Streaming Verification of Outsourced Computation

Graham Cormode
G.Cormode@warwick.ac.uk

Amit Chakrabarti (Dartmouth)
Andrew McGregor (U Mass Amherst)
Michael Mitzenmacher (Harvard)
Justin Thaler (Harvard)
Ke Yi (HKUST)
Big Data Streams

- The data stream model requires computation in small space with a single pass over input data
  - Models large network data, database transactions
- Fundamental challenge of data stream analysis: Too much information to store or transmit
- So process data as it arrives: one pass, small space: the data stream approach.
- Approximate answers to many questions are OK, if there are guarantees of result quality
  - Parameters: space needed, time per update as function of approximation accuracy, probability of error
Data Stream Algorithms

- Many problems solved efficiently in streaming model
  - $F_0$: How many distinct items (out of $10^{18}$ possible)?
  - $HH$: Which items occur most frequently?
  - $H$: What is the (empirical) entropy of the observed dbn?

- But many other natural problems are “hard” in this model
  - Hardness means large amount of space is needed
  - E.g. Was a particular item in the stream?
  - E.g. What is inner product of two vectors?

- **Lower bounds** proved via communication complexity
  - Independent of any assumptions on computational power
Streaming Interactive Proofs

- "Practical" solution: outsource to a more powerful "prover"
  - Fundamental problem: how to be sure that the prover is being honest?
- Prover provides "proof" of the correct answer
  - Ensure that "verifier" has very low probability of being fooled
  - Related to communication complexity Arthur-Merlin model, and Algebrization, with additional streaming constraints
Motivating Applications

- Cloud Computing
  - To save money, and energy, outsource data to a 3rd party
  - But want to know they are honest, without duplicating!
  - Use a streaming interactive proof to verify computation

- Trusted Hardware
  - Hardware components within a (distributed) system (e.g. video card, additional computing cores)
  - Use streaming interactive proofs for (mutual) trust
One Round Model

- One-round model [Chakrabarti, C, McGregor 09]
  - Define protocol with help function $h$ over input length $N$
  - Maximum length of $h$ over all inputs defines *help cost*, $H$
  - Verifier has $V$ bits of memory to work in
  - Verifier uses randomness so that:
    - For all help strings, $\Pr[\text{output} \neq f(x)] \leq \delta$
    - Exists a help string so that $\Pr[\text{output} = f(x)] \geq 1-\delta$
  - $H = 0$, $V = N$ is trivial; but $H = N$, $V = \text{polylog } N$ is not

Data Stream

Streaming Verification of Outsourced Computation
Frequency Moments

- Given a sequence of $m$ items, let $w_i$ denote frequency of item $i$
- Define $F_k = \sum_i |w_i|^k$
  - Core computation in data streams
  - Requires $\Omega(N)$ space to compute exactly
  - Need polynomial space to approximate for $k>2$

- Results: for $h,v$ s.t. $(hv) > N$, exists a protocol with $H = k^2 h \log m$, $V = O(k v \log m)$ to compute $F_k$
  - Lower bounds: $HV = \Omega(N)$ necessary for exact, and $HV = \Omega(N^{1-5/k})$ for approximate $F_k$ computation
Frequency Moments

- Map \([N]\) to \(h \times v\) array
- Interpolate entries in array as a polynomial \(f(x, y)\)
- Verifier picks random \(r\), evaluates \(f(r, j)\) for \(j \in [v]\)
  - Low-degree extension (LDE) of the input
- Prover sends \(s(x) = \sum_{j \in [v]} f(x, j)^k\) (degree \(kh\))
  - Verifier checks \(s(r) = \sum_{j \in [v]} f(r, j)^k\)
  - Output \(F_k = \sum_{i \in [h]} s(i)\) if test passed
- Probability of failure small if evaluated over large enough field
Streaming LDE Computation

- Must evaluate $f(r,i)$ incrementally as $f()$ is defined by stream.
- Structure of polynomial means updates to $(a,b)$ cause

$$f(r,i) \leftarrow f(r,i) + p_{a,b}(r,i)$$

where $p_{a,b}(x,y) = \prod_{i \in [h] \setminus \{a\}} (x-i)(a-i)^{-1} \cdot \prod_{j \in [v] \setminus \{b\}} (y-j)(b-j)^{-1}$
- Lagrange polynomial, can be evaluated in small space

- Can be computed quickly, using appropriate precomputed look-up tables
Applications of Frequency Moments

- Inner products: $x \cdot y = \frac{1}{2} (F_2(x+y) - (F_2(x) + F_2(y)))$
  - Adapt previous protocol to verify directly

- Approximate $F_2$:
  - Methods known to $(1 \pm \varepsilon)$ approximate $F_2$ by computing $F_2$ of a random projection
  - Random projection computable in small space
  - Gives $HV = \Theta(1/\varepsilon^2)$ tradeoff

- Approximate $F_\infty = \max_i m_i$:
  - Observe that $F_\infty^t \leq F_t \leq N F_\infty^t$
  - Pick $t = \log N / \log (1+\varepsilon)$ to get $(1+\varepsilon)$ approx to $F_\infty$
  - Gives $HV = \Theta(1/\varepsilon^3 \text{ poly-log } N)$ tradeoff
Multi-Round Protocol

- **Advantage of one-round protocols**: Prover can provide proof without direct interaction (e.g. publish + go offline)
- **Disadvantage**: Resources still polynomial in input size
- Multi-round protocol improves exponentially [C, Thaler, Yi 12]:
  - Prover and Verifier follow communication protocol
  - $H$ now denotes upper bound on total communication
  - $V$ is verifier’s space, study tradeoff between $H$ and $V$ as before

