# THE 2011 SCHOOL ON APPROXIMABILITY

MATERIALS RESEARCH CENTER (MRC) AUDITORIUM INDIAN INSTITUTE OF SCIENCE, BANGALORE, INDIA

JANUARY 5-9, 2011

The focus of this school will be the approximability of optimization problems. Traditionally, approximability used to be a two-pronged task divided into algorithms and complexity but recently, a lot of cross-fertilization is taking place and the boundaries between algorithms and complexity are blurring. One of the goals of this school would be to further bring down this divide and promote a holistic thinking about approximability, which should lead to exciting new developments in the area.

The school will bring together key contributors in this area to discuss these modern methods to a large research audience. In addition, the school will also have talks and open sessions for sharing new research and directions in approximability.

## The Speakers.

- Sanjeev Arora (Princeton University)
- L. Sunil Chandran (Indian Institute of Science)
- Amit Deshpande (Microsoft Research India)
- Nikhil Devanur (Microsoft Research Redmond)
- Naveen Garg (Indian Institute of Technology Delhi)
- Prahladh Harsha (Tata Institute of Fundamental Research, Mumbai)
- Kamal Jain (Microsoft Research Redmond)
- Ravi Kannan (Microsoft Research India)
- Richard M. Karp (University of California Berkeley)
- Subhash Khot (New York University)
- Amit Kumar (Indian Institute of Technology Delhi)
- Lorenzo Orecchia (University of California Berkeley)
- Vinayaka Pandit (IBM India Research Lab)
- Prasad Raghavendra (Georgia Institute of Technology)
- R. Ravi (Carnegie Mellon University)
- Sambuddha Roy (IBM India Research Lab)
- Saket Saurabh (Institute of Mathematical Sciences, Chennai)
- Mohit Singh (McGill University)
- David Steurer (Microsoft Research New England)
- Madhur Tulsiani (Princeton University/Institute for Advanced Study)

## The Sponsors.

- Microsoft Research India (MSR India)
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## The Organizers.

- Co-Chairs
  - Naveen Garg (Indian Institute of Technology Delhi)
  - Nisheeth K. Vishnoi (Microsoft Research India)
- Support from IISc and MSR India
  - G. Rangarajan (Convener, IISc Mathematics Initiative)
  - Vidya Natampally (Microsoft Research India, Director of Strategy)
  - Ashwani Sharma (Microsoft Research India, Program Manager, External Research)

Table 1: The Program at a Glance

		Jan 5	Jan 6	Jan 7	Jan 8	Jan 9
0900	0930	Registration				
0930	1000	Tea/Coffee				
1000	1100	R. Karp	R. Ravi	S. Khot	R. Ravi	A. Kumar
1100	1130	Tea/Coffee				
1130	1230	S. Arora	S. Arora	P. Raghavendra	S. Khot	N. Garg
1230	1400	Lunch				
1400	1500	D. Steurer	D. Steurer	N. Devanur	P. Raghavendra	
1500	1505	Break				
1505	1605	M. Singh	L. Orecchia	S. Roy	P. Harsha	
1605	1630	Tea/Coffee				
1630	1730	R. Kannan	S. Khot	K. Jain	M. Tulsiani	
1730	1800	A. Deshpande	S. Saurabh	V. Pandit	L. S. Chandran	

#### The Venues.

- Registration: Materials Research Center (MRC) Auditorium, IISc.
- Tea/Coffee: MRC Auditorium, IISc.
- Lunch: Main Guest House Lawns, IISc.
- Professor Karp's Talk: 1000-1100 on Jan 5, 2011 will be at the Faculty Hall, Main Building, IISc. Everyone is expected to be seated by 0955.
- All Other Talks: MRC Auditorium, IISc.

