Thinking Above the Code

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Why Think?

It helps us do most things:

- Hunting a sabre-toothed tiger.
- Building a house.
- Writing a program.
Why Think?

It helps us do most things:
Why Think?

It helps us do most things:

Hunting a sabre-toothed tiger.
Why Think?

It helps us do most things:

Hunting a sabre-toothed tiger.

Building a house.
Why Think?

It helps us do most things:

Hunting a sabre-toothed tiger.

Building a house.

Writing a program.
When to Think

- Hunting a sabre-toothed tiger. Before leaving the cave.
- Building a house. Before beginning construction.
- Writing a program. Before writing any code.
When to Think

Hunting a sabre-toothed tiger.
When to Think

Hunting a sabre-toothed tiger.

Before leaving the cave.
When to Think

Hunting a sabre-toothed tiger.

Before leaving the cave.

Building a house.
When to Think

Hunting a sabre-toothed tiger.

Before leaving the cave.

Building a house.

Before beginning construction.
When to Think

Hunting a sabre-toothed tiger.
  Before leaving the cave.

Building a house.
  Before beginning construction.

Writing a program.
When to Think

Hunting a sabre-toothed tiger.
  Before leaving the cave.

Building a house.
  Before beginning construction.

Writing a program.
  Before writing any code.
How to Think

"Writing is nature's way of letting you know how sloppy your thinking is."

Guindon

To think, you have to write. If you're thinking without writing, you only think you're thinking.
How to Think

“Writing is nature’s way of letting you know how sloppy your thinking is.”

Guindon
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Guindon

To think, you have to write.

If you’re thinking without writing, you only think you’re thinking.
What to Write

Hunting a sabre-toothed tiger.
Writing not invented, dangerous activity.

Building a house.
Draw blueprints.

Writing a program.
Write a
What to Write

Hunting a sabre-toothed tiger.
What to Write

Hunting a sabre-toothed tiger.

Writing not invented, dangerous activity.
What to Write

Hunting a sabre-toothed tiger.

Writing not invented, dangerous activity.

Building a house.
What to Write

Hunting a sabre-toothed tiger.

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Building a house.

 Draw blueprints.
What to Write

Hunting a sabre-toothed tiger.

*Writing not invented, dangerous activity.*

Building a house.

*Draw blueprints.*

Writing a program.
What to Write

Hunting a sabre-toothed tiger.

    Writing not invented, dangerous activity.

Building a house.

    Draw blueprints.

Writing a program.

    Write a blueprint
What to Write

Hunting a sabre-toothed tiger.

Writing not invented, dangerous activity.

Building a house.

Draw blueprints.

Writing a program.

Write a blueprint specification.
Specifications
Specifications

Don’t Panic!
Specifications

Don’t Panic!

This is a blueprint:
Specifications

Don’t Panic!

This is also a blueprint:
A spectrum of blueprints
A spectrum of blueprints
A spectrum of blueprints

Very Detailed
A spectrum of blueprints

Very Detailed
A spectrum of blueprints

Rough Sketch

Very Detailed

[Image of a rough sketch of a garden shed]

[Image of the Golden Gate Bridge]
A spectrum of blueprints

Rough Sketch

Very Detailed
A spectrum of blueprints

Rough Sketch

Ordinary

Very Detailed
A spectrum of specifications
A spectrum of specifications
A spectrum of specifications
A spectrum of specifications

Prose

Mathematical Prose

Formal
Most code is really simple.
A spectrum of specifications

Most code is really simple.

It can be specified with a couple of lines of prose.
Some code is subtle.
A spectrum of specifications

Mathematical Prose

Some code is subtle.

It requires more thought.
A spectrum of specifications

Some code is complex or very subtle or critical.
A spectrum of specifications

Some code is complex or very subtle or critical.

Especially in concurrent/distributed systems.
A spectrum of specifications

Some code is complex or very subtle or critical.

We should use tools to check it.
How to Write a Spec
How to Write a Spec

Writing requires thinking.
A very successful way of thinking.
Science makes mathematical models of reality.

Astronomy:
Reality: planets have mountains, oceans, tides, weather, . . .
Model: planet a point mass with position & momentum.
Like a Scientist
How to Think About Programs

Like a Scientist

A very successful way of thinking.
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How to Think About Programs

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Astronomy:

Reality: planets have mountains, oceans, tides, weather, . . .

Model: planet a point mass with position & momentum.
Computer Science
Computer Science

Reality: Digital systems
Reality: Digital systems

- a processor chip
Computer Science

Reality: Digital systems
  a processor chip
  a game console
Computer Science

Reality: Digital systems

- a processor chip
- a game console
- a computer executing a program

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Computer Science

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Computer Science

Reality: Digital systems
   a processor chip
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Models:
Computer Science

Reality: Digital systems
  a processor chip
  a game console
  a computer executing a program

Models: Turing machines
Computer Science

Reality: Digital systems
    a processor chip
    a game console
    a computer executing a program

Models: Turing machines
    Partially ordered sets of events
Computer Science

Reality: Digital systems
  a processor chip
  a game console
  a computer executing a program

Models: Turing machines
  Partially ordered sets of events
The Two Most Useful Models
The Two Most Useful Models

Functions
The Two Most Useful Models

Functions

Sequences of States
Functions

Model a program as a function mapping input(s) to output(s).

In math, a function is a set of ordered pairs.

Example: the square function on natural numbers \{⟨0, 0⟩, ⟨1, 1⟩, ⟨2, 4⟩, ⟨3, 9⟩, \ldots\}.
Functions

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Example: the *square* function on natural numbers
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\[ \{ \langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 2, 4 \rangle, \langle 3, 9 \rangle, \ldots \} \]
Functions

Model a program as a function mapping input(s) to output(s).

In math, a function is a set of ordered pairs.

Example: the \textit{square} function on natural numbers

\[
\{ \langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 2, 4 \rangle, \langle 3, 9 \rangle, \ldots \} 
\]

\[
square(2) = 4
\]
Functions

Model a program as a function mapping input(s) to output(s).

In math, a function is a set of ordered pairs.

Example: the $\textit{square}$ function on natural numbers

$$\{ \langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 2, 4 \rangle, \langle 3, 9 \rangle, \ldots \}$$

Domain of $\textit{square}$ is $\{0, 1, 2, 3, \ldots\}$ a.k.a. $\textit{Nat}$
Functions

Model a program as a function mapping input(s) to output(s).

