Trajectory estimation problems in vision

– temporal tracking
  • deterministic: Kalman Filter
  • stochastic: Particle Filter

– curve tracing
  • deterministic: LiveWire (Mortensen&Barrett, 95)
  • stochastic: JetStream (Perez et al. 01)

– stereo vision
  • deterministic: Dynamic Programming
  • stochastic: ??

Inferring the cyclopean image
A forward-backward algorithm for stereo matching

• Stereo vision
  – seeing in depth
  – cyclopean vision (cf. double vision)

• Applications to
  – teleconferencing
  – virtual reality

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Virtual reality

(Anandan, Criminisi, Kang, Szeliski, Uyttendaele, Microsoft Research)

Teleconferencing: failure of eye contact

camera

screen

camera
Problem: estimate cyclopean intensity

Parallax: \( L(x+d) = R(x) \)

Given: \( z = (L, R) \), dynamic programming (Baker and Binford, 1981; Ohta and Kanade, 1985; Cox et al., 1996; Belhumeur, 1996) gives:

Estimate parallax: \( \hat{d} = \arg \max_d p(d \mid z) \),

Cyclopean image: \( \hat{I} = I(z, \hat{d}) \).

?? Occlusion
?? Prior on \( d \) i) Epipolarity ii) Ordering

Epipolar geometry
(tutorial by Andrea Fusiello)
**Stereo disparity on epipolar lines**

Intensity on left epipolar line:
\[ L = \{L_m, \ m = 0, \ldots, N\} \]

Intensity on left epipolar line:
\[ R = \{R_n, \ n = 0, \ldots, N\} \]

Matched pair
\[ (L_m, R_n) \]

has "disparity"
\[ d = n - m \]

— measure of parallax.

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**Epipolar matching as optimal path finding**

(Ohta & Kanade, 1985; Cox, Hingorani & Rao, 1996)

Mapping: \( h_s = (m(s), n(s)) \)

Find \( h_s \), \( S \) minimising
\[ C = \sum_{s=0}^{S} C(h_s) \quad \text{(Dijkstra)} \]

where
\[ C(h_s) = C(h_{s-1}) + C_d(h_s, h_{s-1}) + C_o(L_m(s), R_n(s)) \]
Ordering constraint

Nail illusion

Gaze correction: virtual cyclopean camera

Dijkstra solution (10 fps)
Robust virtual cyclopean camera

- Family of paths
- Dijkstra framework won’t do it
- Viterbi framework could work.

JetStream: particle filter

(Perez et al.)
Stereo disparity in cyclopean coordinates

Intensity on left epipolar line:
\[ L = \{L_m, \ m = 0, \ldots, N\} \]

Intensity on left epipolar line:
\[ R = \{R_n, \ n = 0, \ldots, N\} \]

Cyclopean epipolar line is
\[ I = \{I_k, \ k = 0, \ldots, 2N\} \]

Stereo correspondence — warp:
\[ h_k = (m(k), n(k)) = \left( \frac{1}{2}(k - d_k), \frac{1}{2}(k + d_k) \right) \]

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Cyclopean Coordinates

Warp: \[ h_k = (m(k), n(k)) = \left( \frac{1}{2}(k - d_k), \frac{1}{2}(k + d_k) \right) \]
**Posterior distribution for cyclopean image**

**Know:** $p(z \mid I, d)$

**Want:** $p(I \mid z)$

1. Marginalise over $I$ to get $p(z \mid d)$

$\star$ 2. Bayes: $P(d \mid z) \propto p(z \mid d)P(d)$

3. Marginalise over $d$:

$$p(I \mid z) \propto p_0(I) \sum_d p(z \mid I, d)P(d \mid z)$$

4. Choose $\hat{I}$ to minimize $\mathcal{E}_{Iz} \mathcal{L}(I, \hat{I})$ -- robust estimate

$\star$ Obtain marginals for $d$

--- check decomposition of $p(z \mid I, d)$ and $P(d)$

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**Observation model**

Observation density:

$$p(z \mid I, d) = \prod_{k \in \mathcal{K}} p(L_{m(k)}, R_{n(k)} \mid I_k) \prod_{m \notin \mathcal{M}} p_0(L_m) \prod_{n \notin \mathcal{N}} p_0(R_n)$$

**Generative form** (matched)

$L_m = I_k + \mu_m$ (uniform)

$R_n = I_k + \nu_n$ (indep. Gaussians)

**Marginalise** (step 1)

$$p(z \mid d) \propto \prod_{k \in \mathcal{K}} \frac{1}{p_0^k} \mathcal{N} \left( L_{m(k)} - R_{n(k)}; 0, \Sigma_- \right)$$ (improper Gaussian)

Check decomposition of $p(z \mid I, d)$ (for step 3)
Disparity prior

Decomposition (for step 2)

\[ P(d) = P(d_0) \prod_k P(d_k \mid d_{k-1}) \]

For example:

\[
P_d(d_k \mid d_{k-1}) = \begin{cases} 
1 & \text{if } h_{k-1} \text{ is odd and } d_k = d_{k-1} \\
q & \text{if } h_{k-1} \text{ is even and } d_k = d_{k-1} \pm 1 \\
1 - 2q & \text{if } h_{k-1} \text{ is even and } d_k = d_{k-1} \\
0 & \text{otherwise}
\end{cases}
\]
Conventional forward algorithm?