## The Technical Program in Detail.

- Wednesday, January 5, 2011
  - 0900-0930: *Registration*
  - 0915-0945: Tea/Coffee
  - 1000-1100: Richard M. Karp [Keynote Talk]
     Effective Heuristics for NP-Hard Problems Arising in Molecular Biology.
  - 1100-1130: Tea/Coffee
  - 1130-1230: **Sanjeev Arora** [Keynote Talk] Semidefinite Programming and Approximation Algorithms: A Survey.
  - 1230-1400: Lunch
  - 1400-1500: **David Steurer** *Algorithms for Unique Games and Related Problems.*
  - 1500-1505: Break
  - 1505-1605: Mohit Singh
     A Randomized Rounding Approach to the Traveling Salesman Problem.
  - 1605-1630: Tea/Coffee
  - 1630-1730: **Ravi Kannan** *Questions and Algorithms for Tensors.*
  - 1730-1800: **Amit Deshpande** *Approximability of Subspace Approximation.*
- Thursday, January 6, 2011
  - 0930-1000: Tea/Coffee
  - 1000-1100: **R. Ravi**

Matching Based Augmentations for Approximating Connectivity Problems.

- 1100-1130: *Tea/Coffee*
- 1130-1230: Sanjeev Arora
   Algorithms for Approximating Graph Expansion and Connections to Geometry.
- 1230-1400: Lunch
- 1400-1500: **David Steurer** *Graph Expansion and the Unique Games Conjecture.*
- 1500-1505: *Break*
- 1505-1605: **Lorenzo Orecchia** *Fast Spectral Algorithms for Graph Partitioning and Graph Decomposition.*
- 1605-1630: Tea/Coffee
- 1630-1730: **Subhash Khot** *Introduction to Hardness of Approximation and Probabilistically Checkable Proofs.*

1730-1800: Saket Saurabh
 Slightly Superexponential Parameterized Problems.

## - Friday, January 7, 2011

- 0930-1000: Tea/Coffee
- 1000-1100: Subhash Khot
   Inapproximability of NP-complete Problems, Discrete Fourier Analysis, and Geometry.
- 1100-1130: Tea/Coffee
- 1130-1230: **Prasad Raghavendra** How to Round Any CSP?
- 1230-1400: Lunch
- 1400-1500: **Nikhil Devanur** *The Steiner Tree Problem.*
- 1500-1505: Break
- 1505-1605: **Sambuddha Roy** *Primal Dual Approximation Algorithms.*
- 1605-1630: *Tea/Coffee*
- 1630-1730: Kamal Jain
   Online Bipartite Matching via a Primal-Dual Technique for Convex Programs.
- 1730-1800: Vinayaka Pandit
   Local Search Based Approximation Algorithms for Facility Location Problems.

## - Saturday, January 8, 2011

- 0930-1000: Tea/Coffee
- 1000-1100: **R. Ravi** *Iterative Methods in Combinatorial Optimization.*
- 1100-1130: Tea/Coffee
- 1130-1230: **Subhash Khot** *On the Unique Games Conjecture.*
- 1230-1400: Lunch
- 1400-1500: Prasad Raghavendra
   Complexity of Constraint Satisfaction Problems: Exact and Approximate.
- 1500-1505: *Break*
- 1505-1605: Prahladh Harsha
   Inapproximability Results from Label-Cover Variants and Other Hardness Assumptions.
- 1605-1630: Tea/Coffee
- 1630-1730: **Madhur Tulsiani** *Introduction to LP and SDP Hierarchies.*

- 1730-1800: **L. Sunil Chandran** *Rainbow Colouring of Graphs.*
- Sunday, January 9, 2011
  - 0930-1000: Tea/Coffee
  - 1000-1100: **Amit Kumar** *Scheduling to Minimize Weighted Flow-Time.*
  - 1100-1130: Tea/Coffee
  - 1130-1230: Naveen Garg
     Fast Combinatorial Algorithms for Solving Positive Linear Programs.
  - 1230-1400: Lunch

#### The Abstracts.

## 1. Jan 5, 2011. 1000-1100: **Richard M. Karp**

Effective Heuristics for NP-Hard Problems Arising in Molecular Biology.

**Abstract.** The speaker and his colleagues have recently addressed a variety of NP-hard problems arising in genetics and molecular biology. These include global alignment of multiple genomes, inference of ancestral relationships from genomic data, discovery of conserved protein modules from protein-protein interaction data, discovery of disease signatures from gene expression data, and discovery of genetic interactions between protein complexes.

