Secondary Transmission Profile for a Single-band Cognitive Interference Channel

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***Abstract*—In this paper, we consider a model for cognitive interference channel with two flows, a primary and a secondary flow. The objective for the secondary flow is to maximize its own rate by only the utilizing the temporal white spaces caused by the burstiness in primary flow's transmissions *and* without adversely affecting the primary flow's utility. The key aspect analyzed in this paper is the lack of knowledge about start and stop times of primary flow's transmissions. Thus, the secondary flow has to periodically sense available temporal opportunities. We propose a class of protocols which divide the secondary's transmissions in many small bursts (thus making the secondary flow paranoid about overlapping on primary's transmissions). Furthermore, since the secondary cannot sense while transmitting (we assuming half-duplex radios), it has to reduce its transmission power for each subsequent symbol in each component burst. Thus, the proposed power profile for the secondary is significantly different from that in a regular interference channel, where both flows have the same "status."**

I. Introduction

Cognitive wireless systems are a new paradigm to deploy new wireless services without having to replace the legacy devices. Much like any new area, many variations have been proposed and studied in the literature [3]. Some of the earlier work [7], [6], [4] in opportunistic spectrum allocation for cognitive flows were motivated by studies done by FCC [2] showing vast spectral inefficiencies existing in current systems. Many aspects of cognitive radio have been studied including how the knowledge of the primary message at the secondary can be used to code the secondary packets for maximizing its rate [1], [10], finding the information theoretic capacity of the secondary flow with causal or non causal information about the primary transmission [5], or the stability of the queues at both the flows for maximal secondary rate which guarantees a primary throughput [8], etc. The key issue in cognitive systems is the lack of complete information about the current deployments and spectral usage. This lack of knowledge can exist at many time-scales and about different operating parameters of the legacy system. In this paper, we consider a class of single-band interference channel, where the cognitive flow aims to operate during the silence periods of the primary flow, with the aim to cause least disturbance to the ongoing flow.

We consider the topology in Figure 1, where Tx1-Rx1 represents the ongoing primary flow. The transmissions by primary flow are assumed to be bursty (like communication over a walkie-talkie), which leads to white spaces in time. Our objective is to exploit the temporal white spaces for injecting secondary flow's traffic while causing controlled amount of interference to the primary flow. The unknowns captured in our analysis is that of the start and stop times of the primary flow transmissions.

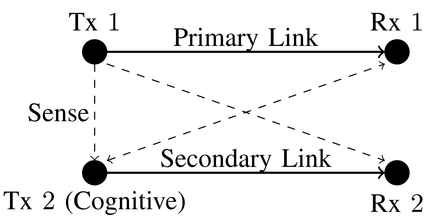

Fig. 1. A two-flow cognitive interference network.

We first propose a multi-burst transmission scheme where the secondary flow alternates between sensing and sending at fast intervals, and in effect breaks the long packet into smaller packet bursts. Second, we show that the optimal solution to the proposed problem formulation of rate-maximization under distortion constraints leads to a decaying power profile for secondary's bursts. That is, the secondary must reduce its power for each subsequent symbol to accommodate for the increasing probability of primary flow starting its transmission. We completely characterize the solution for the special case of 2-symbol long bursts and numerically solve for higher dimensional cases.

We show that for smaller packet arrival rates for the primary or for a *loose* interference constraint (i.e. primary allows more interference), the secondary user is limited by the average power constraint and in such a case the power available to the secondary is distributed evenly between all the symbols of the packet. However when the rate of the primary's arrival is high (hence fewer temporal opportunities) or the interference constraint is strict, the profile takes an exponential-like form, where the power of each subsequent secondary's symbol reduces dramatically. We also show that the multi burst strategy outperforms a single burst scheme which performs no sensing and constantly sends at a constant power. Instead of sensing if the primary is present or not in a particular band, one can also do "soft sensing" and send packets not only when the channel is idle but continuously as a function of the sensed information [9]. This can be thought of as a mix between our single burst and multi-burst strategies, although [9] has no notion of a power profile.

II. Problem Formulation

In this section, we first describe the interference channel model and then formulate the optimization problem for the proposed class of protocols.

