t=1}^{V}\vec{\beta}_t)}{\prod_{t=1}^{V}\Gamma(\vec{\beta}_t)} \cdot \prod_{d=1}^{M} \frac{\prod_{k=1}^{K}\Gamma((\vec{n}_d+\vec{\alpha})_k)}{\Gamma(\sum_{k=1}^{K}(\vec{n}_d+\vec{\alpha})_k} \cdot \frac{\Gamma(\sum_{k=1}^{K}\alpha_k)}{\prod_{k=1}^{K}\Gamma(\alpha_k)})
\end{aligned}
$$

其中，待优化参数为$\vec{n}_k$和$\vec{n}_d$。$\vec{n}_k$是主题k产生单词的计数表，总共有K个不同的$\vec{n}_k$，每个$\vec{n}_k$的维度为词表大小。$\vec{n}_d$是文档d产生主题的计数表，总共有M个不同的$\vec{n}_d$，每个$\vec{n}_d$的维度为主题数。

为了能够进行采样，我们需要计算产生第$i=(m,n)$个词（即:第m个文档的第n个词）的主题为k的条件概率：$p(z_i=k|\vec{z}_{\neg i},\vec{w},\vec{\alpha},\vec{\beta})$.我们定义如下符号：$\vec{n}_{k,\neg i}$表示将主题为$k$的主题到词的计数$\vec{n}_k$中把第$i$个词的计数删掉。故而$(\vec{n}_k+\vec{\beta})_i=(\vec{n}_{k,\neg i}+\vec{\beta})_i+1$，从而有：

$$\begin{aligned}
\frac{\Gamma(\sum_{t=1}^{V}(\vec{n}_{k,\neg i}+\vec{\beta})_t)}{\Gamma(\sum_{t=1}^{V}(\vec{n}_k+\vec{\beta})_t)} &= \frac{\Gamma(\sum_{t=1;t\neq i}^{V}(\vec{n}_k+\vec{\beta})_t+(\vec{n}_{k,\neg i}+\vec{\beta})_i)}{\Gamma(\sum_{t=1;t\neq i}^{V}(\vec{n}_k+\vec{\beta})_t+(\vec{n}_k+\vec{\beta})_i)} \\
&= \frac{\Gamma(\sum_{t=1;t\neq i}^{V}(\vec{n}_k+\vec{\beta})_t+(\vec{n}_{k,\neg i}+\vec{\beta})_i)}{\Gamma(\sum_{t=1;t\neq i}^{V}(\vec{n}_k+\vec{\beta})_t+(\vec{n}_{k,\neg i}+\vec{\beta})_i+1)} \\
&= \frac{1}{\sum_{t=1;t\neq i}^{V}(\vec{n}_k+\vec{\beta})_t+(\vec{n}_{k,\neg i}+\vec{\beta}_i)} = \frac{1}{\sum_{t=1}^{V}(\vec{n}_{k,\neg i}+\vec{\beta})_t}
\end{aligned}$$

同样的$\vec{n}_{m,\neg i}$表示在第m个文档中产生词的主题的计数$\vec{n}_m$中把第i个词对应的主题计数删掉。故而$(\vec{n}_m+\vec{\alpha})_k=(\vec{n}_{m,\neg i}+\vec{\alpha})_k+1$，从而有：

$$\frac{\Gamma(\sum_{k'=1}^{K}(\vec{n}_{m,\neg i}+\vec{\alpha})_{k'})}{\Gamma(\sum_{k'=1}^{K}(\vec{n}_m+\vec{\alpha})_{k'})}=\frac{1}{\sum_{k'=1}^{K}(\vec{n}_{m,\neg i}+\vec{\alpha})_{k'}}$$

