Quantum thermodynamics
— a primer for the curious quantum mechanic

Lídia del Rio, ETH Zurich

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Why quantum thermodynamics?

- Why is thermodynamics so effective?
- Emergent theory?
- Axiomatic formulation?
Why quantum thermodynamics?

- Do quantum systems obey the laws of thermodynamics?
- Correction terms: small or quantum?
- Can we explore new effects?
Why quantum thermodynamics?

- Heat dissipation in (quantum) computers
- Microscopic heat engines
- “Thermodynamics” of relevant parameters at the nano scale?
This lecture

Information and thermodynamics

- Work cost of classical information processing
- Quantum work extraction and erasure

Axiomatic quantum thermodynamics

- Resource theory of thermal operations
- Insights and results
- Directions
Maxwell’s demon

(P, v)
Thermodynamics of information processing

- How much work must we supply to compute a function?
- Must (quantum) computers always dissipate heat?
Szilard boxes

1 bit + heat bath (T) $\Rightarrow$ work $kT \ln 2$
Szilard boxes

1 bit + heat bath ($T$) $\Leftrightarrow$ work $kT \ln 2$
Szilard boxes

1 bit + heat bath ($T$) $\iff$ work $kT \ln 2$

Landauer’s principle [1973]
Information + heat $\iff$ work

- rate: $kT \ln 2$ per bit
Maxwell’s demon
Cost of computations [Bennett 1992]

Must computers dissipate heat?

- Irreversible computation: reversible + erasure
- $\mathcal{E}(\rho_S) = \text{Tr}_{A'}(U \rho_S \otimes \sigma_A U)$
Cost of computations [Bennett 1992]

Must computers dissipate heat?

- Irreversible computation: reversible + erasure
- $\mathcal{E}(\rho_S) = \text{Tr}_{A'}(U \rho_S \otimes \sigma_A U)$
- Reversible computations: free in principle
- Work cost: cost of erasure
Work cost of erasure

\[ kT \ln 2 \text{ per bit} \]

Erasure

- Formatting a hard drive:

\[ 0?10101??1 \rightarrow 000000000 \]

- Resetting a quantum system: \( \rho_S \rightarrow |0\rangle_S \)
Work cost of erasure

\[ kT \ln 2 \text{ per bit} \]

Erasure

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In numbers

- \( k = 1.38 \times 10^{-23} \text{ J/K} \)
- Erasure of 16 TB hard drive at room temperature: 0.4 \( \mu \)J
- Lifting a tomato by 1m on Earth: 1J.
Information compression

Compression length: \( n = H(\rho) \) bits

\[
W(S) = H(S) \ kT \ln 2
\]
(Subjective) side information

\[ W(S|M) = H(S|M) \ kT \ln 2 \]
What about quantum information?

- Szilard box for quantum systems?
  - How do we even measure work?
What about quantum information?

- Szilard box for quantum systems?
  - How do we even measure work?
- How to use quantum memories?
  - Reading $\Rightarrow$ disturbing contents
What about quantum information?

- Szilard box for quantum systems?
  - How do we even measure work?
- How to use quantum memories?
  - Reading $\Rightarrow$ disturbing contents
- Entropy $H(S|M)$ can be negative!
  - but does that mean anything?
Quantum Szilard box

- Manipulating $H$: moving energy level by $\delta E$ costs $\delta E$ if state is occupied
- Thermalizing: system relaxes to $G(T) = \frac{1}{Z} e^{\frac{H}{kT}}$

Semi-classical model [Alicki et al.]
Quantum Szilard box

Quantum model [Skrzypczyk et al.]

- Free unitaries if $[U, H] = 0$
- Explicit heat bath and battery
Quantum Szilard box

Quantum model [Skrzypczyk et al.]