A. Interference Channel

We consider the interference channel shown in Figure 1, which consists of two flows, Tx1-Rx1 and Tx2-Rx2. The channel inputs and outputs are related as,

$$Y_1 = X_1 + X_2 + N_1 \quad (1)$$
$$Y_2 = X_1 + X_2 + N_2, \quad (2)$$

where X_1, X_2 are channel inputs, Y_1, Y_2 are the channel outputs and N_1, N_2 are the Gaussian noise at the receivers.

The transmission time is assumed to be slotted. We assume that the primary traffic is bursty according to Poisson arrivals with rate λ. All transmissions by the primary are assumed to be of equal length of M symbols; we label each M slot transmission as a packet whose arrival distribution (probability mass function) is given by $f(k) = \frac{e^{-\lambda}\lambda^k}{k!}$. The primary is assumed to have an average power constraint of P_1.

Due to the bursty nature of the traffic, primary flow does not send data continuously and hence is silent in between transmissions. These silence periods are used by the secondary flow to inject its own traffic. However, we assume that neither the start or stop times of primary packets (each of which is M time-slot long as stated above) is known to the secondary flow transmitter. We further assume that the secondary flow nodes are equipped with half-duplex radios, which can either send or receive but not both simultaneously.

B. Objective

Our objective is to use the silence times between primary packet transmissions to allow secondary flow to send its data, while ensuring that those transmissions do not "adversely" affect the primary flow transmissions.

To make the problem concrete, we consider the following class of secondary transmissions. We assume that all secondary bursts are of length K symbols. Each packet uses the same power profile such that power $P_2(k)$ is used in in time-slot $k = 1, \ldots, K$. Since the secondary flow is ignorant of the start and stop times of the primary packets, cognitive schemes will (most likely) alternate between sensing for a primary transmission and sending their own packets. Since the secondary's radios are half-duplex, sensing takes time away from transmission of data and hence loses spectral efficiency. We assume that L symbols are sufficient for perfectly sensing primary's transmissions. If the secondary senses before every packet transmission, then it takes $N = L + K$ symbols to send a packet. Secondly, assuming that we code over the same power levels over multiple bursts, it is same as sending K packets. Finally, the pre-log factor as a result of time lost due to primary transmissions is given as $\frac{E[I]}{E[I]+E[B]} = \frac{1}{1+\lambda M}$, since $E[I] = \int_0^\infty t\lambda e^{-\lambda t} = \frac{1}{\lambda} = \frac{1}{\lambda}$ and $E[B] = M$, where, B and I are the busy and idle times for the primary. As a result, the transmission efficiency is $\frac{K}{N}\frac{1}{K}\frac{1}{1+\lambda M} = \frac{1}{N(1+\lambda M)}$.

We measure secondary's utility using mutual information for Gaussian codes, only for those symbols when the primary transmissions are off. This, essentially, assumes that the secondary receiver has genie-aided information about the primary's transmission times to consider only those times-symbols which have no primary interference.

$$U_s = \frac{1}{2N(1+\lambda M)} \sum_{k=1}^{K} \log\left(1 + \frac{P_2(k)}{N_2}\right) \quad (3)$$

The loss in primary flow's performance is measured by increase in its SINR. If the primary transmission starts when the secondary was transmitting, the secondary would not detect it till the next sensing state. Hence, the SINR seen by the primary depends on how many symbols of the last secondary packet interfere with the primary packet and is given by,

$$SINR = \gamma_0 \frac{P_1}{N_1} + \frac{\gamma_1 P_1}{M}\left(\frac{M-1}{N_1} + \frac{1}{P_2(K)+N_1}\right) + \ldots + \frac{\gamma_K P_1}{M}\left(\frac{M-K}{N_1} + \frac{1}{P_2(K)+N_1} + \ldots + \frac{1}{P_2(1)+N_1}\right), \quad (4)$$

where γ_k is the probability that the primary interferes with k symbols of a secondary packet. The primary SINR is constrained to stay above $\eta P_1/N_1$.

