5. *Computational Line Geometry* by Pottmann and Wallner.

Books on Cryptography

1. *Data Privacy and Security* by David Salomon.
2. *Block Error-Correcting Codes: A Computational Primer* by Xambo-Descamps.

Misc Books

1. *Verification of Reactive Systems: Formal Methods and Algorithms* by Schneider.
2. *Information Theory, Inference, and Learning Algorithms* by MacKay.
3. *Finite Automata* by Lawson.
4. *Logic for Learning* by Lloyd.
5. *Combinatorial Designs: Constructions and Analysis* by Stinson.
6. *Selected Papers on Discrete Math* by Donald Knuth.

Review[2] of

The Classical Decision Problem
Authors: Egon Börger, Erich Grädel and Yuri Gurevich

Series: Universitext
Springer-Verlag, 1997
Second printing, 2001
Softcover, x + 482 pages
Reviewer: Dan A. Simovici, (dsim@cs.umb.edu)

1 Overview

This book is dedicated to a comprehensive presentation of the classical decision problem of first-order logic. The centrality of the original decision problem (which can be stated equivalently as the satisfiability problem for formulas, the validity problem for sentences, or the provability of formulas in a sound and complete formal system) has been identified by the founders of mathematical logic. Subsequent developments in mathematical logic and theoretical computer science, especially the formalization of the notion of computable function by Gödel, Kleene, and Herbrand, yielded a negative answer to the classical decision problem and focused the attention of researchers towards the identification of fragments of first-order logic that are decidable or undecidable.

The introduction to the book contains a historical perspective of this research effort that is worth reading first and re-reading often while working through the book. The notion of reduction class that is a mainstay of the treatment of decidability is also introduced in this initial chapter. Löwenheim's decidability result for formulas with unary predicates and his identification of the class of formulas with binary predicates as a reduction class are recalled as well as the work of Herbrand, Skolem, and Bernays.

Traditional fragments of first-order logic (considered as classes of formulas in prenex normal form that are defined by restrictions on the quantifier prefix or alphabet) are considered in detail for their decidability or undecidability; ample historical references are provided.

2 Summary of Contents

The book is divided into two parts, “Undecidable Classes” and “Decidable Classes and Their Complexity”, comprising eight chapters. These parts are followed by an Appendix “Tiling Problems”.

Part I: Undecidable Classes

contains chapters entitled “Reductions”, “Undecidable Standard Classes for Pure Predicate Logic”, “Undecidable Standard Classes with Functions and Equality”, and “Other Undecidable Classes”.

Chapter 2: Reductions introduces the central notion of reduction class for satisfiability as a class of formulas X such that there exists a computable function f that maps every satisfiable formula ϕ into a formula $f(\phi) \in X$.

The first section entitled “Undecidability and Conservative Reductions” begins with the the Church-Turing theorem on the undecidability of the classical decision problem. The argument is classical, presented in a pithy manner and consists in reducing the halting problem for deterministic Turing machines to the Entscheidungsproblem. The presentation is sufficiently general to allow specializations of the argument for several types of logics.

The same section deals with Skolem normal form theorem, Herbrand structures, Horn and Krom formulae (the latter defined as first-order formulas in prenex normal form whose quantifier-free part is a conjunction of clauses that contain at most two constituents). The Aanderaa-Börger reduction result for 2-register machines is presented here.

Trahtenbrot’s result on the recursive inseparability of the sets of all finitely satisfiable, all unsatisfiable and all only infinitely-satisfiable formulas is presented in detail. Conservative reductions and conservative reduction classes are introduced as instruments that sometime simplify reduction proofs. This section concludes with a discussion of the relationship between inseparability and model complexity

The second section of this chapter, “Logic and Complexity” discusses the use of the logical specification of computations for the study of time-restricted or space-restricted hating problems in syntactically defined classes of formulas with solvable decision problems.

Cook-Levin Theorem on the NP-completeness of satisfiability for formulas of propositional logic is presented here, as well as the PSPACE-completeness of the decision problem for quantified propositional logic.

The notions of spectrum of a formula and generalized spectrum (as the class of a sentence in existential second-order logic) lead to a presentation of on Fagin’s Theorem on generalized spectra.

Descriptive complexity theory is discussed starting from the the notion of a logic capturing a complexity class on a class of finite structures. Model-theoretic characterizations of P and NLOGSPACE on ordered structures are given. The final section of this chapter contains Gurevich’s classifiability theorem that is an organizing principle of this area. Using the fact that the classes of prenex formulas form a well-order poset, where the collection of decidable classes is closed downward, it follows that there exist a finite number of minimal undecidable classes which dominates all undecidable classes, and there is a finite number of maximal decidable classes. Specifically, there are sixteen minimal undecidable classes and seven maximal decidable ones.