In math, a function is a set of ordered pairs.

Example: the $square$ function on natural numbers
\[
\{\langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 2, 4 \rangle, \langle 3, 9 \rangle, \ldots \}
\]

To define a function, specify:

Domain of $square = Nat$
Functions

Model a program as a function mapping input(s) to output(s).

In math, a function is a set of ordered pairs.

Example: the *square* function on natural numbers

\[ \{ \langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 2, 4 \rangle, \langle 3, 9 \rangle, \ldots \} \]

To define a function, specify:

**Domain of square** = *Nat*  
\[ \text{square}(x) = x^2 \] for all \( x \) in its domain.
Functions

Model a program as a function mapping input(s) to output(s).

In math, a function is a set of ordered pairs.

Example: the square function on natural numbers
\[
\{ \langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 2, 4 \rangle, \langle 3, 9 \rangle, \ldots \}
\]

Functions in math $\neq$ functions in programming languages.
Model a program as a function mapping input(s) to output(s).

In math, a function is a set of ordered pairs.

Example: the \textit{square} function on natural numbers

\{\langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 2, 4 \rangle, \langle 3, 9 \rangle, \ldots \}  

Functions in math \neq \text{ functions in programming languages.}  
Math is much simpler.
Limitations of the Function Model

- Specifies what a program does, but not how.
- Quicksort and bubble sort compute the same function.
- Some programs don't just map inputs to outputs.
  - Some programs run "forever".
  - Operating systems
Limitations of the Function Model

Specifies what a program does, but not how.
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– Operating systems
The Standard Behavioral Model

A program execution is represented by a behavior. A behavior is a sequence of states. A state is an assignment of values to variables. A program is modeled by a set of behaviors: the behaviors representing possible executions.
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The Standard Behavioral Model

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A behavior is a sequence of states.
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The Standard Behavioral Model

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A program is modeled by a set of behaviors:

the behaviors representing possible executions.
An Example: Euclid’s Algorithm

Computes GCD of $M$ and $N$ by:

– Initialize $x$ to $M$ and $y$ to $N$.
– Keep subtracting the smaller of $x$ and $y$ from the larger.
– Stop when $x = y$.

For $M = 12$ and $N = 18$, one behavior:

$\begin{align*}
  x &= 12, \quad y = 18 \\
  \rightarrow x &= 12, \quad y = 6 \\
  \rightarrow x &= 6, \quad y = 6
\end{align*}$
An Example: Euclid’s Algorithm

An algorithm is an abstract program.
An Example: Euclid’s Algorithm

Computes GCD of $M$ and $N$ by:

– Initialize $x$ to $M$ and $y$ to $N$.
An Example: Euclid’s Algorithm

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– Stop when $x = y$.

For $M = 12$ and $N = 18$, one behavior:

\[
[x = 12, \ y = 18] \rightarrow [x = 12, \ y = 6] \rightarrow [x = 6, \ y = 6]
\]
How to Describe a Set of Behaviors

Theorem

Any set \( B \) of behaviors = all behaviors satisfying a safety property \( S \) \( \cap \) all behaviors satisfying a liveness property \( L \)
How to Describe a Set of Behaviors

Theorem

Any set $B$ of behaviors $\ = \$
How to Describe a Set of Behaviors

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Any set \( \mathcal{B} \) of behaviors =

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How to Describe a Set of Behaviors

Theorem

Any set $\mathcal{B}$ of behaviors =

all behaviors satisfying a safety property $S$

$\cap$ all behaviors satisfying a liveness property $L$

Safety Property:

False iff violated at some point in behavior.
How to Describe a Set of Behaviors

Theorem

Any set \( B \) of behaviors =

all behaviors satisfying a safety property \( S \)
\( \cap \) all behaviors satisfying a liveness property \( L \)

Safety Property:

False iff violated at some point in behavior.

Example: partial correctness.
How to Describe a Set of Behaviors

Theorem

Any set $\mathcal{B}$ of behaviors $=$

all behaviors satisfying a safety property $S$

$\cap$ all behaviors satisfying a liveness property $L$

Liveness Property:

Need to see complete behavior to know if it’s false.
How to Describe a Set of Behaviors

Theorem

Any set $B$ of behaviors =

all behaviors satisfying a safety property $S$

$\cap$ all behaviors satisfying a liveness property $L$

Liveness Property:

Need to see complete behavior to know if it’s false.

Example: termination.
How to Describe a Set of Behaviors

Theorem

Any set $B$ of behaviors =

all behaviors satisfying a safety property $S$
\cap all behaviors satisfying a liveness property $L$

Specify a set of behaviors with

– a safety property
– a liveness property
In practice, specifying safety is more important.
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That’s where errors are most likely to occur.
In practice, specifying safety is more important.

That’s where errors are most likely to occur.

To save time, I’ll ignore liveness.
How to Specify a Safety Property

With two things:
– The set of possible initial states.
– A next-state relation, describing all possible successor states of any state.

What language should we use?

Let's act like scientists.
Let's use math.
How to Specify a Safety Property

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What language should we use?

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Let’s use math.
The Set of Initial States

For Euclid's Algorithm:
\[(x = M) \land (y = N)\]

Only possible initial state:
\[
\begin{bmatrix}
    x = M, \\
    y = N
\end{bmatrix}
\]
The Set of Initial States

Described by a formula.
The Set of Initial States

Described by a formula.

For Euclid’s Algorithm: \((x = M) \land (y = N)\)
The Set of Initial States

Described by a formula.

For Euclid’s Algorithm: \((x = M) \land (y = N)\)

Only possible initial state: \([x = M, y = N]\)
The Next-State Relation

Described by a formula.

For Euclid's Algorithm:

\[(x > y \land x' = x - y \land y' = y) \lor (y > x \land y' = y - x \land x' = x)\]
The Next-State Relation

Described by a formula.

Unprimed variables for current state,
Primed variables for next state.
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Described by a formula.

Unprimed variables for current state,
Primed variables for next state.

For Euclid’s Algorithm:
The Next-State Relation

Described by a formula.

Unprimed variables for current state,
Primed variables for next state.

For Euclid’s Algorithm:  
\[
(x > y) \\
\lor (y > x)
\]
The Next-State Relation

Described by a formula.
Unprimed variables for current state,
Primed variables for next state.