- Free unitaries if $[U, H] = 0$
- Explicit heat bath and battery
Erasure with quantum side information

Memory preservation

Erase the first qubit, preserving the others:

$$
|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}
$$

$$
|\psi\rangle\langle\psi|_{S_1} \otimes \rho_{2,3} \rightarrow |0\rangle\langle0|_{S} \otimes \frac{1}{2} \otimes \rho_{2,3}
$$

$$
\rho_M
$$
Erasure with quantum side information

Memory preservation
Generally: Erase $S$, preserving $M$ (and correlations)
We can still use the memory optimally:

\[ W(S|M) = H(S|M) \ kT \ln 2 \]

where \( H(S|M) = H(SM) - H(M). \) \(^1\)

\(^1\)[LdR et al. 2011]
Erasure with quantum side information

We can still use the memory optimally:

\[ W(S|M) = H(S|M) \, kT \ln 2 \]

where \( H(S|M) = H(SM) - H(M) \). \[1\]

Example

\[ |\psi\rangle = \frac{100\rangle + 111\rangle}{\sqrt{2}} \]

\[ H(S|M) = -1 \]

\[ |\psi\rangle\langle\psi|_{S_1 \otimes \rho_{2,3}} \rightarrow |0\rangle\langle0|_S \otimes \frac{11}{2} \otimes \rho_{2,3} \]

\[ \rho_M \]

\[ ^{1}[\text{LdR et al. 2011}] \]
Erasure with quantum side information

We can still use the memory optimally:

$$W(S|M) = H(S|M) \ kT \ln 2$$

where $H(S|M) = H(SM) - H(M)$. \(^1\)

Example

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\psi\rangle\langle\psi|_{S1} \rightarrow \frac{1}{2} S \otimes \frac{1}{2} I + \text{work } 2kT \ln 2$$

\(^1\)[LdR et al. 2011]
Erasure with quantum side information

We can still use the memory optimally:

\[ W(S|M) = H(S|M) \ kT \ \ln 2 \]

where \( H(S|M) = H(SM) - H(M) \). \(^1\)

Example

\[ \| \psi \| = \frac{100\langle + 11\rangle}{\sqrt{2}} \]

\[ \frac{1}{2}S \rightarrow |0\rangle\langle 0|_S \quad \text{work} \quad kT \ln 2 \]

\(^1\) [LdR et al. 2011]
Erasure with quantum side information

We can still use the memory optimally:

\[
W(S|M) = H(S|M) \ kT \ln 2
\]

where \( H(S|M) = H(SM) - H(M) \). ¹

Example

\[
|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}
\]

Total: \( W(S|M) = -kT \ln 2 = H(S|M) \ kT \ln 2 \)

¹[LdR et al. 2011]
Work cost of computations

Cost of implementing a map $\mathcal{E}$

- $\mathcal{E} : X \rightarrow X'$
- unitary dilation $X \rightarrow X' \otimes E$
- $W = H(E|X')_{\mathcal{E}(\rho)} kT \ln 2$ \[2\]

\[2\] [Faist et al. 2015]
Work cost of computations

Cost of implementing a map $\mathcal{E}$

- $\mathcal{E} : X \rightarrow X'$
- unitary dilation $X \rightarrow X' \otimes E$
- $W = H(E|X')_{\mathcal{E}(\rho)} kT \ln 2$ \(^2\)

In numbers

- AND gate: $1.6 \ kT \ln 2$
- Running a 20 Petaflops computation: $1W$

\(^2\)[Faist et al. 2015]
von Neumann entropy [1932]

Goal: erasure \( (\sum_k p_k \, |\phi_k\rangle\langle\phi_k|)^\otimes N \rightarrow |\phi_1\rangle\otimes N \)
von Neumann entropy [1932]

Goal: erasure \( \left( \sum_k p_k \left| \phi_k \right\rangle \left\langle \phi_k \right| \right) \otimes^N \rightarrow \left| \phi_1 \right\rangle \otimes^N \)
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von Neumann entropy [1932]