$p(\vec{w}_{\neg i}|\vec{z}_{\neg i},\vec{\alpha},\vec{\beta})$ 和$p(\vec{z}_{\neg i}|\vec{\alpha},\vec{\beta})$ 表示将第$i=(m,n)$个词删掉，从而产生一个新的语料。新的语料同原始语料的区别仅仅在于第m个文档中不包含第n个词，故而其产生过程中也不需要生成对应的z_i。我们记第i个term 的ID为t，第i个词的主题为k。

$$\begin{aligned}
p(\vec{w}_{\neg i}|\vec{z}_{\neg i},\vec{\beta}) &= \prod_{k'=1;k'\neq k}^{K}\frac{\triangle(\vec{n}_{k'}+\vec{\beta})}{\triangle(\vec{\beta})})\cdot\frac{\triangle(\vec{n}_{k,\neg i}+\vec{\beta})}{\triangle(\vec{\beta})} \\
p(\vec{z}_{\neg i}|\vec{\alpha}) &= \prod_{d=1;d\neq m}^{M}\frac{\triangle(\vec{n}_d+\vec{\alpha})}{\triangle(\vec{\alpha})}\cdot\frac{\triangle(\vec{n}_{m;\neg i}+\vec{\alpha})}{\triangle(\vec{\alpha})}
\end{aligned}$$

$$
\begin{aligned}
p(z_i = k|\vec{z}_{\neg i}, \vec{w}, \vec{\alpha}, \vec{\beta}) &= \frac{p(\vec{w}, \vec{z}|\vec{\alpha}, \vec{\beta})}{p(\vec{w}, \vec{z}_{\neg i}|\vec{\alpha}, \vec{\beta})} \\
&= \frac{p(\vec{w}|\vec{z}, \vec{\alpha}, \vec{\beta})p(\vec{z}|\vec{\alpha}, \vec{\beta})}{p(\vec{w}_{\neg i}|\vec{z}_{\neg i}, \vec{\alpha}, \vec{\beta})p(\vec{z}_{\neg i}|\vec{\alpha}, \vec{\beta})p(w_i|\vec{z}_{\neg i}, \vec{\alpha}, \vec{\beta})} \\
&= \frac{p(\vec{w}|\vec{z}, \vec{\beta})}{p(\vec{w}_{\neg i}|\vec{z}_{\neg i}, \vec{\beta})p(w_i|\vec{\alpha}, \vec{\beta})} \cdot \frac{p(\vec{z}|\vec{\alpha})}{p(\vec{z}_{\neg i}|\vec{\alpha})} \\
&\propto \frac{p(\vec{w}|\vec{z}, \vec{\beta})}{p(\vec{w}_{\neg i}|\vec{z}_{\neg i}, \vec{\beta})} \cdot \frac{p(\vec{z}|\vec{\alpha})}{p(\vec{z}_{\neg i}|\vec{\alpha})} \Longleftarrow p(w_i|\vec{\alpha}, \vec{\beta}) = C \\
&= \frac{\prod_{k'=1}^{K} \frac{\triangle(\vec{n}_{k'}+\vec{\beta})}{\triangle(\vec{\beta})}}{(\prod_{k'=1;k'\neq k}^{K} \frac{\triangle(\vec{n}_{k'}+\vec{\beta})}{\triangle(\vec{\beta})}) \cdot \frac{\triangle(\vec{n}_{k,\neg i}+\vec{\beta})}{\triangle(\vec{\beta})}} \cdot \frac{\prod_{d=1}^{M} \frac{\triangle(\vec{n}_d+\vec{\alpha})}{\triangle(\vec{\alpha})}}{\prod_{d=1;d\neq m}^{M} \frac{\triangle(\vec{n}_d+\vec{\alpha})}{\triangle(\vec{\alpha})} \cdot \frac{\triangle(\vec{n}_{m;\neg i}+\vec{\alpha})}{\triangle(\vec{\alpha})}} \\
&= \frac{\triangle(\vec{n}_k + \vec{\beta})}{\triangle(\vec{n}_{k,\neg i} + \vec{\beta})} \cdot \frac{\triangle(\vec{n}_m + \vec{\alpha})}{\triangle(\vec{n}_{m;\neg i} + \vec{\alpha})} \\
&= \frac{\Gamma(\sum_{t'=1}^{V} (\vec{n}_{k,\neg i} + \vec{\beta})_{t'}) \cdot \prod_{t'=1}^{V} \Gamma((\vec{n}_k + \vec{\beta})_{t'})}{\prod_{t'=1}^{V} \Gamma((\vec{n}_{k,\neg i} + \vec{\beta})_{t'}) \cdot \Gamma(\sum_{t'=1}^{V} (\vec{n}_k + \vec{\beta})_{t'})} \cdot \frac{\Gamma(\sum_{k'=1}^{K} (\vec{n}_{m,\neg i} + \vec{\alpha})_{k'}) \cdot \prod_{k'=1}^{K} \Gamma(\vec{n}_m + \vec{\alpha})_{k'}}{\Gamma(\sum_{k'=1}^{K} (\vec{n}_m + \vec{\alpha})_{k'}) \cdot \prod_{k'=1}^{K} \Gamma(\vec{n}_{m,\neg i} + \vec{\alpha})_{k'}} \\
&= \frac{\prod_{t'=1}^{V} \Gamma((\vec{n}_k + \vec{\beta})_{t'})}{\prod_{t'=1}^{V} \Gamma((\vec{n}_{k,\neg i} + \vec{\beta})_{t'}) \cdot \sum_{t'=1}^{V} (\vec{n}_{k,\neg i} + \vec{\beta})_{t'}} \cdot \frac{\prod_{k'=1}^{K} \Gamma(\vec{n}_m + \vec{\alpha})_{k'}}{\prod_{k'=1}^{K} \Gamma(\vec{n}_{m,\neg i} + \vec{\alpha})_{k'} \cdot \sum_{k'=1}^{K} (\vec{n}_{k',\neg i} + \vec{\alpha})_{k'}} \\
&= \frac{\Gamma((\vec{n}_k + \vec{\beta})_t)}{\Gamma((\vec{n}_{k,\neg i} + \vec{\beta})_t) \cdot \sum_{t'=1}^{V} (\vec{n}_{k,\neg i} + \vec{\beta})_{t'}} \cdot \frac{\Gamma((\vec{n}_m + \vec{\alpha})_k)}{\Gamma((\vec{n}_{m,\neg i} + \vec{\alpha})_k) \cdot \sum_{k'=1}^{K} (\vec{n}_{k',\neg i} + \vec{\alpha})_{k'}} \\
&= \frac{(\vec{n}_{k,\neg i} + \vec{\beta})_t}{\sum_{t'=1}^{V} (\vec{n}_{k,\neg i} + \vec{\beta})_{t'}} \cdot \frac{((\vec{n}_{m,\neg i} + \vec{\alpha})_k}{\sum_{k'=1}^{K} (\vec{n}_{k',\neg i} + \vec{\alpha})_{k'}}
\end{aligned}
$$