Finally, the secondary user has an average power constraint of P_2. As a result, the power constraint on each packet is given by $P = P_2 N - (1 - e^{-K\lambda})$ (see Remark 2). With the above setup, the optimization problem can be described as

$$\mathbf{P_2} = \arg\max_{P_2(k)} \frac{K}{2N} \sum_{k=1}^{K} \log\left(1 + \frac{P_2(k)}{N_2}\right),$$
$$\text{s. t. } \sum_{k=1}^{K} \frac{a_k}{P_2(k)+N_1} \geq \rho,$$
$$\sum_{k=1}^{K} P_2(k) \leq P, \text{and } P_2(k) \geq 0, k \in [1, K], \quad (5)$$

where, $a_k = \gamma_K + \ldots + \gamma_{K-k+1}$, $\rho = \frac{\sum k\gamma_k - (1-\eta)M}{N_1}$ and γ_k is given by Lemma 1.

C. Protocol Classes

We compare two protocols. First is the sense and transmit protocol described above, where the secondary user is "paranoid" and does not send unless it is sure that the medium is silent. The second class will be continuous transmission policies, where the secondary does not sense and sends continuously at constant power P_2 for the same SINR constraint.

III. Secondary Power Profile

In this section, we first derive some results which form the building blocks for our main results.

A. Basic Properties

Lemma 1 (Probability of interference): If the primary packet arrival process has a Poisson distribution with rate λ and the secondary follows a power profile given by $P_2(k)$ for $1 \leq k \leq K$, the probability that the primary packet interferes with k time symbols of the last secondary burst , is given by,

$$\gamma_k = \begin{cases} 1 - \frac{e^{\lambda}(e^{-L\lambda} - e^{-N\lambda})}{1-e^{-N\lambda}} & k = 0 \\ e^{k\lambda}\left(\frac{e^{-N\lambda}(e^{\lambda}-1)}{1-e^{-N\lambda}}\right) & 1 \leq k \leq K \end{cases}$$

Proof: See Appendix A. ■

Remark 1: It is easy to show that the γ_ks sum to 1.

$$\sum_{k=1}^{K}\gamma_k = \sum_{k=1}^{K} e^k \frac{e^{-N\lambda}(e^{\lambda}-1)}{1-e^{-N\lambda}}$$
$$= e^{-N\lambda}\frac{e^{(K+1)\lambda}-e^{\lambda}}{1-e^{-N\lambda}} = \frac{e^{\lambda}(e^{-L\lambda}-e^{-N\lambda})}{1-e^{-N\lambda}}, \quad (6)$$

and therefore, $\sum_{k=0}^{K}\gamma_k = \gamma_0 + \sum_{k=1}^{K}\gamma_k = 1$.

Lemma 2 (Average secondary bursts): If the primary packet arrival process has a poisson distribution with rate λ, the average number of successful secondary packets of length K that can be sent during an idle channel is given by, $E[p_s] = \frac{e^{-(N-1)\lambda}}{1-e^{-N\lambda}}$, and the average total number of packets is given by, $E[p_t] = \frac{e^{-(L-1)\lambda}}{1-e^{-N\lambda}}$.

Proof: See Appendix B. ■

Remark 2: Let the power alloted to a packet be P. On an average the number of packets lost due to interference is $\frac{1-\gamma_0}{E[p_t]} = 1-e^{-K\lambda}$. Hence, $P = P_2N - (1-e^{-K\lambda})$.

B. Main Results

Theorem 1: The solution to Equation 5 always exists. The partial characterization of the solution is given as follows. When the SINR constraint is inactive, the solution is given by, $P_2(k) = \frac{P}{K}, \quad k \in [1,K]$ and when the average power constraint is inactive and $N_1 = N_2 = N_0$, the solution is given by, $P_2(1) = N_0\left(\frac{a_1}{a_1-(1-\eta)M}-1\right), P_2(i) = 0, \quad i \in [2,K]$.