Goal: erasure \[ (\sum_k p_k |\phi_k\rangle\langle\phi_k|)^{\otimes N} \rightarrow |\phi_1\rangle^{\otimes N} \]

\[
V_k = p_k \quad V \quad \Rightarrow \quad W_k = N_k \ln(V_k/V) = N \quad p_k \ln p_k
\]

\[
\frac{W}{N} = \sum_k p_k \ln p_k \quad \Rightarrow \quad S(\rho) = - \text{Tr}(\rho \ln \rho)
\]
von Neumann entropy [1932]

Goal: erasure \( (\sum_k p_k |\phi_k\rangle\langle\phi_k|)^\otimes N \rightarrow |\phi_1\rangle^\otimes N \)

\[ V_k = p_k \quad V \quad \Rightarrow \quad W_k = N_k \ln(V_k/V) = N \ p_k \ln p_k \]

\[ \frac{W}{N} = \sum_k p_k \ln p_k \quad \Rightarrow \quad S(\rho) = - \text{Tr}(\rho \ln \rho) \]
Why is thermodynamics so effective?

- microscopic details
- Identifies:
  - easy and hard operations
  - freely available resources
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- Efficient exploitation:
  - steam engines, fridges
  - cost of state transformations
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- Operational approach: resource theory
Why is thermodynamics so effective?

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- Efficient exploitation:
  - steam engines, fridges
  - cost of state transformations
- Operational approach: resource theory (just like LOCC)
Resource theories

Operational questions

- Can one achieve $X \rightarrow Y$?
- Monotones: characterizing pre-order
- Useful and useless resources?
Resource theories

Example: LOCC

- **Allowed operations**: local operations and classical communication
- **Monotones**: formation and distillation entanglement, squashed entanglement, . . .
- **Free resources**: separable states. **Currency**: Bell states
Thermodynamics as a resource theory

- Limitations:
  - lack of knowledge: \((N, V, T), (N, V, E), \ldots\)
  - conservation laws: energy, momentum, \ldots
  - limited control of operations
Thermodynamics as a resource theory

- Limitations:
  - lack of knowledge: \((N, V, T), (N, V, E), \ldots\)
  - conservation laws: energy, momentum, \ldots
  - limited control of operations

- Resources: macroscopic descriptions of systems (hot gas, cold bodies)

- Operations: adiabatic, isothermal, \ldots

- Insights: laws of thermodynamics, free energy as a monotone, Carnot efficiency, \ldots

\footnote{[Carathéodory 1909] [Giles 1964] [Lieb and Yngvason 1998, 1999, 2003]}
Thermal operations

- **Resources**: quantum descriptions of systems \((\rho_S, H_S)\)
Thermal operations

- **Resources:** quantum descriptions of systems $(\rho_S, H_S)$
- **Account for entropy:** $U \rho_S U^\dagger$

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4 [Janzing 200] [Brandao et al 2011]
Thermal operations

- Resources: quantum descriptions of systems \((\rho_S, H_S)\)
- Account for entropy: \(U\rho_S U^\dagger\)
- Account for energy: \(U\rho_S U^\dagger, \quad [U, H_S] = 0\)

\(^4\text{[Janzing 200] [Brandao et al 2011]}\)
Thermal operations

- Resources: quantum descriptions of systems \((\rho_S, H_S)\)
- Account for entropy: \(U\rho_SU^\dagger\)
- Account for energy: \(U\rho_SU^\dagger, \quad [U, H_S] = 0\)
- Free environment: \(U(\rho_S \otimes \text{Gibbs}(T))U^\dagger, \quad [U, H_{SB}] = 0\)

