Catalytic Decoupling

Joint work with Mario Berta, Frédéric Dupuis, Renato Renner and Matthias Christandl (arXiv:1605.00514, accepted for publication in PRL)

merged with

Deconstruction and Conditional Erasure of Correlations

Joint work with Mario Berta, Fernando Brandao, and Mark Wilde (arXiv:1609.06994)

Christian Majenz
QMATH, University of Copenhagen

QIP, Microsoft Research, Seattle

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UNIVERSITY OF COPENHAGEN

Introduction:
Decoupling and Erasure

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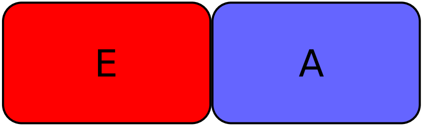

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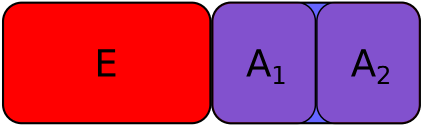

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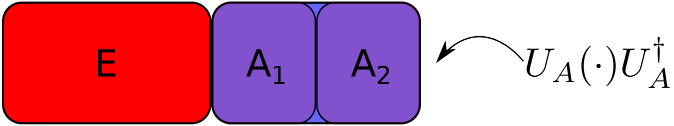

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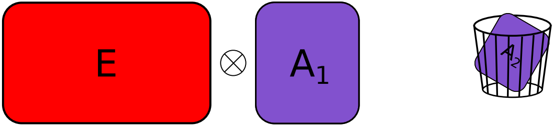

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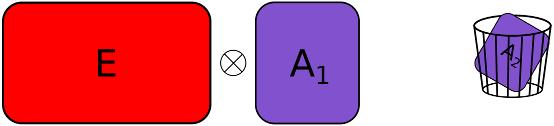

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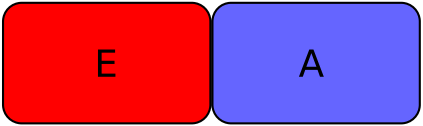

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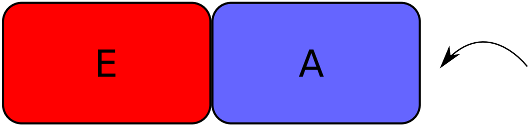

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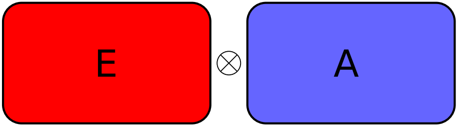

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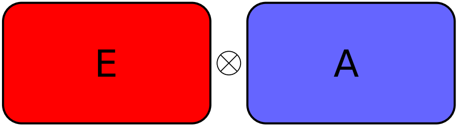

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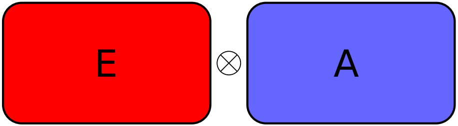

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- decoupling, erasure of correlations: two sides of same coin

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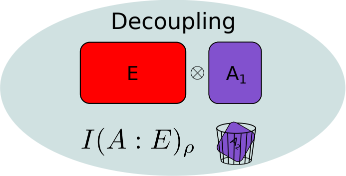

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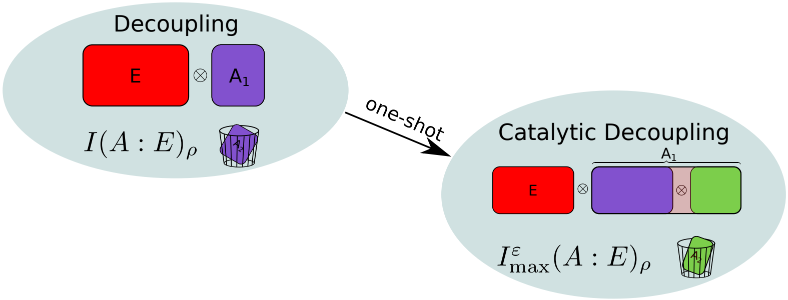