Proof: See Appendix C. ■

The key challenge in solving the general case arises from having to solve a high-order polynomial equation. For the special case where the secondary's burst is only two symbols long, the resulting polynomial is quadratic and can be solved as follows. In this case, we assume one extra symbol is used to sense the channel. Also for simplicity, consider $N_1 = N_2 = 1$. For such a case the optimization problem can be restated as,

$$\begin{aligned}\mathbf{P_2} &= \arg\max_{(x,y)} \ \log(1+x)+\log(1+y)\\ &\text{s. t. } \frac{\gamma_2}{x+1}+\frac{\gamma_1+\gamma_2}{y+1} \ge \gamma_1+2\gamma_2-(1-\eta)M,\\ &\text{and } x+y \le \frac{P_2}{E[p_t]}. \end{aligned} \quad (7)$$

As in Theorem 1, this can also be split into different cases, when the SINR constraint is inactive, when the average power constraint is inactive and when both the constraints are tight.

The SINR constraint $\frac{\gamma_2}{x+1}+\frac{\gamma_1+\gamma_2}{y+1} = \rho$ has an x-intercept of $x_0 = \frac{\gamma_2}{\gamma_2-(1-\eta)M}-1$ and y-intercept of $y_0 = \frac{\gamma_1+\gamma_2}{\gamma_1+\gamma_2-(1-\eta)M}-1$. Similarly, the average power constraint has x-intercept $x_1 = P$ and y-intercept $y_1 = P$. Note that both $x_0 \ge 0, y_0 \ge 0$ and that $x_0 > y_0$. Therefore the average power constraint is slack when $x_0 < x_1$. i.e. when,

$$\frac{\gamma_2}{\gamma_2-(1-\eta)M}-1 < P \quad (8)$$

$$\Rightarrow (\frac{P}{P+1})\frac{1-e^{-\lambda}}{1-e^{-3\lambda}} > (1-\eta)M, \quad (9)$$

$$\Rightarrow \eta < 1-\left(\frac{P}{M(P+1)}\right)\frac{1-e^{-\lambda}}{1-e^{-3\lambda}}. \quad (10)$$

In this case, the solution is given by $x = \frac{\gamma_2}{\gamma_2-(1-\eta)M}-1, y = 0$ which is directly obtained from Theorem 1.

The SINR constraint is slack when either $\rho < 0$ or when it lies above the average power constraint. i.e. when, $P - x + 1 < \frac{\gamma_1+\gamma_2}{\rho-\frac{\gamma_2}{x+1}}$. This can be simplified to get the condition, $\rho x^2 + (\gamma_1+\rho P)x + (\gamma_2-\rho)(P+1) > 0$. This holds for all x if the discriminant of this quadratic equation is negative. i.e. when

$$(\gamma_1+\rho P)^2 < 4\rho(\gamma_2-P)(P+1).$$

In this case, the solution is given by, $x = y = \frac{P}{2}$.

Finally, the solution when both the constraints are tight is found by solving the two constraints simultaneously which gives rise to the following quadratic equation, $\rho x^2 + (\gamma_1 - \rho P)x + \gamma_1 + \gamma_2 - \rho(1+P) = 0$. Hence the solution is given by the root of the quadratic which gives the higher rate.

C. Single Burst Transmission

Now let us consider the case when the secondary does not sense for idle channels and sends at a constant power level of P_2 as a function of the rate of the packet arrivals and the SINR constraint. The SINR constraint dictates that $\frac{P_1}{P_2+N_1} \ge \eta\frac{P_1}{N_1} \Rightarrow P_2 \le (\frac{1}{\eta}-1)N_1$. As the secondary is not sensing or waiting for idle channels, there is no rate penalty as a pre-log multiplier in this case. However, as the secondary is sending packets all the time, it also sees interference from the primary. Hence the rate achieved by the secondary, is now given by,

$$\frac{1}{2(1+\lambda M)}\log\left(1+\frac{P_2}{N_2}\right)+\frac{\lambda M}{2(1+\lambda M)}\log\left(1+\frac{P_2}{P_1+N_2}\right).$$

The fraction of idle time and busy time of the channel is derived in Section II-B. We will numerically compare the performance of this scheme with the multi-burst scheme in the next section.

IV. Numerical Results

Even though it is difficult to find closed form solutions for the general case, we can numerically find the solution to see how the performance of the secondary system gains from the decaying power profile. To understand the shape of the power profile let us first look at the constraint set of the 2-d power profile as shown in Figure 2. For lower packet rates of the primary ($\lambda = 0.6$), the average power constraint is the determining equation for the secondary. This can be seen from the fact that the SINR constraint lies above the average power constraint in Figure 2. Hence the solution lies at point A. But for higher packet rates of the primary, the optimal solution lies at the intersection of both the constraints at point B.