For Euclid’s Algorithm:

\[
\begin{align*}
\text{If } x &> y \\
\text{then } x' & = x - y \\
\text{and } y' & = y
\end{align*}
\]

\[
\text{or}
\begin{align*}
\text{If } y &> x \\
\text{then } y' & = y - x \\
\text{and } x' & = x
\end{align*}
\]
The Next-State Relation

Described by a formula.

Unprimed variables for current state,
Primed variables for next state.

For Euclid’s Algorithm:

\[
\begin{align*}
&\left( x > y \right) \\
&\quad \land x' = x - y \\
&\quad \land y' = y \\
\lor \\
&\left( y > x \right) \\
&\quad \land y' = y - x \\
&\quad \land x' = x
\end{align*}
\]
For Euclid’s Algorithm

Take $M = 12$, $N = 18$

Next:

$(x > y \land x' = x - y \land y' = y) \lor (y > x \land y' = y - x \land x' = x)$

Behavior:

$[x = 12, y = 18] \rightarrow [x = 12, y = 6] \rightarrow [x = 6, y = 6]$
For Euclid’s Algorithm

Take $M = 12$, $N = 18$

Behavior:
For Euclid’s Algorithm

\[ \text{Init: } (x = M) \land (y = N) \]

Behavior:
For Euclid’s Algorithm

\text{Init:} \quad (x = 12) \land (y = 18)

Behavior:

\[ [x = 12, \, y = 18] \]
For Euclid’s Algorithm

\[ \text{Init: } (x = M) \land (y = N) \]

\[ \text{Next: } \begin{cases} \quad (x > y \\ \land \quad x' = x - y \\ \land \quad y' = y) \\ \lor \quad (y > x \\ \land \quad y' = y - x \\ \land \quad x' = x) \end{cases} \]

\[ \text{Behavior: } [x = 12, y = 18] \]
For Euclid’s Algorithm

 INIT: \((x = M) \land (y = N)\)

 NEXT: \(12 > 18\)
\(\land\) \(x' = 12 - 18\)
\(\land\) \(y' = 18\)

\(\lor\)
\(18 > 12\)
\(\land\) \(y' = 18 - 12\)
\(\land\) \(x' = 12\)

Behavior:

\([x = 12, y = 18]\)
For Euclid’s Algorithm

Init: \((x = M) \land (y = N)\)

Next: \(\begin{cases} (12 > 18 \ \text{FALSE} \\
\land \ x' = 12 - 18 \\
\land \ y' = 18) \\
\lor \ (18 > 12 \ \text{TRUE} \\
\land \ y' = 18 - 12 \\
\land \ x' = 12) \end{cases} \)

Behavior:

\([x = 12, y = 18]\)
For Euclid’s Algorithm

Init: \((x = M) \land (y = N)\)

Next: \((12 > 18 \land x' = 12 - 18 \land y' = 18)\) \lor (18 > 12 \land y' = 18 - 12 \land x' = 12)\)

Behavior:

\([x = 12, y = 18]\)
For Euclid’s Algorithm

\textit{Init: } \ (x = M) \land (y = N)

\textit{Next: } \quad (\ \boxed{\begin{array}{l}
12 > 18 \\
\land x' = 12 - 18 \\
\land y' = 18
\end{array} \quad \text{FALSE}})

\vee \quad (\ 18 > 12 \\
\land y' = 18 - 12 \\
\land x' = 12\ )

\text{Behavior: }

[ x = 12, y = 18]
For Euclid’s Algorithm

\[ \text{Init:} \quad (x = M) \land (y = N) \]

\[ \text{Next:} \quad \begin{cases} 
12 > 18 \\
\land x' = 12 - 18 \\
\land y' = 18 
\end{cases} \quad \text{FALSE} \\
\lor \\
\begin{cases} 
18 > 12 \\
\land y' = 18 - 12 \\
\land x' = 12 
\end{cases} \]

Behavior:

\[ [x = 12, y = 18] \rightarrow [x = 12, y = 6] \]
For Euclid’s Algorithm

\[ Init: \ (x = M) \land (y = N) \]

\[ Next: \ (x > y \land x' = x - y \land y' = y) \lor (y > x \land y' = y - x \land x' = x) \]

Behavior:

\[ [x = 12, y = 18] \rightarrow [x = 12, y = 6] \]
For Euclid’s Algorithm

Init: \((x = M) \land (y = N)\)

Next: \((12 > 6 \land x' = 12 - 6 \land y' = 6) \lor (6 > 12 \land y' = 6 - 12 \land x' = 12)\)

Behavior:

\([x = 12, y = 18] \rightarrow [x = 12, y = 6]\)
For Euclid’s Algorithm

\[ \text{Init: } (x = M) \land (y = N) \]

\[ \text{Next: } ( \begin{array}{c} 12 > 6 \quad \text{TRUE} \\ \land x' = 12 - 6 \\ \land y' = 6 \end{array} ) \quad \lor \quad ( \begin{array}{c} 6 > 12 \quad \text{FALSE} \\ \land y' = 6 - 12 \\ \land x' = 12 \end{array} ) \]

Behavior:

\[ [x = 12, y = 18] \rightarrow [x = 12, y = 6] \]
For Euclid’s Algorithm

Init: \((x = M) \land (y = N)\)

Next: \((12 > 6 \land x' = 12 - 6 \land y' = 6) \lor (6 > 12 \land y' = 6 - 12 \land x' = 12)\)

Behavior:

\([x = 12, y = 18] \rightarrow [x = 12, y = 6]\)
For Euclid’s Algorithm

Init: \( (x = M) \land (y = N) \)

Next: \( (12 > 6 \land x' = 12 - 6 \land y' = 6 ) \)
\( \lor (6 > 12 \land y' = 6 - 12 \land x' = 12 ) \)

Behavior:

\([x = 12, y = 18] \rightarrow [x = 12, y = 6] \rightarrow [x = 6, y = 6]\)
For Euclid’s Algorithm

Init: \((x = M) \land (y = N)\)

Next: 

\[
( x > y \\
\land x' = x - y \\
\land y' = y ) \\
\lor \\
( y > x \\
\land y' = y - x \\
\land x' = x )
\]

Behavior:

\([x = 12, y = 18] \rightarrow [x = 12, y = 6] \rightarrow [x = 6, y = 6] \]
For Euclid’s Algorithm

Init: \((x = M) \land (y = N)\)