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Thermal operations

- **Resources:** quantum descriptions of systems \((\rho_S, H_S)\)
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- **Free environment:** \(U(\rho_S \otimes \text{Gibbs}(T))U^\dagger, \quad [U, H_{SB}] = 0\)
- **Free forgetting:** \(\text{Tr}_A[U(\rho_S \otimes \text{Gibbs}(T))U^\dagger], \quad [U, H_{SB}] = 0\)

\[\text{[Janzing 200]} \quad \text{[Brandao et al 2011]}\]
Thermal operations

- Resources: quantum descriptions of systems \((\rho_S, H_S)\)
- Account for entropy: \(U\rho_S U^\dagger\)
- Account for energy: \(U\rho_S U^\dagger, [U, H_S] = 0\)
- Free environment: \(U(\rho_S \otimes \text{Gibbs}(T))U^\dagger, [U, H_{SB}] = 0\)
- Free forgetting: \(\text{Tr}_A[U(\rho_S \otimes \text{Gibbs}(T))U^\dagger], [U, H_{SB}] = 0\)
- Toy model \(^4\)

\(^4\) [Janzing 200] [Brandao et al 2011]
Why thermal states?

\[ G(T) = \frac{1}{Z} e^{-\frac{H}{kT}} \]

Landauer's principle
Why thermal states?

\[ G(T) = \frac{1}{Z} e^{-\frac{H}{kT}} \]

Noise models
Why thermal states?

\[ G(T) = \frac{1}{Z} e^{-\frac{H}{kT}} \]

Reduced description

- Large system composed of independent parts: \( H = H_S + H_E \)
- Energy shell \( \Omega_E \) of fixed energy
- Global state of maximal entropy: \( \mathbb{1}_{\Omega_E} / d_{\Omega_E} \)
- \( \rho_S = G_S(T(E)) \)
Why thermal states?

\[ G(T) = \frac{1}{Z} e^{-\frac{H}{kT}} \]

Typicality of thermalization\(^5\)

- \( d_S \ll d_\Omega \)
- **Static thermalization**: for most global states and most subsystems,
  \[ \rho_S \approx \text{Tr}_E(\mathbb{1}_\Omega / d_\Omega) = G_S(T) \]
- **Decoupling** (also with side quantum information).
- Also if \( S \) corresponds to observable.

\(^5\)Review: [Gogolin & Eisert 2016]
Why thermal states?

\[ G(T) = \frac{1}{Z} e^{-\frac{H}{kT}} \]

Typicality of thermalization\(^5\)

\(^5\)Review: [Gogolin & Eisert 2016]
Why thermal states?

\[ G(T) = \frac{1}{Z} e^{-\frac{H}{kT}} \]

Typicality of thermalization\(^5\)

- **Evolution towards thermal state:** if \( H \) is rich, \( \rho_S(t) \approx G_S(T) \) for most times \( t \) and initial states.
- **Time scales:** under study

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\(^5\)Review: [Gogolin & Eisert 2016]
Why thermal states?

\[ G(T) = \frac{1}{Z} e^{-\frac{H}{kT}} \]

Complete passivity

Intuition: only free state that does not trivialize the resource theory

- Allowed operations: unitaries
- Allowed many copies of a state
- Cannot extract energy \( \implies G(T)^\otimes n \)
Why thermal states?

\[ G(T) = \frac{1}{Z} e^{-\frac{H}{kT}} \]

Complete passivity

Intuition: only free state that does not trivialize the resource theory

- Allowed operations: unitaries
- Allowed many copies of a state
- Cannot extract energy \( \implies G(T)^{\otimes n} \)

Still a spherical cow...
Insights: noisy operations

Case $H = 0$

- Pre-order: majorization\(^6\) $\rho \rightarrow \sigma \iff \rho \prec \sigma$,

$$\rho \prec \sigma \iff \sum_{i=1}^{k} \lambda_i(\rho) \geq \sum_{i=1}^{k} \lambda_i(\sigma)$$

\(^6\) Review: [Gour et al (2013)]
Insights: noisy operations

Case $H = 0$

- Pre-order: majorization\(^6\) $\rho \rightarrow \sigma \iff \rho \prec \sigma$,

\[
\rho \prec \sigma \iff \sum_{i=1}^{k} \lambda_i(\rho) \geq \sum_{i=1}^{k} \lambda_i(\sigma)
\]