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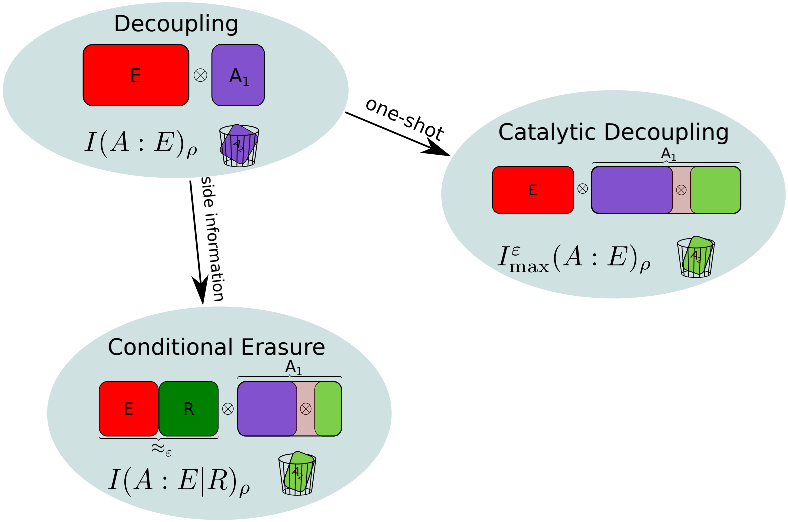
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Theorem (Dupuis, Berta, Wullschleger, Renner '10)

Let ρ_{AE} *be a bipartite quantum state, and let* $\mathcal{H}_A \cong \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2}$ *such that*

$$\log |A_2| \geq \frac{1}{2} \left(H^{\varepsilon}_{\max}(A)_\rho - H^{\varepsilon}_{\min}(A|E)_\rho \right) - \mathcal{O}\left(\log \frac{1}{\varepsilon} \right).$$

Then $\exists U_A$ *such that*

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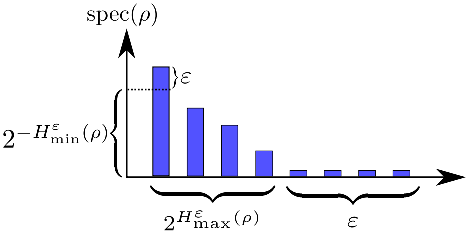

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but there are product states with $H^{\varepsilon}_{\max}(A)_{\rho} - H^{\varepsilon}_{\min}(A|E)_{\rho} = \mathcal{O}(\log|A|) \Rightarrow$ suboptimal for applications like state merging

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- tailored techniques

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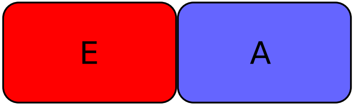

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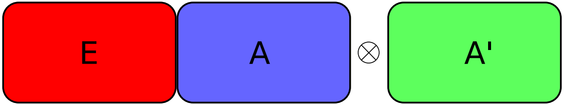

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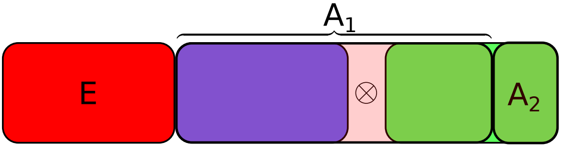

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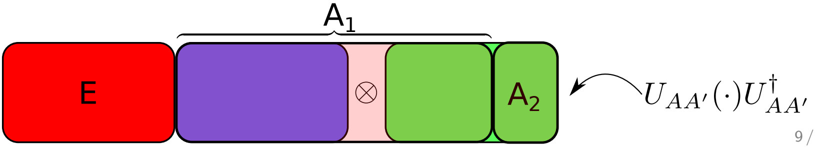

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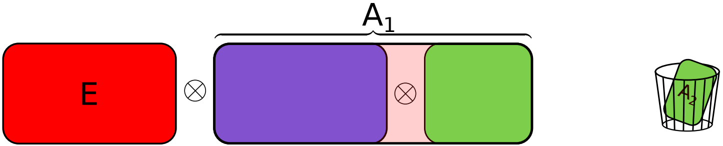

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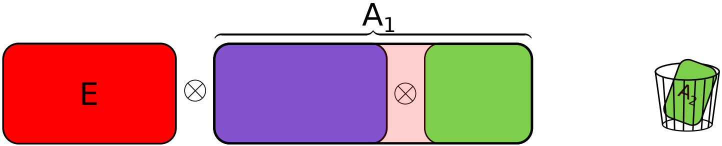