The talk will describe successful heuristic algorithms for two of these problems:

- (a) Identifying Siblings Given a population of individuals and noisy data about which pairs are siblings, identify the true sibling pairs using the assumption that siblinghood is a transitive relation (joint work with Bonnie Kirkpatrick and Shuai Li).
- (b) The Colorful Subgraph Problem Given a graph in which each vertex is labeled with a color, find, if one exists, a connected subgraph containing exactly one vertex of each color. In one interpretation of the problem the vertices represent proteins, the edges represent protein-protein interactions and the colors represent the functional classifications of the proteins (joint work with colleagues at Berkeley and Tel Aviv University).

The talk will close with some general observations about the construction and evaluation of heuristic algorithms for NP-hard problems.

## 2. Jan 5, 2011. 1130-1230: Sanjeev Arora

Semidefinite Programming and Approximation Algorithms: A Survey.

**Abstract.** Computing approximately optimal solutions is an attractive way to cope with NP-hard optimization problems. In the past decade or so, semidefinite programming or SDP (a form of convex optimization that generalizes linear programming) has emerged as a powerful tool for designing such algorithms, and the last few years have seen a profusion of results (worst-case algorithms, average case algorithms, impossibility results, etc).

This talk will be a survey of this area and these recent results. We will see that analysing semidefinite program draws upon ideas from a variety of other areas, and has also led to new results in mathematics.

At the end we will touch upon work that greatly improves the running time of SDP-based algorithms, making them potentially quite practical.

The survey will be essentially self-contained.

## 3. Jan 5, 2011. 1400-1500: David Steurer

Algorithms for Unique Games and Related Problems.

**Abstract.** I will discuss several algorithms for Unique Games with approximation guarantees independent of the alphabet size and instance size.

The first algorithms of this kind applied to restricted classes of constraint graphs, e.g., expanders (Arora et al., 2008), ruling out that these instance classes are hard in the sense of the Unique Games Conjecture.

Developing these ideas further led to a subexponential algorithm for general instances of Unique Games (Arora, Barak, Steurer, 2010). This algorithm demonstrates that hardness results based on the Unique Games Conjecture cannot rule out subexponential algorithms unlike many NP-hardness results.

## 4. Jan 5, 2011. 1505-1605: Mohit Singh

A Randomized Rounding Approach to the Traveling Salesman Problem.

**Abstract.** I will outline a  $(3/2 - \varepsilon)$ -approximation algorithm for the graphical traveling salesman problem where the goal is to find a shortest tour in an unweighted graph G. This is a special case of the metric traveling salesman problem when the underlying metric is defined by shortest path distances in G. The result improves on the 3/2-approximation algorithm due to Christofides.

Similar to Christofides, our algorithm first finds a spanning tree whose cost is upper-bounded by the optimum and then it finds the minimum cost Eulerian augmentation of that tree. The main difference is in the selection of the spanning tree. Except in certain cases where the solution of LP is nearly integral, we select the spanning tree randomly by sampling from a maximum entropy distribution defined by the linear programming relaxation.

Despite the simplicity of the algorithm, the analysis builds on a variety of ideas such as properties of strong Rayleigh measures from probability theory, graph theoretical results on the structure of near minimum cuts, and the integrality of the T-join polytope from polyhedral theory.

This is joint work with Shayan Oveis Gharan and Amin Saberi.

## 5. Jan 5, 2011. 1630-1730: Ravi Kannan

*Questions and Algorithms for Tensors.* 

**Abstract.** Minimizing or maximizing a quadratic form  $(\sum_{ij} A_{ij} x_i x_j)$  over unit length vectors x, can be done using Linear Algebra. No such nice structure or algorithms exist for cubic forms  $(\sum_{ijk} A_{ijk} x_i x_j x_k)$ . Not much is known about how well we can approximate cubic and higher forms. These and other problems concerning tensors, their applications, available algorithms and open questions will be described.