Next: \((6 > 6 \quad \text{FALSE})
\land x' = 6 - 6
\land y' = 6\)
\lor (6 > 6 \quad \text{FALSE})
\land y' = 6 - 6
\land x' = 6\)

Behavior:

\([x = 12, y = 18] \rightarrow [x = 12, y = 6] \rightarrow [x = 6, y = 6]\)
For Euclid’s Algorithm

Init: \( (x = M) \land (y = N) \)

Next:
\[
\begin{align*}
( & 6 > 6 \\
\land & x' = 6 - 6 \\
\land & y' = 6)
\end{align*}
\]
FALSE

\[
\begin{align*}
( & 6 > 6 \\
\land & y' = 6 - 6 \\
\land & x' = 6)
\end{align*}
\]
FALSE

Behavior:

\([x = 12, y = 18] \rightarrow [x = 12, y = 6] \rightarrow [x = 6, y = 6]\)
For Euclid’s Algorithm

\[ \text{Init}: \quad (x = M) \land (y = N) \]

\[ \text{Next}: \quad \begin{align*}
   &\quad \left( 6 \geq 6 \\
   &\quad \land \ x' = 6 - 6 \\
   &\quad \land \ y' = 6 \ ight) \quad \text{FALSE} \\
   \lor \quad \begin{align*}
   &\quad \left( 6 > 6 \\
   &\quad \land \ y' = 6 - 6 \\
   &\quad \land \ x' = 6 \ ight) \quad \text{FALSE}
\end{align*}
\]

\[ \text{Behavior}: \quad [x = 12, \ y = 18] \rightarrow [x = 12, \ y = 6] \rightarrow [x = 6, \ y = 6] \]
Euclid’s Algorithm

For any values of $x$ and $y$, there are unique values of $x'$ and $y'$ that make $Next$ true.

Euclid's algorithm is deterministic.

To Model Nondeterminism

Allow multiple next states for a current state.

Multiple assignments of values to primed variables that make $Next$ true for a single assignment of values to unprimed variables.
Euclid’s Algorithm

For any values of $x$ and $y$, there are unique values of $x'$ and $y'$ that make $Next$ true.
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To Model Nondeterminism
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Allow multiple next states for a current state.
Euclid’s Algorithm

For any values of $x$ and $y$, there are unique values of $x'$ and $y'$ that make $Next$ true.

Euclid’s algorithm is deterministic.

To Model Nondeterminism

Allow multiple next states for a current state.

Multiple assignments of values to primed variables that make $Next$ true for a single assignment of values to unprimed variables.
What About Formal Specs?

Need them only to apply tools.

Require a formal language.
What About Formal Specs?

Need them only to apply tools.
What About Formal Specs?

Need them only to apply tools.

Require a formal language.
The Language: TLA+
The Language: TLA+

This

\[\text{Init: } (x = M) \land (y = N)\]

\[\text{Next: } (x > y)
\land x' = x - y
\land y' = y \]
\[\lor (y > x)
\land y' = y - x
\land x' = x \]
The Language: TLA+

becomes this

\[
\begin{align*}
\text{Init} & \triangleq (x = M) \land (y = N) \\
\text{Next} & \triangleq (x > y \\
& \quad \land x' = x - y \\
& \quad \land y' = y) \\
& \lor (y > x \\
& \quad \land y' = y - x \\
& \quad \land x' = x)
\end{align*}
\]
The Language: TLA^+ 

plus declarations 

CONSTANTS $M, N$

VARIABLES $x, y$

$Init \triangleq (x = M) \land (y = N)$

$Next \triangleq (x > y$
\[\land x' = x - y\]
\[\land y' = y\]
\[\lor (y > x$
\[\land y' = y - x\]
\[\land x' = x)\]
The Language: TLA⁺

plus some boilerplate.

MODULE Euclid

EXTENDS Integers

CONSTANTS M, N

VARIABLES x, y

Init ≜ (x = M) \land (y = N)

Next ≜ ( x > y \\
\land x' = x - y \\
\land y' = y ) \\
\lor ( y > x \\
\land y' = y - x \\
\land x' = x )
The Language: TLA+

You type

------------------ MODULE Euclid -------------------
EXTENDS Integers

CONSTANTS M, N

VARIABLES x, y

Init == (x = M) \&\&( y = N)

Next ==  (  x > y
         /
         \&\&  x’ = x - y
         /
         \&\&  y’ = y )

    \&(  y > x
         /
         \&\&  y’ = y - x
         /
         \&\&  x’ = x )
You can model check TLA+ specs.
You can model check TLA⁺ specs.

– Checks all executions of a small model.
You can model check TLA\(^+\) specs.

– Checks all executions of a small model.

– Extremely effective and fairly easy.
You can model check TLA+ specs.

You can write formal correctness proofs and check them mechanically.
You can model check TLA+ specs.

You can write formal correctness proofs and check them mechanically.

– Hard work.
You can model check TLA+ specs.

You can write formal correctness proofs and check them mechanically.

But math works only for toy examples.
You can model check TLA+ specs.

You can write formal correctness proofs and check them mechanically.

But math works only for toy examples.

To model real systems, you need a real language with types, procedures, objects, etc.
You can model check TLA+ specs.

You can write formal correctness proofs and check them mechanically.

**Wrong**

But math works only for toy examples.

To model real systems, you need a real language with types, procedures, objects, etc.

Wrong
We have used TLA+ on 10 large complex real-world systems. In every case TLA+ has added significant value, either preventing subtle serious bugs from reaching production, or giving us enough understanding and confidence to make aggressive performance optimizations without sacrificing correctness. Executive management are now proactively encouraging teams to write TLA+ specs for new features and other significant design changes. In annual planning, managers are now allocating engineering time to use TLA+.

Chris Newcombe
Amazon Engineer

November, 2013
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[W]e have used TLA\textsuperscript{+} on 10 large complex real-world systems. In every case TLA\textsuperscript{+} has added significant value, either preventing subtle serious bugs from reaching production, or giving us enough understanding and confidence to make aggressive performance optimizations without sacrificing correctness. Executive management are now proactively encouraging teams to write TLA\textsuperscript{+} specs for new features and other significant design changes. In annual planning, managers are now allocating engineering time to use TLA\textsuperscript{+}.

Chris Newcombe
Amazon Engineer
November, 2013
The XBox 360 Memory System

Writing a TLA+

spec caught a bug that would not
otherwise have been found.