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Theorem (CM, Berta, Dupuis, Renner, Christandl)

Let $\rho_{AE} \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_E)$ *be a quantum state. Then, for any* $0 \leq \varepsilon' < \varepsilon$ *catalytic decoupling with error* ε *can be achieved with remainder system size*

$$\log |A_2| \approx \frac{1}{2} I_{\max}^{\varepsilon'}(E : A)_\rho.$$

Conversely catalytic decoupling is impossible whenever

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- max-mutual information: $I_{\max}(A:B)_\rho = \min_{\sigma_B} D_{\max}(\rho_{AB} \| \rho_A \otimes \sigma_B)$
- $D_{\max}(\rho\|\sigma) = \min\{\lambda \in \mathbb{R} | 2^\lambda \sigma \geq \rho\}$

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- max-mutual information: $I_{\max}(A : B)_\rho = \min_{\sigma_B} D_{\max}(\rho_{AB} \| \rho_A \otimes \sigma_B)$
- $D_{\max}(\rho \| \sigma) = \min\{\lambda \in \mathbb{R} | 2^\lambda \sigma \geq \rho\}$
- Two proofs, one using the techniques from Anshu et al. and Berta et al. respectively

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- Unitary randomizing and partial trace models equivalent with ancilla

Conditional Erasure

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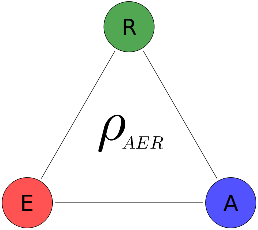

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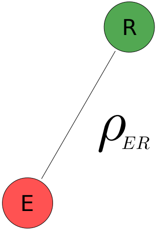

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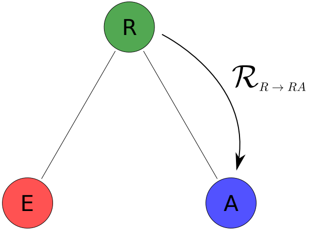

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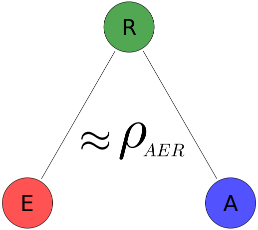

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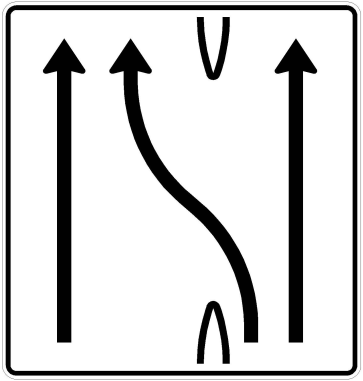
- Characterization for pure states: Noise $\gg I(A:E|R)$ necessary in general (Wakakuwa et al. '15)
- obvious solution in the classical case: condition on R!

State redistribution (SRD)



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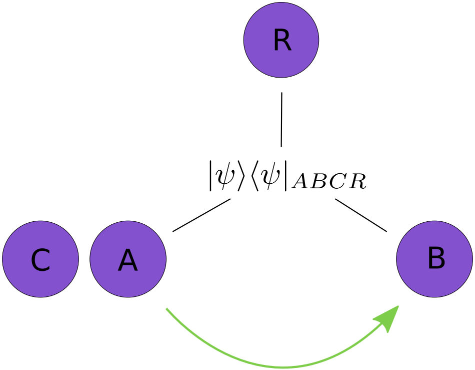

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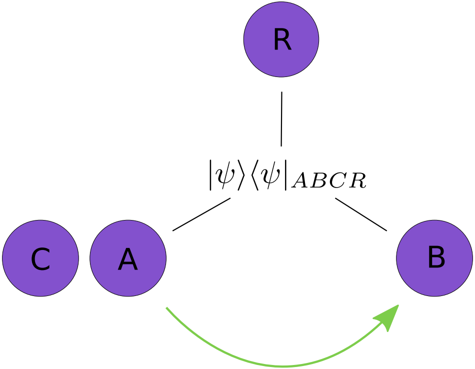