Figure 3 shows the dependence of the secondary rate with respect to the rate of primary packet arrivals. For smaller values of η, i.e. when the primary allows for large interference, the determining equation for the secondary is the average power constraint and in this case all the curves for $\eta < 0.7$ coincide on the solid curve shown in Figure 3. The curve for $\eta = 0.95$ changes slope at the point where the determining

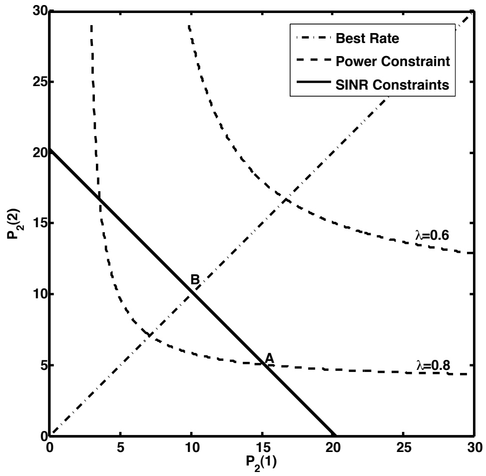

Fig. 2. *Constraint sets for a three burst secondary packet size for different primary packet statistics,* $\eta = 0.87, P_2 = 7$ *and* $N_1 = N_2 = 1$.

equation changes from the average power constraint to the intersection of both the constraints. It can be clearly seen that the multi-burst system outperforms the single burst scheme.

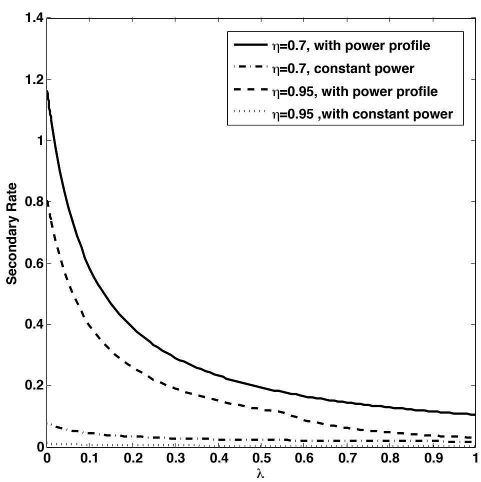

Fig. 3. *Primary achievable rate for the multi-burst and the single burst strategy under the same interference constraints as a function of* λ.

Figure 4 shows some of the power profiles for P_2 with $N = 3$ and $N = 4$, for $L = 1$. For the 2-d case, at lower values of η, the the power profile is uniform as the only constraint is the average power constraint as seen in Figure 2 too. For higher η and λ, the profile starts to look like an exponential decay.

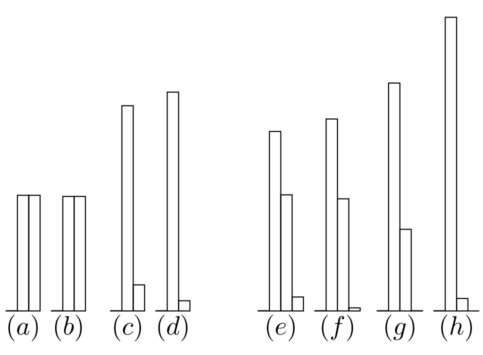

Fig. 4. *2-d power profiles with parameters: (a)* $\eta = 0.85, \lambda = 0.4$*, (b)* $\eta = 0.85, \lambda = 0.8$*, (c)* $\eta = 0.9, \lambda = 0.4$*, (d)* $\eta = 0.9, \lambda = 0.8$ *and similarly for (e), (f), (g) and (h) for 3-d power profiles.*

V. Conclusions

We analyzed the interference channel with a primary link having bursty data and a secondary link which intelligently injects packets into the temporal white spaces by sending small burst of packets with a power profile matched to the packet arrival rate of the primary. We formulated the general problem as an optimization problem with SINR and average power constraints and derived the parameters of the system. For the special case of a 2 symbol power profile, we saw that the multi-burst strategy outperforms a single burst non-sensing scheme. We saw that when the only constraint on the system is the average power constraint, the power profile distributes equally between all the time symbols but when the SINR constraint or both of them are tight, the power profile is skewed toward the start of the packet.