## 6. Jan 5, 2011. 1730-1800: Amit Deshpande

Approximability of Subspace Approximation.

**Abstract.** Given a data set of points in large dimensions, the subspace approximation problem asks for a linear subspace that "fits" these points well. In this talk, we'll focus on particular subspace-fits defined using  $\ell_p$  norms, which generalize the ordinary least-squares fits. We'll give approximation algorithms for these problems based on semidefinite/convex relaxation and randomized rounding. Moreover, we'll show that it's hard to do any better (unless the Unique Games Conjecture is false).

Based on joint work with Madhur Tulsiani and Nisheeth K. Vishnoi.

## 7. Jan 6, 2011. 1000-1100: R. Ravi

Matching Based Augmentations for Approximating Connectivity Problems.

**Abstract.** We describe a very simple idea for designing approximation algorithms for connectivity problems: For a spanning tree problem, the idea is to start with the empty set of edges, and add

matching paths between pairs of components in the current graph that have desirable properties in terms of the objective function of the spanning tree problem being solved. Such a matching augments the solution by reducing the number of connected components to roughly half their original number, resulting in a logarithmic number of matching iterations. A logarithmic performance ratio results for the problem by appropriately bounding the contribution of each matching to the objective function by that of an optimal solution. In this survey talk, we trace the initial application of these ideas to traveling salesperson problems through a simple tree pairing observation down to more sophisticated applications for constrained spanning tree and buy-at-bulk type network design problems.

A write-up summarizing this talk appears in the proceedings of LATIN 2006.

## 8. Jan 6, 2011. 1130-1230: Sanjeev Arora

Algorithms for Approximating Graph Expansion and Connections to Geometry.

**Abstract.** Partitioning a graph into two (or more) large pieces while minimizing the size of the "interface" between them is a fundamental combinatorial problem. Graph partitions or separators are central objects of study in the theory of Markov chains, geometric embeddings and are a natural algorithmic primitive in numerous settings, including clustering, divide and conquer approaches, PRAM emulation, VLSI layout, and packet routing in distributed networks. This talk surveys geometric approaches to approximating graph expansion that have led to new results in computer science and mathematics.

#### 9. Jan 6, 2011. 1400-1500: **David Steurer**

*Graph Expansion and the Unique Games Conjecture.* 

**Abstract.** I will discuss a close connection between the Unique Games Conjecture and the approximability of graph expansion.

The Small-Set Expansion Hypothesis is a natural assertion about the approximability of the expansion of small sets in graphs. This hypothesis turns out to imply the Unique Games Conjecture (Raghavendra, Steurer, 2010), giving the first sufficient condition for the truth of the conjecture based on a problem other than Unique Games.

The connection between graph expansion and the Unique Games Conjecture also has further consequences for integrality gaps and the approximability of basic graph partitioning problems.

## 10. Jan 6, 2011. 1505-1605: Lorenzo Orecchia

Fast Spectral Algorithms for Graph Partitioning and Graph Decomposition.

**Abstract.** The main object of this talk is a graph decomposition that partitions a graph in a number of internally-well connected pieces (almost expanders) while ensuring that only a small number of edges connect different pieces. In the first part of the talk, I will discuss applications of this decomposition to some fundamental algorithmic problems, including graph sparsification and the solution of certain systems of linear equations. In the second part, I will focus on how to produce such a decomposition very fast, i.e. in nearly-linear time, using spectral algorithm to approximate the balanced cut-problem. Finally, I will present a recent result in this direction, which is based on a novel semidefinite programming approach to the design of spectral algorithms.

#### 11. Jan 6, 2011. 1630-1730: Subhash Khot

Introduction to Hardness of Approximation and Probabilistically Checkable Proofs.

**Abstract.** This talk will provide a basic introduction to the area, in particular the connection between the "PCP Theorem" and hardness of approximation. The talk will be based on Lectures 1-3 available at: http://www.cs.nyu.edu/khot/pcp-course.html

#### 12. Jan 6, 2011. 1730-1800: Saket Saurabh

Slightly Superexponential Parameterized Problems.

