TIME-CORRELATED NOISE IN QUANTUM COMPUTATION
Motivation
Fault-tolerant computation

- computing requires isolation & control
- maybe no such qubits occur "naturally"
- fault-tolerance: generic approach
- noise has to be weak & weakly correlated in spacetime
- here: arbitrary correlations in time
Fabrication faults
Fabrication faults

fabrication faults: known / unknown
operations: flexible / fixed
Fabrication faults

fabrication faults: known / unknown
operations: flexible / fixed
Noise model
Stochastic noise
Local stochastic noise

\[ P(1 \land 5) \leq \lambda^2 \]
Spatially local stochastic noise

\[ P(\Box \land \Box) \leq x^2 \]
Quantum memories based on single-shot error correction exhibit an error threshold under spatially local stochastic noise.
Single-shot error correction
Error correction

logical qubit \rightarrow physical qubits

extra d.o.f. \rightarrow check ops

syndrome extraction \rightarrow decoding \rightarrow correction
Error correction

ideal

noisy

syndrome extraction → decoding → correction
Error correction

single-shot if quantum-local
(analogous to LOCC)

syndrome extraction
local, quantum

decoding
global, classical

correction
local, quantum
Error correction

ideal

noisy

other
Topological codes
Topological codes

**local** check operators

local indistinguishability
2D codes

errors: strings

syndrome: endpoints
2D codes
2D codes

Spatially local (and Markovian), e.g.

\[ P(x_{i,t} \cap x_{i,t}) = \lambda^2 \]
Single-shot codes

D = 4

D = 3
4D codes

errors: membranes
syndrome: loops
3D codes

errors: strings
syndrome: endpoints

\[ P(e) \leq \Delta \]
Subsystem codes

logical qubit

physical qubits

extra d.o.f. \{ check ops
gauge d.o.f. \}
Gauss law

charge

flux

charge

field

error

syndrome

gauge

syndrome
3D gauge color codes

tetrahedron = qubit
edge = gauge op
vertex = check op
X & Z type
Confinement

check ops

unconfined

gauge ops

confined!
Result
Quantum memory

Perfect encoding and decoding to test the quality of the quantum memory: alternated noise and noisy error correction
Noise

\[ P(1 \land 3 \land 7) \leq \lambda^3 \]
Noisy error correction

\[ P \left( \bigwedge_{i=1}^{3} e_i \bigwedge \right) \leq 3^{-3} \]
Quantum memory

For error rates below a threshold

\[ p(\text{error}) \leq a t + b \]

where \( a \) & \( b \) decrease exponentially with the system size.
DISCUSSION

- Universal computation probably straightforward
- $D < 3$
- Known fabrication faults
- Fully local (CA) error correction
- The physics of gauge color codes. Gapless phases? Confinement?
Local operations

- Transversal
- Local
- Quantum-local

Not universal
Not universal?

Universal + EC

but + EC = universal

Fault tolerance!
Th.

with $\epsilon$ the error rate of
d = 2
\( d = 4 \)
localized measurement errors yield localized residual noise
\[
\begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1/\sqrt{2} & 1/\sqrt{2} \\
-1/\sqrt{2} & -1/\sqrt{2}
\end{bmatrix}
\]

\[
d = 2
\]