Time-stamped observations

\[ z = (z_1, \ldots, z_k, \ldots) \]

Forward probabilities

\[ \alpha_k(d_k) \equiv P(d_k, z_1, \ldots z_k) \]

Forward algorithm

\[ \alpha_k(d_k) \propto \sum_{d_{k-1}} \alpha_{k-1}(d_{k-1}) p(d_k | d_{k-1}) p_z(z_k | d_k) \]

But observations are not time-stamped – two streams:

\[ z = (L_1, \ldots, L_M, R_1, \ldots, R_N) \]

(observation history?)

History for stereo observations

‘Past’ depends not only on time \( k \) but also on value \( d_k \)
Forward and backward probabilities

Define past observations:
\[ z_{h_k} \equiv \left( L_1, \ldots, L_{m(k)}, R_1, \ldots, R_{n(k)} \right) \quad \text{(even } h_k \text{)} \]
\[ z_{h_k} \equiv \left( L_1, \ldots, L_{m(k)-1/2}, R_1, \ldots, R_{n(k)-1/2} \right) \quad \text{(odd } h_k \text{)} \]
and future observations: \[ z^{h_k} \equiv z \backslash z_{h_k} \]

Forward probabilities: \[ \alpha_k(d_k) \equiv p(d_k, z_{h_k}) \]
Backward probabilities: \[ \beta_k(d_k) \equiv p(z^{h_k} \mid d_k) \]

Forward step

Recall observation density
\[ p(z \mid d) \propto \prod_{k \in K} \frac{1}{\sqrt{2\pi \Sigma_k}} \exp \left( -\frac{1}{2} (L_{m(k)} - R_{n(k)})^2 \right) \]
\[ f(d_k, d_{k-1}) = \frac{l}{1} \quad \text{for } k \in K \]
\[ \text{otherwise} \]
Forward step
\[ \alpha_k(d_k) \propto \sum_{d_{k-1}} \alpha_{k-1}(d_{k-1}) p(d_k \mid d_{k-1}) f(d_k, d_{k-1}) \]
...Forward step

\[
\alpha_k(d_k) \propto \sum_{d_{k-1}} \alpha_{k-1}(d_{k-1}) p(d_k|d_{k-1}) f(z; d_k, d_{k-1})
\]

... Posterior distribution for cyclopean image

1. Marginalise over I to get \( p(z \mid d) \)
2. Bayes: \( P(d \mid z) \propto p(z \mid d) P(d) \)
3. Marginalise over \( d \):

\[
p(I \mid z) \propto p_0(I) \sum_d p(z \mid I, d) P(d \mid z)
\]

4. Choose \( \hat{I} \) to minimize \( \mathcal{E}_{1\|k} \mathcal{L}(I, \hat{I}) \)

- Assume pointwise loss, so require only

\[
p(I_k \mid z) \quad \text{for each} \quad i \in k
\]
... Posterior distribution for cyclopean image

\[ p(I \mid z) \propto p_0(I) \sum_d p(z \mid I, d) P(d \mid z) \]

\[ \prod_{k \in K} \frac{1}{2} \ p(L_{m(k)}, R_{n(k)} \mid I_k) \]

\[ g(d_k, d_{k-1}, I_k ; z) \]

so \[ p(I_k \mid z) \propto p_0(I_k) \sum_{d_k, d_{k-1}} g(\ldots) P(d_k, d_{k-1} \mid z) \]

\[ \sim \alpha_{k-1}(d_{k-1}) \ P_d(d_k \mid d_{k-1}) \ \beta_k(d_k) \ f(d_k, d_{k-1}) \]

---

Posterior expectations of cyclopean intensity
FB: disparity distributions

FB: c=0.005, q=0.3

row 87
row 90

disparity
FB vs. Dijkstra cf. ground truth

Haloing around occlusions
Substituting background into occlusions

Cyclopean Smoothing --- open questions

- Non-cyclopean virtual views
- Regression to fronto-parallel
- Explicit occlusion labels/process
- Intra-scan-line constraints
- Robust intensity estimators
Labelling of occlusion

Left view

Right view

Occluded regions

Disparity $d(x')$

(data: Scharstein & Szeliski 2002)