**Abstract.** A central problem in parameterized algorithms is to obtain algorithms with running time  $f(k) \cdot n^{O(1)}$  such that f is as slow growing function of the parameter k as possible. In particular, the first natural goal is to make f(k) single-exponential, that is,  $c^k$  for some constant c. This has led to the development of parameterized algorithms for various problems where f(k) appearing in their running time is of form  $2^{O(k)}$ . However there are still plenty of problems where the "slightly super-exponential" f(k) appearing in the best known running time has remained non single-exponential even after a lot of attempts to bring it down. A natural question to ask is whether the f(k) appearing in the running time of the best-known algorithms is optimal for any of these problems.

In this talk, we examine parameterized problems where f(k) is  $k^{O(k)} = 2^{O(k \log k)}$  in the best known running time and for a number of such problems, we show that the dependence on k in the running time cannot be improved to single exponential, unless exponential time hypothesis (ETH) fails.

#### 13. Jan 7, 2011. 1000-1100: Subhash Khot

Inapproximability of NP-complete Problems, Discrete Fourier Analysis, and Geometry.

**Abstract.** This article gives a survey of recent results that connect three areas in computer science and mathematics: (1) (Hardness of ) computing approximate solutions to NP-complete problems. (2) Fourier analysis of boolean functions on boolean hypercube. (3) Certain problems in geometry, especially related to isoperimetry and embeddings between metric spaces.

#### 14. Jan 7, 2011. 1130-1230: Prasad Raghavendra

How to Round Any CSP?

**Abstract.** A large number of interesting combinatorial optimization problems like Max3SAT, MaxCut fall under the class of constraint satisfaction problems. Recent work identifies a semidefinite programming relaxation that yields the optimal approximation ratio for every CSP, under the Unique Games Conjecture.

In this talk, we will present the optimal SDP relaxation, and an efficient rounding scheme that achieves the integrality gap of this optimal SDP, unconditionally for every CSP. This optimal SDP relaxation is stronger or equivalent to any relaxation used in literature to approximate a CSP. Thus, irrespective of the truth of UGC, this yields a generic algorithm that for every CSP, achieves an approximation at least as good as the best known algorithm in literature.

#### 15. Jan 7, 2011. 1400-1500: Nikhil Devanur

The Steiner Tree Problem.

**Abstract.** The Steiner Tree problem is as follows: given an undirected graph with edge costs and a subset of the vertices called the required vertices, find a tree of minimum cost that connects all the required vertices. This is one of the most fundamental problems in combinatorial optimization. I will survey various Linear Programming relaxations for this problem. This includes the Bidirected cut relaxation, determining whose integrality gap is a famous open problem. The Hypergraphic relaxation gives the best known approximation algorithm. I will also show a geometric relaxation, that is equivalent to the bidirected cut relaxation and based on this a 4/3 approximation algorithm for graphs with no edges between any two non-required vertices, called quasi-bipartite graphs.

## 16. Jan 7, 2011. 1505-1605: Sambuddha Roy

Primal Dual Approximation Algorithms.

**Abstract.** The primal dual paradigm in algorithms is an oft-used design technique for approximation algorithms. The talk will mostly be a survey of various primal dual methods as applied to linear integer programs. We will discuss certain representative scenarios where primal dual methods do apply. However we will also discuss certain scenarios where the paradigm does not apply directly, but some extension works.

## 17. Jan 7, 2011. 1630-1730: Kamal Jain

Online Bipartite Matching via a Primal-Dual Technique for Convex Programs.

**Abstract.** Greedy algorithms and Primal-Dual algorithms are two popular combinatorial techniques for designing approximation algorithm. Each of these techniques has its own focus, greedy algorithms focus on picking the optimal next step and a certificate of performance guarantee is constructed at the end of the algorithm. On the other hand Primal-Dual algorithms focus on building the certificate of performance guarantee even if it means picking suboptimal next steps. I will present the following three kinds of teachings:

- (a) A merge of these two techniques.
- (b) A dual program of convex programs suitable for primal-dual type of analysis.
- (c) Using (a) and (b), extending the merged technique to convex programs from the linear programs.