That bug would have caused every XBox 360 to crash
after 4 hours of use.
The XBox 360 Memory System

Writing a TLA+ spec caught a bug that would not otherwise have been found.
The XBox 360 Memory System

Writing a TLA\(^+\) spec caught a bug that would not otherwise have been found.

That bug would have caused every XBox 360 to crash after 4 hours of use.
You can learn about TLA$^+$ on the web.
You can learn about TLA+ on the web.

Today, I’ll talk about informal specs, starting with an example.
TLATEX — the TLA$^+$ pretty-printer
The input:

\[ \text{Foo} \Rightarrow \land a = b \\
\land ccc = d \]
The input:

\[ \text{Foo} \Rightarrow \land a = b \land ccc = d \]

The naive output:

\[ \text{Foo} \Rightarrow \land a = b \land ccc = d \]
The input:

\[
\text{Foo} \Rightarrow \begin{array}{c}
\land \\
\land \\
\land
\end{array}
\begin{array}{c}
a \\
ccc \\
ccc
\end{array} = \begin{array}{c}b \\
d \\
d
\end{array}
\]

The naive output:

\[
\text{Foo} \Rightarrow \land a = b \\
\land ccc = d
\]

The user probably wanted these aligned.
The input:

\[
\text{Foo} \Rightarrow \begin{aligned}
&\land a = b \\
&\land ccc = d
\end{aligned}
\]

The user probably wanted these aligned.

The right output:

\[
\text{Foo} \Rightarrow \land a = b \\
\land ccc = d
\]
The input:

\( \forall \aaa + \bb = \cc \)

\( \forall \iii = \jj \times \kk \)
TLATeX — the TLA\(^+\) pretty-printer

The input:

\[
\begin{align*}
\forall \ aaa + bb &= c \\
\forall \ iii &= jj \times k
\end{align*}
\]

The naive output:

\[
\begin{align*}
\land \ aaa + bb &= c \\
\land \ iii &= jj \times k
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The input:

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\begin{align*}
\land & \quad \text{aaa} + \text{bb} = \text{c} \\
\land & \quad \text{iii} = \text{jj} \times \text{k}
\end{align*}
\]

The naive output:

\[
\begin{align*}
\land & \quad \text{aaa} + \text{bb} = \text{c} \\
\land & \quad \text{iii} = \text{jj} \times \text{k}
\end{align*}
\]

The user probably didn’t want these aligned.
The input:

\[ \begin{array}{c}
\land \ a a a + b b = c \\
\land \ i i i = j j \ast k
\end{array} \]

The right output:

\[ \begin{array}{c}
\land \ a a a + b b = c \\
\land \ i i i = j j \ast k
\end{array} \]

The user probably didn’t want these aligned.
There is no precise definition of correct alignment.
There is no precise definition of correct alignment.
We can’t specify mathematically what the user wants.
There is no precise definition of correct alignment. We can’t specify mathematically what the user wants.

If we can’t specify correctness, specification is useless.
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Wrong.
There is no precise definition of correct alignment.
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If we can’t specify correctness, specification is useless.

Wrong.

Not knowing what a program should do means we have to think even harder.
There is no precise definition of correct alignment.

We can’t specify mathematically what the user wants.

If we can’t specify correctness, specification is useless.

Wrong.

Not knowing what a program should do means we have to think even harder.

Which means that a spec is even more important.
My Spec

6 rules plus definitions (in comments).

Example:
A left-comment token is LeftComment aligned with its covering token.
6 rules plus definitions (in comments).
My Spec

6 rules plus definitions (in comments).

Example:

A left-comment token is LeftComment aligned with its covering token.
My Spec

6 rules plus definitions (in comments).

Example:

A left-comment token is LeftComment aligned with its covering token.

Defined term.
Why did I write this spec?

- It was a lot easier to understand and debug 6 rules than 850 lines of code.
- I did a lot of debugging of the rules, aided by debugging code to report what rules were being used.
- The few bugs in implementing the rules were easy to catch.
- Had I just written code, it would have taken me much longer and not produced formatting as good.
Why did I write this spec?

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I did a lot of debugging of the rules
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Had I just written code, it would have taken me much longer and not produced formatting as good.
Why not a formal spec?
Why not a formal spec?

Getting it right not that important.
Why not a formal spec?

Getting it right not that important.

It didn’t have to work on all corner cases.
Why not a formal spec?

Getting it right not that important.
It didn’t have to work on all corner cases.

There were no tools that could help me.
What is Typical About This Spec

- The spec is at a higher-level than the code.
- It could have been implemented in any language.
- No method or tool for writing better code would have helped to write the spec.
- No method or tool for writing better code would have made the spec unnecessary.
- It says nothing about how to write code.
- You write a spec to help you think about the problem before you think about the code.
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You write a spec to help you think about the problem before you think about the code.
What is Not Typical About This Spec

It's quite subtle.

95% of the code most people write requires less thought; specs that are shorter and simpler suffice.

It's a set of rules.

A set of rules/requirements/axioms is usually a bad spec.

It's hard to understand the consequences of a set of rules.
What is Not Typical About This Spec

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A set of rules/requirements/axioms is usually a bad spec.

It’s hard to understand the consequences of a set of rules.
Specifying How to Compute a Function

Specifying what the pretty-printer should do was hard. Implementing the spec was easy. Specifying what a sorting program should do is easy. Figuring out how to implement it efficiently is hard (if no one has shown you). It requires thinking, which means writing a specification.
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Specifying what the pretty-printer should do was hard. Implementing the spec was easy.

Specifying what a sorting program should do is easy. Figuring out how to implement it efficiently is hard (if no one has shown you).

It requires thinking, which means writing a specification.
An example: Quicksort
An example: Quicksort

An example: Quicksort


For simplicity, assume the $A[i]$ are numbers.
It uses a procedure $\text{Partition}(lo, hi)$. 
It uses a procedure $\text{Partition}(lo, hi)$.

It uses a procedure $\text{Partition}(lo, hi)$.


