Stress Tensor Computation

The stress tensor on a primal triangle (projected to 2D) is computed as

\[
\sigma = [-e_2^\perp + e_3^\perp, e_1^\perp - e_3^\perp] \left[ e_1^\perp, e_2^\perp \right]^{-1},
\]

where \(e_i\) and \(e_i^\perp\) \((i = 1, 2, 3)\) are the directed primal and dual edge vectors shown in Fig. 1, and \(\perp\) denotes rotating a vector 90° counter-clockwise.

In the continuous setting, suppose the height field of a self-supporting surface is \(s(x,y)\) and the Airy stress function has Hessian \(\hat{M} = \begin{pmatrix} m_{22} & -m_{12} \\ -m_{12} & m_{11} \end{pmatrix}\). Then the stress tensor is given by [1]

\[
\sigma = -M \frac{g}{\det g},
\]

where \(M = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix}\) and \(g = \begin{pmatrix} 1+s_x^2 & s_x s_y \\ s_x s_y & 1+s_y^2 \end{pmatrix}\) is the induced metric on \(s(x,y)\).

References