The presentation will be self-contained and will be done at a slow speed.

#### 18. Jan 7, 2011. 1730-1800: Vinayaka Pandit

Local Search Based Approximation Algorithms for Facility Location Problems.

**Abstract.** Local search is used frequently by practitioners for several important combinatorial optimization problems. It has also been successfully used to develop approximation algorithms for a few optimization problems. It has turned out to be a very powerful technique for facility location problems. In this talk, we will present the basic ideas behind analyzing local search heuristics from the point of approximation ratio, present an analysis of a very popular heuristic for the *k*-median

problem. We will also highlight the capacitated facility location problems for which, so far, local search is the only known technique to yield non-trivial approximation ratios and highlight interesting directions in this context.

## 19. Jan 8, 2011. 1000-1100: R. Ravi

Iterative Methods in Combinatorial Optimization.

**Abstract.** I will describe a simple iterative method for proving a variety of results in combinatorial optimization. It is inspired by Jain's iterative rounding method for designing approximation algorithms for survivable network design problems, and augmented with a relaxation idea in the work of Lau, Naor, Salvatipour and Singh in their work on designing the approximation algorithm for its degree bounded version. Its application was further refined in work of Bansal, Khandekar and Nagarajan on degree-bounded directed network design.

There are two key ingredients in the application of the iterative method using a linear programming formulation of the problem. The first is that if any extreme point solution is characterized by few linearly independent tight constraints, then the number of nonzeros in the support of the solution is also bounded by this number and hence an averaging argument implies that one of these nonzeros has sufficiently high value. The second is the particular way in which we can show that different formulations have small number of linearly independent tight constraints at any extreme point solution, which usually involves ideas like uncrossing tight constraints. Once we have an understanding of a problem using this line of attack, it can be very useful in incorporating additional constraints; even if the original problem is polynomial-time solvable, the method can be useful in designing an approximation algorithm for the version with constraints that render it NP-hard.

In this talk, I will start by reviewing some key background in linear programming. Then I will describe the application of the first key ingredient to showing the integrality of the classical assignment problem, and its adaptation to designing an approximation algorithm for the NP-hard generalized assignment version. Then I will move move to showing the integrality of spanning tree polyhedron using the second ingredient of uncrossing constraint and extend the approach to designing approximation algorithms for its degree constrained version. If time permits, I will show how Jain's original proof can be simplified by the new perspective.

This talk describes work of all the above authors interpreted in collaboration with Lau, Nagarajan and Singh.

#### 20. Jan 8, 2011. 1130-1230: Subhash Khot

*On the Unique Games Conjecture.* 

**Abstract.** This talk will survey recently discovered connections between the Unique Games Conjecture and computational complexity, algorithms, discrete Fourier analysis, and geometry. The talk and a written survey are available at: http://cs.nyu.edu/khot/publications.html

## 21. Jan 8, 2011. 1400-1500: Prasad Raghavendra

Complexity of Constraint Satisfaction Problems: Exact and Approximate.

**Abstract.** Is there a common explanation for 2SAT being solvable in polynomial time, and Max2SAT being approximable to a 0.91 factor? More generally, it is natural to wonder what characterizes the complexity of exact constraint satisfaction problems (CSP) like 2SAT and what determines the approximation ratios for MaxCSPs like Max2SAT.

A Constraint Satisfaction Problem (CSP) such as 3-SAT, Horn-SAT is specified by a set of predicates. In the exact version, the goal is to find out if an instance of the CSP is completely satisfiable or unsatisfiable. In the approximate version, the goal is to find an assignment that satisfies the maximum number of constraints.

The algebraic dichotomy conjecture attempts to characterize CSPs for which the exact version is easy, while the Unique Games Conjecture yields the correct thresholds of approximability for CSPs. Although posed independently, the two conjectures point to the same underlying characterization for the complexity of a CSP. Specifically, they assert that the complexity of CSP is determined by the existence of "non-trivial polymorphisms" - operations that combine assignments to the CSP in to new assignments. In this talk, we will expand on this emerging connection between the complexities of approximate and exact CSPs.