For this example, we don’t care how this procedure is implemented.
Let’s specify Quicksort in pseudo-code.

```plaintext
procedure Partition(lo, hi)
{
Pick pivot in lo...hi−1;
return pivot;
}

procedure QS(lo, hi)
{
if (lo < hi)
{p := Partition(lo, hi);
QS(lo, p);
QS(p+1, hi); }
}

main
{QS(0, N−1); }

But is it really Quicksort?
```
procedure Partition(lo, hi) {
    Pick pivot in lo . . . (hi − 1);
    Permute A[lo], . . . , A[hi] to make
    return pivot; }

But is it really Quicksort?
procedure $Partition(lo, hi)$ {
    Pick $pivot$ in $lo \ldots (hi - 1)$;
    Permute $A[lo], \ldots, A[hi]$ to make
        $A[lo], \ldots, A[pivot] \leq A[pivot + 1], \ldots, A[hi]$;
    return $pivot$; }

procedure $QS(lo, hi)$ { if $(lo < hi)$ { $p := Partition(lo, hi)$;
    $QS(lo, p)$;
    $QS(p + 1, hi)$; } }
procedure Partition(lo, hi) {
    Pick pivot in lo ...(hi − 1);
    Permute A[lo],..., A[hi] to make
    return pivot; }

procedure QS(lo, hi) { if (lo < hi) { p := Partition(lo, hi);   
    QS(lo, p);
    QS(p + 1, hi); } }  

main { QS(0, N − 1) ; }
procedure Partition(lo, hi) {
    Pick pivot in lo . . . (hi − 1);
    Permute \( A[lo] \), . . . , \( A[hi] \) to make
    return pivot; }

procedure QS(lo, hi) { if (lo < hi) { p := Partition(lo, hi);
    QS(lo, p);
    QS(p + 1, hi); } } }

main { QS(0, N − 1); }
procedure $Partition(lo, hi)$ {
  Pick $pivot$ in $lo \ldots (hi - 1)$;
  return $pivot$; }

procedure $QS(lo, hi)$ { if $(lo < hi)$ { $p := Partition(lo, hi)$;
  $QS(lo, p)$;
  $QS(p + 1, hi)$; } }

main { $QS(0, N - 1)$; }

Informal: no formal syntax, no declarations, …
procedure Partition(lo, hi) {
    Pick pivot in lo ... (hi - 1);
    Permute A[lo], ..., A[hi] to make
        A[lo], ..., A[pivot] \leq A[pivot + 1], ..., A[hi];
    return pivot; }

procedure QS(lo, hi) { if (lo < hi) { p := Partition(lo, hi);
    QS(lo, p);
    QS(p + 1, hi); } }

main { QS(0, N - 1) ; }

Informal: no formal syntax, no declarations, ...
Easy to understand.
procedure Partition(lo, hi) {
    Pick pivot in lo . . . (hi − 1);
    Permute A[lo], . . . , A[hi] to make
    return pivot; }

procedure QS(lo, hi) {
    if (lo < hi) {
        p := Partition(lo, hi);
        QS(lo, p);
        QS(p + 1, hi); }
    }

main { QS(0, N − 1); }

But is it really Quicksort?
It’s the way Quicksort is almost always described.
It’s the way Quicksort is almost always described.

But recursion is not a fundamental part of Quicksort.
It’s the way Quicksort is almost always described.

But recursion is not a fundamental part of Quicksort.

It’s just one way of implementing divide-and-conquer.
It’s the way Quicksort is almost always described.

But recursion is not a fundamental part of Quicksort.

It’s just one way of implementing divide-and-conquer.

It’s probably not the best way for parallel execution.
Problem: Write a non-recursive version of Quicksort.
Problem: Write a non-recursive version of Quicksort.

Almost no one can do it in 10 minutes.
Problem: Write a non-recursive version of Quicksort.

Almost no one can do it in 10 minutes.

They try to “compile” the recursive version.
Solution:

Maintain a set $U$ of index ranges on which Partition needs to be called.

Initially, $U$ equals $\{\langle 0, N-1 \rangle \}$

We could write it in pseudo-code, but it's better to simply write Init and Next directly.
Solution:

Maintain a set $U$ of index ranges on which $Partition$ needs to be called.
Solution:

Maintain a set $U$ of index ranges on which $\text{Partition}$ needs to be called.

Initially, $U$ equals $\{\langle 0, N - 1 \rangle \}$
Solution:

Maintain a set $U$ of index ranges on which $Partition$ needs to be called.

Initially, $U$ equals $\{\langle 0, N - 1 \rangle\}$

We could write it in pseudo-code, but it’s better to simply write $Init$ and $Next$ directly.
**Init:** \[ A = \text{any array of numbers of length } N \]
\[ \land \ U = \{\langle 0, N - 1 \rangle\} \]
Init: $A = \text{any array of numbers of length } N$
$\wedge U = \{\langle 0, N - 1 \rangle\}$

Before writing $Next$, let’s make a definition:
$$\text{Init: } A = \text{any array of numbers of length } N$$
$$\land U = \{\langle 0, N - 1 \rangle\}$$

Before writing \textit{Next}, let's make a definition:

$$\text{Partitions}(B, \text{pivot}, \text{lo}, \text{hi}) \triangleq$$
the set of arrays obtained from \(B\) by permuting
\(B[\text{lo}], \ldots, B[\text{hi}]\) such that \ldots
**Init:** \[ \begin{align*} A &= \text{any array of numbers of length } N \\ U &= \{\langle 0, N - 1 \rangle\} \end{align*} \]

Before writing *Next*, let’s make a definition:

\[ \text{Partitions}(B, \text{pivot}, \text{lo}, \text{hi}) \triangleq \begin{align*} \text{the set of arrays obtained from } B \text{ by permuting } \\ B[\text{lo}], \ldots, B[\text{hi}] \text{ such that } \ldots \end{align*} \]

*Next*:

A relation between old values of \( A, U \) and new values \( A', U' \).
\[
U \neq \{\}
\]

\[
\text{Pick any } \langle b, t \rangle \text{ in } U:
\]

\[
\text{IF } b \neq t \text{ THEN Pick any } p \text{ in } b.(t - 1):
\]

\[
A' = \text{Any element of Partitions}(A, p, b, t)
\]

\[
U' = U \text{ with } \langle b, t \rangle \text{ removed and } \langle b, p \rangle \text{ and } \langle p + 1, t \rangle \text{ added}
\]

\[
\text{ELSE } A' = A \text{ and } U' = U \text{ with } \langle b, t \rangle \text{ removed}
\]