Chapter 3: Undecidable Standard Classes for Pure Predicate Logic This chapter is the part I enjoyed most. Building on Gurevich Classifiability Theorem, the main result of the chapter shows that a prefix-class of formulas without function symbols or equality is undecidable if it contains one of nine special classes of formulas. An inspired diagram placed in the introduction of the chapter summarizes various reductions between the classes involved and helps the reader get a clear picture of the aims of this chapter.

The first section entitled "The Kahr Class" ($[\forall\exists\forall, (\omega, 1)]$ in the notation of the authors) begins with the undecidability of domino problems introduced by Wang; actually, a stronger result on the recursive inseparability of the sets of domino problems that admit no tiling and those who admit a periodic tiling of $Z \times Z$ or $N \times N$. A new proof of this result is contained in a special appendix. The domino problem is used to prove that the Kahr class is a conservative reduction class. In turn, Kahr class plays a crucial role in proving the conservative reduction property of the other classes of interests. The rest of the chapter proves the conservative reduction properties for other classes of formulas.

Chapter 4: Undecidable Standard Classes with Functions or Equality contain results that establish the minimal undecidable prefix vocabulary classes of the full first-order logic, that is, formulas with relation symbols, function symbols, and equality. Supplementing the list of reduction classes of pure predicate logic, the list of classes established in this chapter (together with the decidability results established in chapters 6 and 7) gives a complete classification of prefix classes in the full first-order logic with respect to the decidability of the satisfiability.

Chapter 5: Other Undecidable Classes examines classes that are defined not only by the alphabet or prefix structure but also by the syntax of the quantifier-free part.

The undecidability of several prefix classes of Krom formulae without functions or equality is established (which is preserved even when restricted to Horn formulae) and conservative reduction classes are identified among the Krom and Horn formulae with function symbols or equality. Undecidability results are obtained for classes of formulas that have only a small number of atomic subformulae (important for logic programming). An example of the type of results presented here is the proof of Wirsing's result that the class of universal formulas containing only one equality and one inequality is a conservative reduction class. Other classes discussed in this chapter include formulas with a limited number of variables and conjunctions of prefix-vocabulary classes.

Part II: Decidable Classes and Their Complexity

Chapter 6: Standard Classes with Finite Model Property deals with a description of the standard classes of formulas for which the satisfiability and the finite satisfiability problems are decidable. The main result is a complete description of the seven maximal decidable classes. The decidability of five of these classes (which have the finite model property) is discussed in this chapter.

After introducing some novel techniques for proving complexity results (e.g. a new tiling problem on a torus) the decidability of several classical classes is shown. There classes were shown decidable before the undecidability of the general problem was shown by Church and Turing. The chapter concludes with a description of standard classes of "modest complexity", that is of prefix classes whose decision problems are in P, NP, or co-NP.

Chapter 7: Monadic Theories and Decidable Standard Classes with Infinity Axioms discusses the decidability of the remaining two maximal decidable classes (that is, of first-order logic with equality, one unary function symbol and monadic predicates, and the Shelah class that consists of formulas with at most one universal quantifier, at most one function symbol and arbitrary relation symbols, with equality). The proof of the decidability of the first class, due to one of the authors, is a simplification of Rabin's argument. It uses Games and Tree Automata. The argument for the Shelah class is an expansion of the original paper where this class was introduced.

Chapter 8: Other Decidable Classes is reserved for several special decidable classes that are important to computer scientists working in databases, logic programming, etc. An improvement of Mortimer's result for formulas with two variables is given.

In a section dedicated to unification such result as the log-space completeness for P of the unification problem, the P-completeness of the satisfiability for Herbrand formulas or the NP-completeness of the validity for the positive fragment of first-order logic are shown. The final section of this chapter deals with decidable classes of Krom formulas; various decidability results (e.g., the decidability of the Aanderaa-Lewis class) and techniques are presented.

3 Opinion

This book is an essential reference for any researcher in logic, complexity, and artificial intelligence.

The annotated bibliography of 549 titles is an enormous help for every researcher interested in decidability; it contains in a very concentrated form an enormous survey of the literature on the classical decision problem and enhances the role of the books as a reference source. Historical references that are placed at the end of each chapter are very enjoyable and help the reader follow the literature and gain a perspective of the field.

Occasional minor lapses, such as the inversion of the direction of reduction on page in the second paragraph on p. 29 can be easily spotted and do not diminish the value of this book.

In the preface the authors stress the efforts made to "combine the features of a research monograph and a textbook". It is my opinion that they succeeded in producing an excellent reference book for researchers in the field, and for advanced doctoral students in theoretical computer science and logic. However, I have my reservations on its use as a textbook. The material requires a lot more mathematical sophistication than the "basic knowledge of the language of first-order logic", as the authors claim. The presentation is very terse, with complex details left to a multitude of exercises (some very challenging) sprinkled throughout the text. In fairness to the authors, it seems to me that in view of the vast material included in this book, no other approach was realistic.