## 22. Jan 8, 2011. 1505-1605: Prahladh Harsha

Inapproximability Results from Label-Cover Variants and Other Hardness Assumptions.

**Abstract.** In this talk, I'll survey some of the inapproximability results that are obtained by various strengthening of the label cover problem (e.g., label cover with quasi-randomness property [Khot-Holmerin, Khot]), label cover with sub-constant error [Moshkovitz-Raz, Dinur-Harsha] and other hardness assumptions (e.g. Feige's random-3SAT assumption [Feige, Alekhnovich]).

## 23. Jan 8, 2011. 1630-1730: Madhur Tulsiani

Introduction to LP and SDP Hierarchies.

**Abstract.** I will give overview of various hierarchies which strengthen linear and semidefinite programs by adding increasingly larger local constraints. I will describe few techniques for arguing about them and survey some results proved over the past few years.

#### 24. Jan 8, 2011. 1730-1800: L. Sunil Chandran

Rainbow Colouring of Graphs.

**Abstract.** Consider an edge coloring  $c: E(G) \to N$ , (not necessarily proper) of a graph G. A rainbow path between two vertices is a path such that no two edges in the path have the same colour. The colouring c is called a rainbow (edge) colouring of G if there is a rainbow path between every pair of vertices in G with respect to c. The rainbow connection number rc(G) of a graph G, is the minimum number of colours required in a rainbow coloring of G. For example the  $rc(K_n) = 1$ , for the complete graph  $K_n$  on n vertices, rc(T) = n - 1 for a tree T on n vertices. Note that rc(G) is defined only when G is connected.

In this talk, we will discuss about two recent results regarding rainbow connection number, from our research group. Both the results leads to approximation algorithms for rainbow coloring special classes of graphs.

(1) We show that for every bridgeless graph G with radius  $r, rc(G) \le r(r+2)$ . This bound is the best possible for rc(G) as a function of r, not just for bridgeless graphs, but also for graphs of any stronger connectivity. We further show that for every bridgeless graph G with radius r and chordality (size of a largest induced cycle)  $k, rc(G) \le rk$ . The proof is constructive and leads to an (r+2)-factor approximation algorithm in the former case, and k-factor approximation algorithm in the latter case.

(2) We show that for every connected graph G, with minimum degree at least 2, the rainbow connection number is upper bounded by  $\gamma_c(G)+2$ , where  $\gamma_c(G)$  is the connected domination number of G. Bounds of the form  $diameter(G)\leqslant rc(G)\leqslant diameter(G)+c$ ,  $1\leqslant c\leqslant 4$ , for many special graph classes follow as easy corollaries from this result. This includes interval graphs, AT-free graphs, circular arc graphs, threshold graphs, and chain graphs all with minimum degree at least 2 and connected. From the proof, we can get additive c-factor approximation algorithms where c is a constant, for all the above special classes.

## 25. Jan 9, 2011. 1000-1100: Amit Kumar

Scheduling to Minimize Weighted Flow-Time.

**Abstract.** In this talk, we will survey known results for the off-line and the on-line problem of minimizing weighted flow-time on unrelated machines. In this setting, we are given a set of machines and jobs arrive over time. Each job needs to be processed by one of the machines – the running time of a job depends on which machine it is scheduled on. The goal is to minimize the total (weighted) waiting times of the jobs. We describe a natural linear programming relaxation for this problem which plays a crucial role in designing algorithms for several special cases of this problem (even in the online setting). In fact, integrality gap examples for this relaxation have also been used for proving hardness of approximation for some special cases of this problem. We shall finish by describing open problems in several special settings of this problem.

## 26. Jan 9, 2011. 1130-1230: Naveen Garg

Fast Combinatorial Algorithms for Solving Positive Linear Programs.

**Abstract.** This talk will be about multiplicative update methods for the fast solution of linear programs that arise as first steps in approximation algorithms for some NP-hard problems. I will present a unifying framework and illustrate its power by computing the Held-Karp bound for the traveling salesman problem.