Team Coverage Games

Yoram Bachrach, Pushmeet Kohli, Vladimir Kolmogorov
Microsoft Research Cambridge, University College London
{yobach,pushmeet}@micorosft.com, v.kolmogorov@cs.ucl.ac.uk

ABSTRACT

Team Coverage Games (TCGs) are a representation of cooperative games, where the value a coalition generates depends on both individual contributions of its members and synergies between them. The synergies are expressed in terms of the importance of the agents in various teams. TCGs model the synergy as a reduction in utility that occurs when team members are missing, causing the team not to achieve its full potential. We focus on the case where the utility reduction incured is a concave function of the importance of the missing team members and analyze the domain from a computational game theoretic perspective.

Categories and Subject Descriptors

F.2 [**Theory of Computation**]: Analysis of Algorithms and Problem Complexity;
I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent Systems*;
J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economics*

General Terms

Algorithms, Theory, Economics

Keywords

Computational complexity, Cooperative Game Theory

1. INTRODUCTION

Game theory analyzes and provides models for many types of interaction between self-interested agents. Using such analysis to automate such interactions has immediately raised the question of computational complexity. Cooperative games consider coalitions of agents, each capable of achieving a certain utility. This utility is generated by all the coalition's agents *together*. Representation languages for cooperative games define the value generated by each coalition.

Cooperative game theory characterizes possible gain distributions through *solution concepts*, such as the core [5], the least-core and the nucleolus [7]. We suggest a representation for cooperative games called *Team Coverage Games* (TCGs), where the value a coalition generates depends both on the utility generated by each of its members, and on the coverage of various agent teams. If a team of agents is not covered by a coalition, the value generated by that coalition is reduced, as a function of the importance of the missing team members. We provide algorithms for finding the optimal coalition which generates the highest utility and for computing the core, ϵ-core and least-core of TCGs. We believe that TCGs can help model many interactions, while allowing tractably computing solutions.

Cite as: Team Coverage Games, Yoram Bachrach, Pushmeet Kohli, Vladimir Kolmogorov, *Proc. of 10th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS 2011)*, Yolum, Tumer, Stone and Sonenberg (eds.), May, 2–6, 2011, Taipei, Taiwan, pp. XXX–XXX.

1.1 The Team Coverage Game Model

We propose a model where agents operate in teams, but and only achieve their full contribution in the presence of other team members. If members of a team are missing, the agent can only contribute part of the full contribution she makes in the presence of the whole team. TCGs have n agents, $I = \{1, 2, \ldots, n\}$, each having an individual (possibly negative) contribution u_i which it supplies to the coalition.

Given a coalition C, if some agents are missing for a team t_j (so agents $T_j \setminus C \neq \emptyset$ of t_j are missing), the utility is reduced due to the degredation in that team's performance. This models the utility loss of the coalition due to breaking the well-formed teams. We model this team coverage loss by assigning each member $i \in T_j$ of the team a weight, $w_{i,j}$ indicating the agent's importance to the team t_j. If $i \notin T_j$ then $w_{i,j} = 0$. We denote the total weight of a subset $T' \subseteq T_j$ of team members as $w(T', t_j) = \sum_{i \in T'} w_{i,j}$, which indicates the total importance of the members of T' to team t_j. The reduction in utility due to missing members in team t_j is expressed as a function of the importance of missing members. Note that $T_j \setminus C$ are the missing members of team t_j in coalition C. The total importance of the missing members is $w(T_j \setminus C, t_j)$. We use a team *consistency* function $f_j : \mathbb{R} \to \mathbb{R}$ mapping the total weight (importance) of the missing members to the decrease in the coalition's utility.

The coalition's value depends on both its members' individual contributions and the coverage of teams. Coalition C's utility given the teams $t_1, \ldots, t_k$ is: $U(C) = \sum_{i \in C} u_i - \sum_{t_j \in T} f_j\left(w(T_j \setminus C, t_j)\right)$. We represent any coalition $C \subseteq I$ of agents using boolean indicator variables $x_i \in \{0, 1\}, i \in I$, one variable per agent, where $x_i = 1$ if agent $i \in C$, and $x_i = 0$ if $i \notin C$. Any vector $\mathbf{x} \in \{0, 1\}^{|I|}$ represents a coalition. The utility of any coalition $\mathbf{x}$ can be written as: $U(\mathbf{x}) = \sum_{i \in I} u_i x_i - \sum_{t_j \in T} f_j\left(\sum_{i \in I} w_{i,j}(1 - x_i)\right)$

We define *cardinal* and *threshold* TCGs. In *Cardinal TCGs (CTCG)* the value of a coalition is simply its utility. In *Threshold TCGs (TTCG)* a coalition wins if it obtains a utility higher than a threshold k, and loses otherwise. A CTCG has the characteristic function $v(C) = U(C)$. A TTCG has the characteristic function (using a fixed threshold $r \in \mathbb{R}$) where $v(C) = 1$ if $U(C) > r$ and otherwise $v(C) = 0$.

We now discus optimal coalitions and core issues in CTCGs. The problem of finding the coalition with the highest utility is CTCG-OPT-COALITION: Given a TCG G, find C^* with highest utility, i.e. a coalition C^* such that for any $C' \neq C^*$ we have $U(C') \leq U(C^*)$. Solving this problem requires finding: $\mathbf{x}^* = \arg\max_{\mathbf{x}} \sum_{i \in I} u_i x_i - \sum_{t_j \in T} f_j \left(\sum_{i \in I} w_{i,j}(1 - x_i) \right)$. We show this problem is generally hard, but tractable for submodular consistency functions.