Next:
Next:

\[ U \neq \{\} \quad \text{Stop if } U = \{\} \]

\[ \land \]
Next:

\[ U \neq {} \]
\[ \land \text{ Pick any } \langle b, t \rangle \text{ in } U : \]
\textit{Next:}

\[ U \neq \{\} \]
\[ \land \text{ Pick any } \langle b, t \rangle \text{ in } U : \]
\[ \text{IF } b \neq t \]
\[ \text{THEN} \]

\[ \text{ELSE} \]
Next:

\[ U \neq \{\} \]

\[
\land \text{ Pick any } \langle b, t \rangle \text{ in } U : \\
\text{ IF } b \neq t \\
\text{ THEN Pick any } p \text{ in } b .. (t - 1) : \\
\]

ELSE
Next:

\[ U \neq \{\} \]
\[ \wedge \text{Pick any } \langle b, t \rangle \text{ in } U : \]
\[ \text{IF } b \neq t \]
\[ \quad \text{THEN Pick any } p \text{ in } b .. (t - 1) : \]
\[ A' = \text{Any element of } Partitions(A, p, b, t) \]

ELSE
Next:

\[ U \neq \{\} \]

\[ \land \text{ Pick any } \langle b, t \rangle \text{ in } U : \]

IF \( b \neq t \)

THEN Pick any \( p \) in \( b \ldots (t-1) \):

\[ A' = \text{Any element of } \text{Partitions}(A, p, b, t) \]

\[ \land \ U' = U \text{ with } \langle b, t \rangle \text{ removed and} \]
\[ \langle b, p \rangle \text{ and } \langle p+1, t \rangle \text{ added} \]

ELSE
Next:

\[ U \neq \{\} \]

∧ Pick any \(\langle b, t \rangle\) in \(U\):

IF \(b \neq t\)

THEN Pick any \(p\) in \(b \ldots (t-1)\):

\[ A' = \text{Any element of } Partitions(A, p, b, t) \]

∧ \(U' = U\) with \(\langle b, t \rangle\) removed and \(\langle b, p \rangle\) and \(\langle p+1, t \rangle\) added

ELSE \(A' = A\)
Next:

\[ U \neq \{\} \]

\& Pick any \( \langle b, t \rangle \) in \( U \):

IF \( b \neq t \)

THEN Pick any \( p \) in \( b \ldots (t-1) \):

\[ A' = \text{Any element of } Partitions(A, p, b, t) \]

\& \( U' = U \) with \( \langle b, t \rangle \) removed and \( \langle b, p \rangle \) and \( \langle p+1, t \rangle \) added

ELSE \( A' = A \)

\& \( U' = U \) with \( \langle b, t \rangle \) removed
Why can (almost) no one find this version of Quicksort?
Why can (almost) no one find this version of Quicksort?

Their minds are stuck in code.
Why can (almost) no one find this version of Quicksort?

Their minds are stuck in code.

They can’t think at a higher level.
Next:
\[ U \neq \{ \} \]
\[ \land \text{ Pick any } \langle b, t \rangle \text{ in } U : \]
\[ \text{IF } b \neq t \]
\[ \text{THEN } \text{ Pick any } p \text{ in } b \ldots (t - 1) : \]
\[ A' = \text{Any element of } \text{Partitions}(A, p, b, t) \]
\[ \land \ U' = U \text{ with } \langle b, t \rangle \text{ removed and} \]
\[ \langle b, p \rangle \text{ and } \langle p + 1, t \rangle \text{ added} \]
\[ \text{ELSE } A' = A \]
\[ \land \ U' = U \text{ with } \langle b, t \rangle \text{ removed} \]

Easy to write this as a formula.
Next:

\[ U \neq \{\} \]

\[ \land \text{ Pick any } \langle b, t \rangle \text{ in } U : \]

IF \( b \neq t \)

THEN Pick any \( p \) in \( b \ldots (t-1) \):

\[ A' = \text{Any element of } Partitions(A, p, b, t) \]

\[ \land U' = U \text{ with } \langle b, t \rangle \text{ removed and } \langle b, p \rangle \text{ and } \langle p+1, t \rangle \text{ added} \]

ELSE \( A' = A \)

\[ \land U' = U \text{ with } \langle b, t \rangle \text{ removed} \]

Pick an arbitrary value
Next:
\[ U \neq {} \]
\[ \land \ \exists \langle b, t \rangle \in U : \]
\[ \text{IF } b \neq t \]
\[ \text{THEN } \text{Pick any } p \text{ in } b \ldots (t-1) : \]
\[ A' = \text{Any element of } \text{Partitions}(A, p, b, t) \]
\[ \land \ U' = U \text{ with } \langle b, t \rangle \text{ removed and } \langle b, p \rangle \text{ and } \langle p+1, t \rangle \text{ added} \]
\[ \text{ELSE } A' = A \]
\[ \land \ U' = U \text{ with } \langle b, t \rangle \text{ removed} \]

Pick an arbitrary value is existential quantification.
Next:

\[ U \neq \{\} \]
\[ \land \ \exists \langle b, t \rangle \in U : \]
\[ \text{IF } b \neq t \]
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Or sometimes
Next:

\[ U \neq \{\} \]
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\[ \text{IF } b \neq t \]
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\[ \land \ U' = U \text{ with } \langle b, t \rangle \text{ removed and } \langle b, p \rangle \text{ and } \langle p+1, t \rangle \text{ added} \]
\[ \text{ELSE } \]
\[ A' = A \]
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Or sometimes even simpler.
Next:

\[ U \neq \{\} \]
\[ \land \ \exists \langle b, t \rangle \in U : \]

IF \( b \neq t \)

THEN \( \exists p \in b \ldots (t-1) : \)

\[ A' \in \text{Partitions}(A, p, b, t) \]
\[ \land \ U' = U \ \text{with} \ \langle b, t \rangle \ \text{removed and} \]

ELSE \( A' = A \)

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And so on.
Next:

\[ U \neq \{\} \]
\[ \land \exists \langle b, t \rangle \in U : \]
\[ \text{IF } b \neq t \]
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\[ A' \in \text{Partitions}(A, p, b, t) \]
\[ \land \ U' = (U \setminus \{\langle b, t \rangle\}) \cup \{\langle b, p \rangle, \langle p + 1, t \rangle\} \]

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A TLA\(^+\) formula.
Next:

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ELSE \[ A' = A \]
\[ \land U' = U \setminus \{\langle b, t \rangle\} \]

If you prefer pseudo-code...
PlusCal

Like a toy programming language.
Algorithm appears in a comment in a TLA module.
An expression can be any TLA expression.
Constructs for nondeterminism.
Compiled to an easy to understand TLA spec.
Can apply TLA tools.
PlusCal

Like a toy programming language.
PlusCal

Like a toy programming language.