Data Stream

```
V  Proof  H
```

Streaming Verification of Outsourced Computation
Multi-Round Frequency Moments

Now index data using \( \{0,1\}^d \) in \( d = \log N \) dimensional space

- Verifier picks one \( (r_1 \ldots r_d) \in [p]^d \), and evaluates \( f^k(r_1, r_2, \ldots r_d) \)
- Round 1: Prover sends \( g_1(x_1) = \sum_{x_2 \ldots x_d} f^k(x_1, x_2 \ldots x_d) \), V sends \( r_1 \)
- Round i: Prover sends \( g_i(x_i) = \sum_{x_{i+1} \ldots x_d} f^k(r_1, r_2 \ldots r_{i-1}, x_i, x_{i+1} \ldots x_d) \)
  Verifier checks \( g_{i-1}(r_{i-1}) = g_i(0) + g_i(1) \), sends \( r_i \)
- Round d: Prover sends \( g_d(x_d) = f^k(r_1, \ldots r_{d-1}, x_d) \)
  Verifier checks \( g_d(r_d) = f^k(r_1, r_2, \ldots r_d) \)
Multi-Round Frequency Moments

- **Correctness**: prover can’t cheat last round without knowing $r_d$
- Then can’t cheat round $i$ without knowing $r_i$...
  - Similar to protocols from “traditional” Interactive Proofs
- Inductive proof, conditioned on each later round succeeding
- **Bounds**: $O(k^2 \log N)$ total communication, $O(k \log N)$ space
- $V$’s incremental computation possible in small space, via
  $$\prod_{j=1}^{d} (r_j + \text{bit}(j, i)(1-2r_j))$$
- Intermediate polynomials relatively cheap for helper to find
General Computations

- Want to be able to solve more general computations
- **Framework**: “Interactive Proofs for Muggles”, STOC’08
  Goldwasser, Kalai, Rothblum [GKR08]
- **Idea**: computations modeled by arithmetic circuits
  - Arranged into layers of addition and multiplication gates
- (Super)Round $i$: Prover claims value of LDE of layer $i$ at $r_i$
  Run multiround IP to reduce to a claim about layer $i-1$ at $r_{i-1}$
- Start with claimed output, end with LDE of input
  - Verifier can check against own calculated LDE

Streaming Verification of Outsourced Computation
Putting GKR08 into practice

- Verifier needs an LDE of the “wiring polynomial” of the circuit
  - E.g. \( \text{add}(a, b, c) = 1 \) iff gate \( a \) at layer \( i \) has inputs \( b, c \) from layer \( i-1 \)
  - Looks costly to evaluate directly, need to sum LDE over \( n^3 \) values?
  - Use the multilinear extension of the \text{add()} and \text{mult()} polynomials
  - Each gate contributes one term to the sum, so linear in circuit size

- Linear in circuit size is still slow – same as evaluating the circuit!
  - Take advantage of regularity in common wiring patterns
  - E.g. binary tree: compute contribution of all gates at once
  - Also holds for circuits for FFT, Matrix multiplication etc.
Include some “shortcut” gates in addition to add, mult
- Wide-sum $\bigoplus$: add up a large number of inputs
  - Only needs a single sum-check protocol
- Exponentiation: raise to a constant power ($x^8$, $x^{16}$)
  - More efficient than repeated self-multiplication

Choose the right field size for computations
- Work modulo a large Mersenne prime allows efficient arithmetic
# Experimental Results

<table>
<thead>
<tr>
<th>Problem</th>
<th>Gates</th>
<th>Size (gates)</th>
<th>P time</th>
<th>V time</th>
<th>Rounds</th>
<th>Comm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_2$</td>
<td>$+, \times$</td>
<td>0.4M</td>
<td>8.5 s</td>
<td>.01 s</td>
<td>986</td>
<td>11.5 KB</td>
</tr>
<tr>
<td>$F_2$</td>
<td>$+, \times, \oplus$</td>
<td>0.2M</td>
<td>6.5 s</td>
<td>.01 s</td>
<td>118</td>
<td>2.5 KB</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$+, \times$</td>
<td>16M</td>
<td>552.6 s</td>
<td>.01 s</td>
<td>3730</td>
<td>87.4 KB</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$+, \times, x^8, \oplus$</td>
<td>8.2M</td>
<td>432.6 s</td>
<td>.01 s</td>
<td>1310</td>
<td>51.0 KB</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$+, \times, x^{16}, \oplus$</td>
<td>6.2M</td>
<td>441.2 s</td>
<td>.01 s</td>
<td>1024</td>
<td>56.8 KB</td>
</tr>
<tr>
<td>PMwW</td>
<td>$+, \times, x^8, \oplus$</td>
<td>9.6M</td>
<td>482.2 s</td>
<td>.01 s</td>
<td>1513</td>
<td>56.1 KB</td>
</tr>
</tbody>
</table>

- (Relatively) efficient results for frequency moments, pattern matching with wildcards (PMwW)
Further Recent Enhancements

- Prover’s work is data parallel: can take use of GPU for acceleration [Thaler et al. HotCloud 2012]
- Further tricks shave log factors off prover’s effort [Thaler, Crypto 2013]
- Reduce dependency on domain size when data is sparse [Chakrabarti et al., 2013]
- Use crypto tools to handle three party model (data owner, server, clients) [Cormode et al., SIGMOD 2013]
Open Questions

- **Lower bounds** for multi-round versions of the protocols
  - May need new communication complexity models
- **Characterize problems** that can be solved in this model
  - NP is known to be solvable with $H = \text{poly}(N), V = \log N$ [Lipton 90]
  - But we want $H=O(N)$, and ideally $H=o(N)$
- **Use** these protocols
  - Protocols seem practical, but are they compelling?
  - For what problems are protocols most needed?