Theorem 1 (CTCG-OPT-COALITION is NP-hard). *Finding the maximal value coalition $\mathbf{x}^*$ is NP-hard for general team consistency functions f.*

Theorem 2. *CTCG-OPT-COALITION is polynomially solvable for submodular consistency functions.*

Algorithms for minimizing submodular functions have a high complexity. Some submodular functions can be minimized efficiently by solving an ST-Min-Cut problem. In particular, certain forms relying on concave functions can be minimized [6] and Theorem 2 relies on this method.

We now turn to considering core related problems. It is known that the core is non-empty for *convex* games [8], i.e. games with supermodular functions v satisfying $\forall_C v(C) \geq 0$ and $v(\emptyset) = 0$. However, in CTCGs for some coalitions C we might have $v(C) < 0$, and specifically $v(\emptyset)$ can also be negative. We now generalize the result in [8] as follows.

Theorem 3. *If v is supermodular, $v(\emptyset) \leq 0$ and $v(C^*) = \max_C v(C) \geq 0$ then the core is non-empty.*

Theorem 3 is *constructive*: it allows constructing a core imputation from C^*, when the theorem's condition hold.

We now consider the ϵ-core. The excess of C as $d(C) = v(C) - p(C)$. The CTCG-ME problem is: Given a CTCG, an imputation $p = (p_1, \ldots, p_n)$ and $q \in \mathbb{R}$, test whether $max_{C \subseteq I} d(C) \leq q$. The TCG-$\epsilon$-CORE-MEMBERSHIP (TCG-ECM) problem is: Given a CTCG G, ϵ and an imputation $p = (p_1, \ldots, p_n)$, test whether p is in the ϵ-core. Theorem 4 shows we can solve TCG-ECM in polynomial time, using a linear program that finds a violated ϵ-core constraint.

Theorem 4. *CTCG-ME and TCG-ECM are in P.*

Another important proble is finding an impuation in the ϵ-core. The TCG-ϵ-CORE-FIND-IMPUTATION (TCG-ECFI) problem is: Given a TCG and ϵ, find an imputation $p = (p_1, \ldots, p_n)$ in the ϵ-core if one exists, or reply that no such imputation exists. We show a tractable algorithm for TCG-ECFI based on the above method for finding the maximal excess coalition, using a technique similar to the one used in [4] for weighted voting games.

Theorem 5. *TCG-ECFI is in P.*

Theorem 5 allows finding the least-core, using a binary search on the minimal ϵ making the ϵ-core non-empty.

We now provide results for the threshold version TTCG, where a coalition wins if its utility is higher than k.

Theorem 6. *In submodular TTCGs, finding vetoers and computing the core are in P.*

Theorem 7. *Any problem that is computationally hard for Weighted Voting Games is also hard for TTCGs.*

Although TTCGs appear to be similar to CTCGs, the two differ in computational complexity. In CTCGs we can compute the least-core in polynomial time, but in TTCGs even computing the maximal excess is NP-hard. Finding the maximal excess coalition is NP-complete in weighted voting games [4], so hardness follows for TTCGs through Theorem 7. Transforming a TTCG to a weighted voting game creates agents with potentially *different* individual contributions. The maximal excess problem remains hard even in domains with *identical* individual contribution, and with only *pair teams* (i.e. each team has at most two agents).

Theorem 8. *In TTCGs, finding the minimally paid winning coalition for an imputation is NP-hard, even with identical individual contribution and pair teams.*

2. CONCLUSIONS AND RELATED WORK

We proposed the *Team Coverage Games (TCG)* representation. TCGs have some similarities with other game forms, such as classes are based on skills [3, 1]. However, TCGs are a "softer" version of such games replacing "hard" constrains with a "punishment" for missing members. Another somewhat similar analysis is [2]. It studies coalitional stability, but focuses on overlapping coalitions. Several questions are open for future research. First, our analysis has focused on the core and the least-core. It would be interesting to examine other solutions. The relation between TCGs and WVGs allows translating hardness results for WVGs to TTCGs. However, CTCGs do not generalize WVGs so computational results for CTCGs must be derived some other way. Finally, we have relied on submodularity, and it would be interesting to see which results apply to more general settings.

3. REFERENCES

[1] Yoram Bachrach and Jeffrey S. Rosenschein. Coalitional skill games. In *AAMAS 2008*, pages 1023–1030, Estoril, Portugal, May 2008.

[2] G. Chalkiadakis, E. Elkind, E. Markakis, M. Polukarov, and N.R. Jennings. Stability of overlapping coalitions. *ACM SIGecom Exchanges*, 8(1):9, 2009.

[3] N.R. Devanur, M. Mihail, and V.V. Vazirani. Strategyproof cost-sharing mechanisms for set cover and facility location games. *Decision Support Systems*, 39(1):11–22, 2005.

[4] E. Elkind, L.A. Goldberg, P. Goldberg, and M. Wooldridge. Computational complexity of weighted threshold games. In *Proceedings of the 22nd national conference on Artificial intelligence-Volume 1*, pages 718–723. AAAI Press, 2007.

[5] Donald Bruce Gillies. *Some theorems on n-person games.* PhD thesis, Princeton University, 1953.

[6] P. Kohli, L. Ladicky, and P. H. S. Torr. Robust higher order potentials for enforcing label consistency. In *IJCV*, 2009.

[7] David Schmeidler. The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics*, 17(6):1163–1170, 1969.

[8] L.S. Shapley. Cores of convex games. *International Journal of Game Theory*, 1(1):11–26, 1971.