Appendix A
Proof of Lemma 1

We will denote the inter-arrival time, expressed in symbols as T_s. Hence, if $T \in (i, i+1], T_s = i+1$, and $p_s = \lfloor \frac{T_s}{N} \rfloor$. By definition, p_s secondary bursts can be sent before the next primary packet arrives and in some cases, the $(p_s+1)^{th}$ secondary packet interferes with the primary data. Let T_s^L denote how many symbols are left after the p_s^{th} secondary burst. Note that T_s^L can take values only between 0 and $N-1$. Just after the p_s^{th} burst, the secondary listens for L symbols, hence if the primary packet starts during this interval, there will be no interference. i.e. $\gamma_0 = Pr[T_s^L \in [0, L]]$. If the primary packet starts just after the L symbols, the secondary user will get to know about the primary packet only when it senses next and hence the primary will interfere with the whole secondary burst.

Therefore, $\gamma_K = Pr[T_s^L = L+1]$ and with a similar argument, $\gamma_k = Pr[T_s^L = L+K-k+1] = Pr[T_s^L = N-k+1]$, for $1 \le k \le K$. Note that $T_s^L = N-k+1$, if $T_s = iN-k+1$, for any integer $i > 0$, where T_s is the number of symbols since the last primary packet ended. If we consider time to start from $t = 0$, then the i^{th} time slot is when $n-1 \le t \le n$. Hence, for the above values of T_s^L, $iN-k-1 \le T < iN-k$, where T is the inter-arrival time which has an exponential distribution. Therefore, for any $1 < k \le K$,

$$\gamma_k = \sum_{i=1}^{\infty} Pr[iN-k-1 \le T < iN-k] = \sum_{i=1}^{\infty} \lambda \int_{iN-k-1}^{iN-k} e^{-\lambda T} dT$$
$$= e^{k\lambda}(e^{\lambda}-1) \sum_{i=1}^{\infty} -e^{-iN\lambda} = e^{k\lambda} \left(\frac{e^{-N\lambda}(e^{\lambda}-1)}{1-e^{-N\lambda}} \right). \qquad (11)$$

Similarly,

$$\gamma_0 = \sum_{i=0}^{\infty} Pr[iN \le T < iN+L-1] +$$
$$\sum_{i=0}^{\infty} Pr[(i+1)N-1 \le T < (i+1)N]$$
$$= \sum_{i=0}^{\infty} \left[e^{-iN\lambda} - e^{-(iN+L-1)\lambda} + e^{-((i+1)N-1)\lambda} - e^{-(i+1)N\lambda} \right]$$
$$= 1 - \frac{e^{\lambda}(e^{-L\lambda} - e^{-N\lambda})}{1-e^{-N\lambda}}. \qquad (12)$$

Appendix B
Proof of Lemma 2

Let T and T_s be as defined in Appendix A. The inter-arrival time has an exponential distribution with parameter λ $\left(f_T(t) = \lambda e^{-\lambda t}, t \ge 0\right)$. Any packet that arrives during a given time slot is sent over the channel by the primary user in the next time slot.

To find $E[p_s]$, we have to first find the probability distribution of p_s. Keeping in mind that if the last secondary burst interferes with the primary, it is not counted in p, notice that,

$$\begin{aligned} p_s = 0 &\text{ if } T_s = 0, \dots, N-1 \\ &\Rightarrow T \in [0, N-1] \\ p_s = 1 &\text{ if } T_s = N, \dots, 2N-1 \\ &\Rightarrow T \in (N-1, 2N-1] \\ \vdots \quad & \quad \vdots \\ p_s = i &\text{ if } T_s = iN, \dots, iN-1 \\ &\Rightarrow T \in (iN-1, (i+1)N-1] \end{aligned} \tag{13}$$