由Dirichlet分布的期望公式得：

$$
\begin{aligned}
\hat{\varphi}_{kt} &= \frac{(\vec{n}_{k,\neg i} + \vec{\beta})_t}{\sum_{t'=1}^{V} (\vec{n}_{k,\neg i} + \vec{\beta})_{t'}} \\
\hat{\theta}_{mk} &= \frac{((\vec{n}_{m,\neg i} + \vec{\alpha})_k}{\sum_{k'=1}^{K} (\vec{n}_{k',\neg i} + \vec{\alpha})_{k'}}
\end{aligned}
$$

给定$\vec{\alpha}$作为先验，我们使用模型来产生了一批数据，这批数据除了没有产生第m个文档的第n个词，其余同原始数据一样，即$\vec{z}_{\neg i}$以及$\vec{w}_{\neg i}$，那么$\vec{\theta}_m$的后验概率仍然为一个Dir概率分布，故而有$p(\vec{\theta}_m|\vec{z}_{\neg i}, \vec{w}_{\neg i}) = Dir(\vec{\theta}_m|\vec{n}_{m,\neg i} + \vec{\alpha})$。同理$p(\vec{\varphi}_k|\vec{z}_{\neg i}, \vec{w}_{\neg i}) = Dir(\vec{\varphi}_k|\vec{n}_{k,\neg i} + \vec{\beta})$。基于这两个公式，我们可以用另一个方法进行证明：