- Monotones: Schur-convex functions, e.g.

\[
D^\alpha(\rho_S \| \mathbb{1}_S/d_S),
\]

entropies $H(\rho), H_\alpha(\rho), \ldots$

- Classically: $D^\alpha(\rho \| \sigma) = \frac{\text{sgn} \alpha}{\alpha - 1} \log \sum_i p_i^\alpha q_i^{1-\alpha}$

\(^6\text{Review: [Gour et al (2013)]}\)
Lorenz curves

1. Sort eigenvalues of $\rho$: $p_1 \geq p_2 \geq \cdots \geq p_d$

2. Build step function: $f_\rho(x) = p_i(\rho)$ for $i - 1 \leq x < i$
Lorenz curves

1. Integrate to get Lorenz curve:

\[ g_\rho(x) = \int_0^d x f_\rho(x') \, dx' \]

2. Pre-order: \( \rho \rightarrow \sigma \iff g_\rho(x) \geq g_\sigma(x), \forall x \in [0, 1] \)
Insights: thermal operations

General Hamiltonian

- Pre-order: thermo-majorization (for block-diagonal states!)
- Monotones: e.g. relative entropy to thermal state

\[ D^\alpha(\rho \| G(T)), \]

free energies...

- Rescaled Lorenz curves\(^7\)

\(^7\)[Renes] [Horodecki & Oppenheim]
Rescaled Lorenz curves
For block-diagonal states,

1. Rescale eigenvalues: \( r_i = p_i \ e^{\beta E_i} \) and sort them.
2. Build step function:
   \[
   f_\rho(x) = r_i \quad \text{for} \quad \sum_{k<i} e^{-\beta E_k} \leq x < \sum_{k\leq i} e^{-\beta E_k}
   \]
3. Integrate to get Lorenz curve:
   \[
   g_\rho(x) = \int_0^x f_\rho(x') \ dx'
   \]
4. Pre-order: \( \rho \rightarrow \sigma \iff g_\rho(x) \geq g_\sigma(x), \ \forall \ x \in [0,1) \)
Recovering the second law

Free energies as monotones

\[ \rho \rightarrow \sigma \implies F^\alpha(\rho, T) \geq F^\alpha(\sigma, T), \forall \alpha, \text{ where} \]

\[ F^\alpha(\rho, T) = kT \left[ D^\alpha(\rho \| G(T)) - \log Z \right] \]

\[^8[Brandao \, et \, al \, 2014] \]
Recovering the second law

Free energies as monotones

$\rho \rightarrow \sigma \implies F^\alpha(\rho, T) \geq F^\alpha(\sigma, T), \forall \alpha$, where$^8$

$$F^\alpha(\rho, T) = kT \left[D^\alpha(\rho||G(T)) - \log Z\right]$$

$\Leftarrow$ for classical states

$^8\text{[Brandao et al 2014]}$
Recovering the second law

Free energies as monotones

\[ \rho \rightarrow \sigma \implies F^\alpha(\rho, T) \geq F^\alpha(\sigma, T), \quad \forall \alpha, \] where

\[ F^\alpha(\rho, T) = kT \left[ D^\alpha(\rho || G(T)) - \log Z \right] \]

\[ \iff \text{for classical states} \]

\[ \text{In particular} \]

\[ F^1(\rho, T) = \text{Tr}(H \rho) - kTS(\rho). \]

\[ ^8 \text{[Brandao et al 2014]} \]
Recovering the second law

Free energies as monotones

\[ \rho \to \sigma \implies F^\alpha(\rho, T) \geq F^\alpha(\sigma, T), \ \forall \alpha, \ \text{where}^8 \]

\[ F^\alpha(\rho, T) = kT \left[ D^\alpha(\rho \| G(T)) - \log Z \right] \]

\[ \iff \text{for classical states} \]

\[ \Downarrow \text{In particular} \]

\[ F^1(\rho, T) = \text{Tr}(H \rho) - kT S(\rho). \]

\[ \Downarrow \text{Free energies: rescaling of } D^\alpha(\rho \| G(T)) \text{ such that} \]

\[ F^1(\mid E \rangle \langle E \mid, T) = E \]