$$\begin{aligned} \text{Hence, } Pr(p_s = i) &= \int_{iN-1}^{(i+1)N-1} \lambda e^{-\lambda T} dT \\ &= e^{-(iN-1)\lambda} - e^{-((i+1)N-1)\lambda} \\ &= e^{-iN\lambda}(e^{\lambda} - e^{-(N-1)\lambda}). \end{aligned} \tag{14}$$

$$\begin{aligned} \text{Therefore, } E[p_s] &= \sum_{i=0}^{\infty} iPr(p_s = i) \\ &= \sum_{i=0}^{\infty} ie^{-iN\lambda}(e^{\lambda} - e^{-(N-1)\lambda}) \\ &= (e^{\lambda} - e^{-(N-1)\lambda}) \sum_{i=0}^{\infty} ie^{-iN\lambda} \\ &= \frac{e^{-N\lambda}(e^{\lambda} - e^{-(N-1)\lambda})}{(1-e^{-N\lambda})^2} = \frac{e^{-(N-1)\lambda}}{1-e^{-N\lambda}}. \end{aligned} \tag{15}$$

Now, in order to calculate the total number packets sent on average, notice that we only calculated the packets that did not interfere with the primary transmission. During any idle interval, if the primary doesn't interfere with the secondary at all, then the average number of packets that the secondary can send is $\gamma_0 \sum_i iPr(p_s = i)$. If the primary interferes with one time-slot of the secondary packet, then the secondary sends one extra packet and similarly for all other cases. Note that because there is no error in sensing, the secondary always sends at most one packet which interferes with the primary. Hence, the total number of packets is given by,

$$\begin{aligned} E[p_t] &= \gamma_0 E[p_s] + \gamma_1(E[p]+1) + \ldots + \gamma_K(E[p]+1) \\ &= (\gamma_0 + \gamma_1 + \ldots + \gamma_K)E[p_s] + (\gamma_1 + \ldots + \gamma_K) \\ &= E[p_s] + (1-\gamma_0) = \frac{e^{-(L-1)\lambda}}{1-e^{-N\lambda}}. \end{aligned} \tag{16}$$

In the last step we used the fact that $\gamma_0 + \ldots + \gamma_K = 1$ as shown in Lemma 1.

APPENDIX C
PROOF OF THEOREM 1

The optimization problem can be re-written as, For simplicity of notation put $x_k = P_2(k)$ and lets call utility function $f(x)$, the SINR constraint $g_1(x)$ and the average power constraint $g_2(x)$. As $f(\mathbf{x})$ is a strictly concave function and the constraint set is compact (since $g_1(\mathbf{x}) \leq \rho$ and $g_2(\mathbf{x}) \leq P$ are closed and bounded sets), by the Extreme Value Theorem, Equation 5 has at least one solution. Also, as $f((\mathbf{x}))$ is concave, $g_1((\mathbf{x}))$ is concave and $g_2((\mathbf{x}))$ linear, the KKT conditions are necessary and sufficient for this problem.

As there is a non-negativity constraint, the modified Lagrangian is given by,

$$\begin{aligned} \mathscr{L}(\mathbf{x}) &= f(\mathbf{x}) - \mu_1 g_1(\mathbf{x}) - \mu_2 g_2(\mathbf{x}) \\ &= \sum_{k=1}^{K} \log(1 + \frac{x_k}{N_2}) + \mu_1 \sum_{k=1}^{K} \frac{a_k}{x_k + N_1} - \mu_2 \sum_{k=1}^{K} x_k \end{aligned} \tag{17}$$

Therefore, the KKT conditions for this modified Lagrangian are given by ($k = 1, \ldots, K$ and $i = 1, 2$),

$$\begin{aligned} &\mathscr{L}_k^{'}(\mathbf{x}^*) \leq 0, x_k \geq 0 \text{ and } x_k\left(\mathscr{L}_k^{'}(\mathbf{x}^*)\right) = 0, \\ &g_i(\mathbf{x}^*) \leq c_i, \mu_i \geq 0 \text{ and } \lambda_i\left(g_i(\mathbf{x}^*) - c_i\right) = 0; \end{aligned} \tag{18}$$