$$
\begin{aligned}
p(z_i = k|\vec{z}_{\neg i}, \vec{w}) &= p(z_i = k|\vec{z}_{\neg i}, w_i = t, \vec{w}_{\neg i}) \\
&= \frac{p(z_i = k, w_i = t|\vec{z}_{\neg i}, \vec{w}_{\neg i})}{p(w_i = t)} \\
&\propto p(z_i = k, w_i = t|\vec{z}_{\neg i}, \vec{w}_{\neg i}) \\
&= \int p(z_i = k, w_i = t, \vec{\theta}_m, \vec{\varphi}_k|\vec{z}_{\neg i}, \vec{w}_{\neg i})d\vec{\theta}_m d\vec{\varphi}_k \\
&= \int p(z_i = k, \vec{\theta}_m|\vec{z}_{\neg i}, \vec{w}_{\neg i})p(w_i = t, \vec{\varphi}_k|\vec{z}_{\neg i}, \vec{w}_{\neg i})d\vec{\theta}_m d\vec{\varphi}_k \\
&= \int p(z_i = k|\vec{\theta}_m)p(\vec{\theta}_m|\vec{z}_{\neg i}, \vec{w}_{\neg i})p(w_i = t|\vec{\varphi}_k)p(\vec{\varphi}_k|\vec{z}_{\neg i}, \vec{w}_{\neg i})d\vec{\theta}_m d\vec{\varphi}_k \\
&= \int p(z_i = k|\vec{\theta}_m)p(\vec{\theta}_m|\vec{z}_{\neg i}, \vec{w}_{\neg i})d\vec{\theta}_m \int p(w_i = t|\vec{\varphi}_k)p(\vec{\varphi}_k|\vec{z}_{\neg i}, \vec{w}_{\neg i})d\vec{\varphi}_k \\
&= \int p(z_i = k|\vec{\theta}_m)Dir(\vec{\theta}_m|\vec{n}_{m,\neg i} + \vec{\alpha})d\vec{\theta}_m \cdot \int p(w_i = t|\vec{\varphi}_k)Dir(\vec{\varphi}_k|\vec{n}_{k,\neg i} + \vec{\beta})d\vec{\varphi}_k \\
&= \int \theta_{mk} Dir(\vec{\theta}_m|\vec{n}_{m,\neg i} + \vec{\alpha})d\vec{\theta}_m \cdot \int \varphi_{kt} Dir(\vec{\varphi}_k|\vec{n}_{k,\neg i} + \vec{\beta})d\vec{\varphi}_k \\
&= \mathbf{E}(\theta_{mk}) \cdot \mathbf{E}(\varphi_{kt}) \\
&= \hat{\theta}_{mk} \cdot \hat{\varphi}_{kt}
\end{aligned}
$$

⊡ initialization:

zero all count variables, $n_m^{(k)}$, n_m, $n_k^{(t)}$, n_k

for *all documents* $m \in [1, M]$ **do**

  for *all words* $n \in [1, N_m]$ *in document* m **do**

    sample topic index $z_{m,n} = k \sim Mult(1/K)$

    increment document-topic count:$n_m^{(k)} + 1$

    increment document-topic sum: $n_m + 1$

    increment topic-term count: $n_k^{(t)} + 1$

    increment topic-term sum: $n_k + 1$

  end

end

⊡ Gibbs sampling over burn-in period and sampling period:

while *not finished* **do**

  for *all documents* $m \in [1, M]$ **do**

    for *all words* $n \in [1, N_m]$ *in document* m **do**

      ⊡ for the current assignment of k to a term t for word $w_{m,n}$:

      decrement counts and sums: $n_m^{(k)} - 1$, $n_m - 1$, $n_k^{(t)} - 1$, $n_k - 1$;

      ⊡ multinomial sampling (decrements from previous step):

      sample topic index $\tilde{k} \sim p(z_i | \vec{z}_{\neg i}, \vec{w})$;

      ⊡ use the new assigment of $z_{m,n}$ to the term t for word $w_{m,n}$:

      increment counts and sums: $n_m^{(\tilde{k})} - 1$, $n_m - 1$, $n_{\tilde{k}}^{(t)} - 1$, $n_{\tilde{k}} - 1$;

    end

  end

  ⊡ Check convergence and read out parameters:

  if *converged and L sampling iterations since last read out* **then**

    ⊡ the different parameters read outs are averaged:

    read out parameters set $\vec{\phi}$

    read out parameters set $\vec{\theta}$

  end

end