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\(^8[\text{Brandao et al 2014}]\)
Recovering thermodynamics

Third law
Cannot cool to ground state with finite resources.\(^9\)

\[^9\text{Masanes & Openheim 2014} \text{[Janzing] [Wilming (in prep.)]}\]
\[^10\text{LdR et al 2011}\]
\[^11\text{Reeb & Wolf 2013} \text{[Woods et al 2015]}\]
Recovering thermodynamics

**Third law**
Cannot cool to ground state with finite resources.\(^9\)

**Landauer’s principle**
- Work cost of erasing \(S\) in the presence of \(M\) costs \(^{10}\)
  \[
  W \approx kT \ H(S|M)_\rho
  \]
- Single-shot: \(H^\varepsilon\), finite-size effects\(^{11}\) limit efficiency

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\(^{9}\)[Masanes & Openheim 2014] [Janzing] [Wilming (in prep.)]

\(^{10}\)[LdR et al 2011]

\(^{11}\)[Reeb & Wolf 2013] [Woods et al 2015]
Recovering thermodynamics

Fluctuation theorems

- Crooks’ and Jarzinsky’s relations: prob. violation exponentially suppressed
- Beyond two-measurement setting for coherent processes,\textsuperscript{12} e.g. $|+\rangle \rightarrow |0\rangle$

\textsuperscript{12}[Elouard et al 2015] [Åberg 2016] [Perarnau-Llobet, et al (2016)]
Multiple conserved quantities

- Multiple$^{13}$ conserved quantities $A_1, A_2, \ldots$
- Allowed operations: $U : [U, A_i] = 0$, $\forall i$

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$^{13}$ [Vaccaro and Barnett 2011] [Lostaglio et al 2015] [Guryanova et al 2015] [Yunger Halpern et al 2015] [Perarnau-Llobet et al 2015]
Multiple conserved quantities

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- Generalized Gibbs state

\[ G(\beta_1, \beta_2, \ldots) = e^{\beta_1 A_1 + \beta_2 A_2 + \ldots} \]

- Typical thermalization and passivity results hold

\textsuperscript{13} [Vaccaro and Barnett 2011] [Lostaglio et al 2015] [Guryanova et al 2015] [Yunger Halpern et al 2015] [Perarnau-Llobet et al 2015]
Multiple conserved quantities

- Multiple\textsuperscript{13} conserved quantities $A_1, A_2, \ldots$
- Allowed operations: $U : [U, A_i] = 0, \forall i$
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  \[ G(\beta_1, \beta_2, \ldots) = e^{\beta_1 A_1 + \beta_2 A_2 + \ldots} \]
- Typical thermalization and passivity results hold
- Monotones: $D^\alpha(\rho||G(\beta_1, \beta_2, \ldots))$
- First protocols for conversion between $A_1, A_2, \ldots$

\textsuperscript{13} [Vaccaro and Barnett 2011] [Lostaglio et al 2015] [Guryanova et al 2015] [Yunger Halpern et al 2015] [Perarnau-Llobet et al 2015]
Coherence: quantum-quantum thermodynamics?

Needed to implement unitaries (laser).

\[ \Delta \rho = \text{Tr}(1_2 (\Delta + \Delta^\dagger) \rho) \]
\[ \Delta = \sum_n |n+1\rangle \langle n| \]
\[ M : \text{lowest occupied energy level} \]

Unbounded coherence reservoir:

Back-action of implementing operations: stretching of \( \rho \)

No degradation of \( \Delta \)

Can always pump up \( M \) with energy

More realistic reservoirs:

Protocol for work extraction from coherent states

Operationally restoring reservoir: \( (\Delta \rho, M \rho) = (\Delta \rho', M \rho') \)

\[ \text{[Åberg 2014]} \]
\[ \text{[Korzekwa et al 2016]} \]
Coherence: quantum-quantum thermodynamics?