Algorithm appears in a comment in a TLA\textsuperscript{+} module.
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Can apply TLA\(^+\) tools.
Programs that Run Forever

I've been talking about programs that compute a function. Programs that run forever usually involve concurrency:

– Operating systems.
– Distributed systems.

Few people can get them right by just thinking (and writing). I'm not one of them.

We need tools to check what we do. Use TLA+ PlusCal.
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We need tools to check what we do.

Use TLA⁺/PlusCal.
Most code is really simple.
/** CLASS ResourceFileReader *

* A ResourceFileReader returns an object for reading a resource file, which is a file kept in the same directory as the tlatex.Token class. The constructor takes a file name as argument. The object’s two public methods are

* getLine() : Returns the next line of the file as a string. Returns null after the last line.

* close() : Closes the file.
Why did I write that spec?

To be sure I knew what the code should do before writing it. Without writing a spec, I only thought I knew what it should do. Later, I didn’t have to read the code to know what it did.

General rule: A spec of what the code does should say everything that anyone needs to know to use the code.
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How the code worked was too simple to require a spec.
What programmers should know about thinking.
What everyone should know about thinking.
What everyone should know about thinking.

Everyone thinks they think.
What everyone should know about thinking.

Everyone thinks they think.

If you don’t write down your thoughts, you’re fooling yourself.
What programmers should know about thinking.
What programmers should know about thinking.

You should think before you code.
What programmers should know about thinking.

You should write before you code.
What programmers should know about thinking.

You should write before you code.

A spec is simply what you write before coding.
What code should you specify?

Any piece of code that someone else might want to use or modify. It could be:
- An entire program or system.
- A class.
- A method.
- A tricky piece of code in a method.
What code should you specify?

Any piece of code that someone else might want to use or modify.
What code should you specify?

Any piece of code that someone else might want to use or modify in a month.
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It could be:

- An entire program or system.
- A class.
- A method.
- A tricky piece of code in a method.
What should you specify about the code?

What it does.

Everything anyone needs to know to use it.

Maybe: how it does it.

The algorithm / high-level design.
What should you specify about the code?

What it does.
What should you specify about the code?

What it does.

Everything anyone needs to know to use it.
What should you specify about the code?

What it does.

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Maybe: how it does it.
What should you specify about the code?

What it does.

Everything anyone needs to know to use it.

Maybe: how it does it.

The algorithm / high-level design.
How should you think about / specify the code?

Above the code level.
In terms of states and behaviors.
Mathematically, as rigorously / formally as necessary.
Perhaps with pseudo-code or PlusCal if specifying how.
How should you think about / specify the code?

Above the code level.
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Perhaps with pseudo-code or PlusCal if specifying how.
How do you learn to write specs?

By writing formal specs.
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How do you learn to write formal specs?

You learned to write programs by writing them, running them, and correcting your errors. You can learn to write formal specs by writing them, “running” them with a model checker, and correcting your errors. TLA+ may not be the best language for your formal specification needs. But it’s great for learning learning to think mathematically.
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But it’s great for learning learning to think mathematically.
How do you connect the spec to the code?

Comments connecting mathematical concepts and their implementation.

Example:
Mathematical concept: graph
Implementation: array of node objects & array of link objects
How do you connect the spec to the code?

Comments connecting mathematical concepts and their implementation.
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Example:

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Comments connecting mathematical concepts and their implementation.

Example:

Mathematical concept: graph
Implementation: array of node objects & array of link objects
What about coding?

Nothing I have said implies anything about how you should code. You still have to think while you code. What you write while coding is code. I have nothing to say about how you should code. Use any programming language you want. Use any coding methodology you want: test-driven development, agile programming, . . .
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You’ll still have to test and debug.
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Writing specs is an additional step.
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It may save time by catching errors early, when they’re easier to correct.
You’ll still have to test and debug.

Writing specs is an additional step.

It may save time by catching errors early, when they’re easier to correct.

It will improve your programming, so you write better programs.
Why are programmers reluctant to write specs?
Why are programmers reluctant to write specs?

Writing is hard.
Why is writing hard?

Writing requires thinking.

Thinking is hard.

It's easier to think your thinking.

Writing is like running.
The less you do it, the slower you are.

You have to strengthen your writing muscles.

It takes practice.

It's easier to find an excuse not to.
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What if the spec is wrong?

Maybe you made a mistake.
Maybe the requirements change,
or an enhancement is needed.
The code will have to be changed,
maybe even before the program is finished.
This eventually happens to all useful programs.
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This eventually happens to all useful programs.
In an ideal world, a new spec would be written and the code completely rewritten.
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In the real world, the code is patched and maybe the spec is updated.
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In the real world, the code is patched and maybe the spec is updated.

If this is inevitable, why write specs?
Reason 1

Whoever has to modify the code will be grateful for every word or formula of spec you write. And "whoever" may be you. That's why you should update the spec when changing the code.
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Whoever has to modify the code will be grateful for every word or formula of spec you write.

And “whoever” may be you.

That’s why you should update the spec when changing the code.
Reason 2

Every time code is patched, it becomes a little uglier, harder to understand, and harder to maintain. If you don’t start with a spec, every piece of the code you write is a patch. The program starts out ugly, hard to understand, and hard to maintain.

“No battle was ever won according to plan, but no battle was ever won without one.”

Dwight D. Eisenhower
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Every time code is patched, it becomes a little uglier, harder to understand, and harder to maintain.

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Some people will tell you that writing specs is a waste of time.
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In some situations it is.
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In some situations it is. Sometimes there’s no need to think about what you’re doing.
Some people will tell you that writing specs is a waste of time.

In some situations it is. Sometimes there’s no need to think about what you’re doing.

But remember: when they’re telling you not to write a spec, they’re really telling you not to think.
Thinking doesn’t guarantee that you won’t make mistakes.
Thinking doesn’t guarantee that you won’t make mistakes. Not thinking usually guarantees that you will.
To find out more about TLA⁺, go to my home page and click on:

*The TLA Web Page*
To find out more about TLA$^+$, go to my home page and click on:

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Thank You