Rainbow Coloring of Graphs

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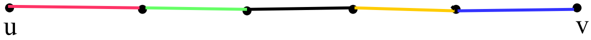
- Consider an edge coloring, not necessarily proper.
- A path between two vertices is a rainbow path if no two edges in this path have the same color.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Examples

- The complete graph K_n: $rc(K_n) = 1$.
- A path on vertices, P_n: $rc(P_n) = n - 1$.
- A Tree on n vertices T_n: $rc(T_n) = n - 1$.
- The complete bipartite graph $K_{n,n}$: $rc(K_{n,n}) = 2$.
- The cycle C_n: $rc(C_n) = \lceil n/2 \rceil$.

An obvious upper bound for $rc(G)$

- Consider a spanning tree T of G.
- Rainbow color T, and then give any used color to other edges.
- This is a rainbow coloring of G using $n-1$ colors. So, $rc(G) \leq n-1$.

The obvious lower bounds for rc(G)

- The diameter of G.
- The number of bridges in G.

Algorithmic Question

- Computing the rainbow connection number of an arbitrary graph is NP-Hard. (Chakraborty et.al. 2009)
- It is interesting to find good approximation algorithms.

This presentation is based on the following two recent papers from our group.

- Rainbow Connection Number and Connected Dominating Sets (L. S. Chandran, A. Das, Deepak Rajendraprasad, Nitin M. Vema)
- Rainbow Connection Number and Radius (M. Basavaraju, L. S. Chandran, D. Rajendraprasad, Arun Ramaswamy).

Previous Work

Most of the previous work in this area attempts to come up with a bound for $rc(G)$ in terms of n and the minimum degree δ.
A lower bound would be $3(n-2)/(\delta+1)-1$.

- Introduced by Chartrand, Johns, McKeon, Zhang, 2008.
- (Cairo et. al. 2008) If $\delta \geq 3$, then $rc(G) < 5n/6$.
- (Cairo et. al. 2008) For general δ, $rc(G) \leq c.n \log \delta/\delta$, where c is a constant.
- Schiermeyer proved for $\delta > 3$, $rc(G) \leq 3n/4$ and conjectured for general case, $rc(G)$ may be at most $3n/(\delta+1)$
- when $\delta \geq \epsilon n$, for some constant ϵ, Chakraborty showed that there exists a large constant C such that $rc(G) \leq C$.
- (Krivelevich and Yuster, 2010.) $rc(G) \leq 20n/\delta$.

Connected Dominating set and Rainbow coloring

- Can a small connected dominating set in G help to get small value for $rc(G)$? We may try the following intuitive approach.
- First get a rainbow coloring of the connected dominating set.
- Then color the remaining edges in such a way that for each vertex x outside there are two disjoint rainbow colored paths (rainbow colored using different set of colors).

The idea of Krivelevitch and Yuster

- First devides G into subgraphs G_1 and G_2.
- Finds 2-step dominating sets in G_1 and G_2.
- Make it a connected set D in G.
- From a vertex x outside D, reach D using the path in G_1 and reach back from D to x using the path in G_2.
- The total colors used is $rc(D) + 4$.

Our Improvement

- Krivelevitch has split the graph G into two graphs G_1 and G_2.
- We will show that this is not necessary.

Two-way dominating set

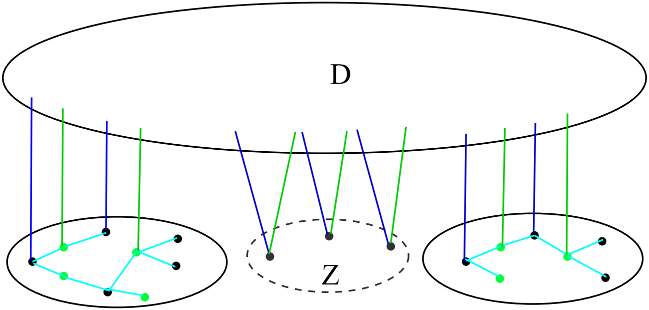
A dominating set D is called a two-way dominating set if every pendant vertex belongs to D. if $G[D]$ is connected then it is called a connected two-way dominating set.

In terms of two-way dominating set

$$rc(G) \leq rc(G[D]) + 3$$

The idea

- Let D be the two-way dominating set. Color the edges of $G[D]$ using $rc(G[D]) = k$ colors.
- We group the vertices of $V' = V - D$ into three sets X, Y, Z.
- Z is the set of isolated vertex in $G[V']$
- X, Y form a bipartition based on a spanning forest of the induced subgraph on $V' - D$.


D
Z

- $X - D$ edges are given color $k + 1$, $Y - D$ edges are given color $k + 2$. Exactly one edge from z to D is given color $k + 1$, for $z \in Z$. Edges within $G[V']$ are colored $k + 3$.

For every connected graph G, with minimum degree $\delta \geq 2$,
$rc(G) \leq \gamma_c(G) + 2$

lower and upper bounds for the some graph classes.

- Interval graphs: $diam(G) \leq rc(G) \leq diam(G) + 1$
- AT-free graphs: $diam(G) \leq rc(G) \leq diam(G) + 3$
- Circular Arc Graphs: $diam(G) \leq rc(G) \leq diam(G) + 4$.

Approximation algorithms for these graph classes

- Interval graphs: Additive 1 factor.
- AT-free graphs: Additive 3 factor.
- Circular Arc Graphs: Additive 4 factor.

two-way two-step dominating set

A two-step dominating set D is called a two-way two-step dominating set if (1) every pendant vertex belongs to D and (2) every vertex in $N^2(D)$ has at least 2 neighbors in $N^1(D)$.

Settles the conjecture of Shearmayer

- A similar but little more detailed strategy allows us settle the conjecture of Shearmayer: $rc(G) \leq 3n/(\delta + 1) + 3$

Based on Radius

- $rc(G) \leq r(r+2)$, where r is the radius.
- This is bound is also shown to be tight.

Based on radius and the largest isometric cycle.

- Let t be the length of the largest isometric cycle in G. Then $rc(G) \leq rt$
- Let k be the chordality of G. Then $t \leq k$. Thus $rc(G) \leq rk$.
- Example: For chordal graphs, $rc(G) \leq 3r$. Thus we have a 3-factor approximation algorithm for chordal graphs.
- In fact, it is known that the diameter of a chordal graph is about 2r. So, we have a 1.5 factor approximation algorithm.

Thank You

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