$$\begin{aligned} &\text{i.e., } \frac{1}{x_k + N_2} - \frac{\mu_1 a_k}{(x_k+N_1)^2} - \mu_2 \leq 0, x_k \geq 0 \\ &\qquad \text{and } x_k\left(\frac{1}{x_k+N_2} - \frac{\mu_1}{(x_k+N_1)^2} - \mu_2\right) = 0, \\ &\sum_{k=1}^{K} \frac{a_k}{x_k+N_1} \geq \rho, \mu_1 \geq 0 \text{ and } \mu_1\left(\sum_{k=1}^{K} \frac{a_k}{x_k+N_1} + \rho\right) = 0, \\ &\sum_{k=1}^{K} x_k \leq P, \mu_2 \geq 0 \text{ and } \mu_2\left(\sum_{k=1}^{K} x_k - P\right) = 0. \end{aligned} \tag{19}$$

To solve these $K+2$ simultaneous equations for $x_1, \ldots, x_K, \mu_1, \mu_2$, let us consider different cases.

Case I: SINR constraint is inactive
The SINR constraint is inactive when $\mu_1 = 0$ and $\mu_2 \neq 0$. This can happen when either the right hand side of the SINR constraint is negative ($\rho < 0$) or when $g_1(x)$ lies completely above $g_2(x)$. From Equation 19, $\mu_2 = \frac{1}{x_1+N_2} = \ldots = \frac{1}{x_K+N_2}$, which implies, $P_2(k) = \frac{P_2}{KE[p_t]}, i \in \{1, 2, \ldots, K\}$ and $\mu_2 = \frac{K}{P+KN_2}$.

Case II: Average power constraint is inactive
The average power constraint is inactive when $\mu_1 \neq 0$ and $\mu_2 = 0$. When $N_1 = N_2 = N_0$, using Equation 19, $\mu_1 = \frac{x_1+N_0}{a_1} = \ldots = \frac{x_K+N_0}{a_K}$, which implies, $P_2(k) = \frac{Ka_k}{\rho} - N_o$ and $\mu_1 = \frac{K}{\rho}$. However, note that this is in fact the minima of the curve (bordered Hessian of the Lagrangian is negative), hence, the maxima lies on one of the endpoints. The intercepts of the SINR constraint with the axes are given by $x_i^0 = \frac{a_i}{\rho - \frac{a_1}{N_0} - \cdots - \frac{a_K}{N_0} + \frac{a_i}{N_0}} - N_0 = N_0(\frac{a_i}{a_i-(1-\eta)M} - 1)$. Note that $x_1^0 > x_2^0 > \cdots > x_K^0$. Hence, the maxima is obtained at $\mathbf{x} = (x_1^0, 0, \cdots, 0)$.

REFERENCES

[1] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Transactions On Information Theory*, vol. 52, no. 5, May 2006.

[2] Federal Communications Commission Spectrum Policy Task Force, "Report of the spectrum efficiency working group," *Technical Report 02-135*, November 2002.

[3] A. J. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," 2008, unpublished.

[4] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal On Selected Areas In Communications*, vol. 23, no. 2, February 2005.

[5] S. A. Jafar and S. Srinivasa, "Capacity limits of cognitive radio with distributed and dynamic spectral activity," *IEEE Journal On Selected Areas In Communications*, vol. 25, pp. 529–537, 2007.

[6] A. Jovicic and P. Viswanath, "Cognitive radio: an information-theoretic perspective," May 2006.

[7] J. Mitola, "Cognitive radio: An integrated agent architecture for software defined radio," Ph.D. dissertation, KTH, Stockholm, Sweden, December 2000.

[8] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Stable throughput of cognitive radios with and without relaying capability," *IEEE Transactions On Information Theory*, vol. 55, no. 12, pp. 2351–2360, December 2007.

[9] S. Srinivasa and S. Jafar, "Soft sensing and optimal power control for cognitive radio," in *Global Telecommunications Conference, 2007. GLOBECOM '07. IEEE*, 26-30 Nov. 2007, pp. 1380–1384.

[10] W. Wu, S. Vishwanath, and A. Arapostathis, "Capacity of a class of cognitive radio channels: interference channels with degraded message sets," *IEEE Transactions On Information Theory*, vol. 53, no. 11, pp. 4391–4399, November 2007.