Needed to implement unitaries (laser).

Catalytic coherence?

- Relevant properties of coherence reservoir $\rho$: $(\Delta_{\rho}, M_{\rho})$
  - $\Delta_{\rho} = \text{Tr}(\frac{1}{2}(\Delta + \Delta^\dagger)\rho)$ coherence: $\Delta = \sum_n |n + 1\rangle\langle n|$
  - $M$: lowest occupied energy level

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14[Åberg 2014]
15[Korzekwa et al 2016]
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- Unbounded coherence reservoir: $^{14}$
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$^{14}$[Åberg 2014]
$^{15}$[Korzekwa et al 2016]
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- Unbounded coherence reservoir:\(^{14}\)
  - back-action of implementing operations: stretching of $\rho$
  - no degradation of $\Delta$
  - can always pump up $M$ with energy

- More realistic reservoirs:\(^{15}\)
  - protocol for work extraction from coherent states
  - operationally restoring reservoir: $(\Delta_\rho, M_\rho) = (\Delta_\rho', M_\rho')$

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\(^{14}\)[Åberg 2014]

\(^{15}\)[Korzekwa et al 2016]
Clocks and control

\[ [U, H_0] = 0, \quad U = e^{-i t H_U} \]

- \( H_U(t) \Rightarrow \text{control} \)
- Effort of building and keeping control systems
- Clocks and controls are out of equilibrium systems
- Fairer book-keeping: give agents little control, explicit clocks

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17[Wilming et al 2014 ]
18[Woods et al 2016 ]
Clocks and control

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First steps

- Ideal clock (particle in a line) \( \implies \) catalytic, perfect \( U \)
- Thermal contact\(^\text{17}\)
- Dimension bounds and clock degradation\(^\text{18}\)

\(^{16}\) [Brandao et al (2011)] [Malabarba et al 2014]
\(^{17}\) [Wilming et al 2014 ]
\(^{18}\) [Woods et al 2016 ]
Autonomous thermal engines

Carnot efficiency\textsuperscript{19} (fine-tuned gaps)

\textsuperscript{19}[Skrzypczyk et al (various)]
Open questions

Clocks and control

- Designs for efficient clocks (theory and experiment)
- Combine with insights from reference frames
- Further restrictions (no fine-tuning of baths, Hamiltonians)
- Clean framework (how much control to give the agent?)
- Relation to coherence (again!)
- Relation to time in foundations
Open questions

Realistic resource theories

- Operational notions of temperature: baths beyond Gibbs
- Finite-size effects
- Beyond weak coupling
- Realistic resource descriptions for experimentalists
- Towards operational resource theories

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20 [Farshi et al (in prep)]
23 [Yunger Halpern (2015)]
Open questions

Generalized probability theories

- GPTs: apply von Neumann’s operational approach to entropy
- Relate thermodynamics on different physical theories

AdS/CFT

- Notions of thermalization
- Black hole entropy & information paradox

\[\text{[Barnum et al 2015]}\]
Thank you for your attention!

Reviews

Thermo QIP 2017

Talks

- Monday 3pm Carlo Sparaciari
- Wednesday 9am Jonathan Oppenheim
- Friday 2pm Michael Kastoryano, then Kohtaro Kato

Posters

Monday

- 35 A sufficient set of gates for thermodynamics
- 36 Fundamental energy cost for quantum measurement
- 38 Thermal Operations under Partial Information - An operational derivation of Jaynes Principle
- 39 Thermalization and Return to Equilibrium on Finite Quantum Lattice Systems

Tuesday

- 52 Autonomous quantum machines and finite sized clocks