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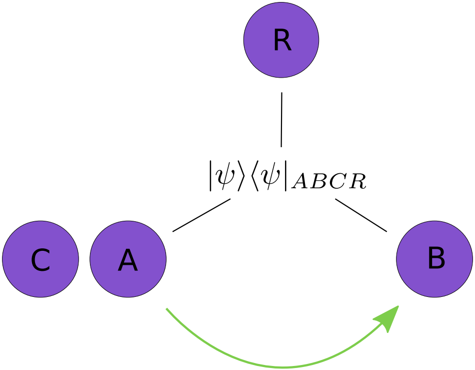

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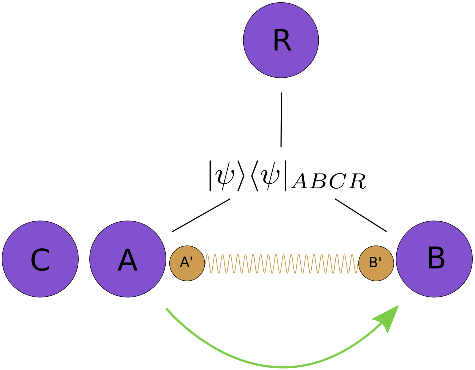

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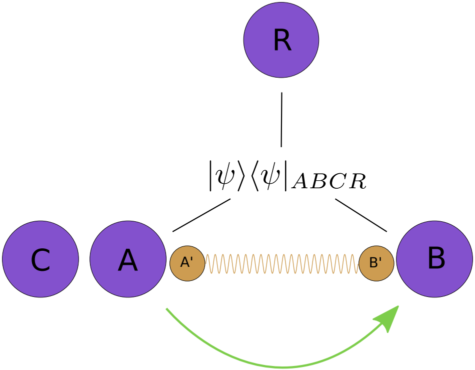
- optimal comunication rate $I(A:R|C)$ (Devetak and Yard '06)



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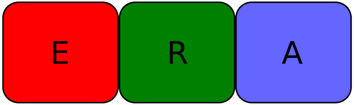

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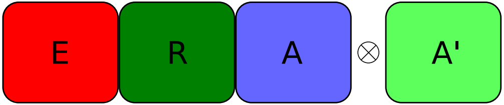

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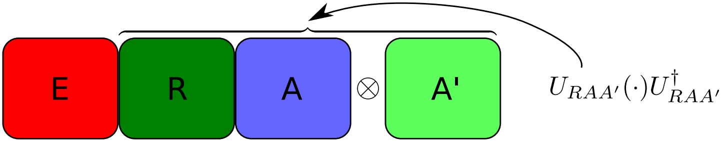

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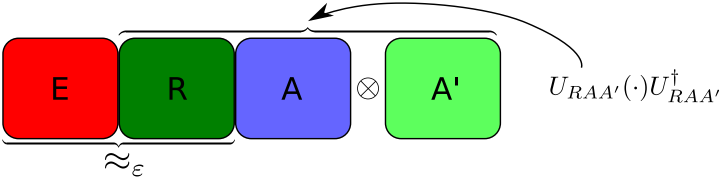

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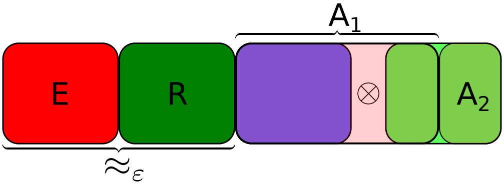

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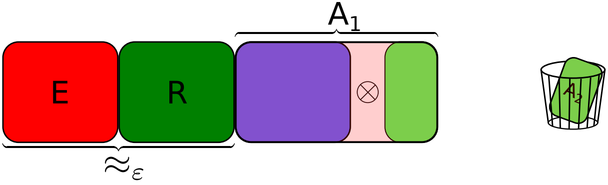

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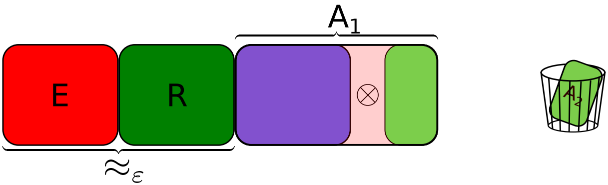

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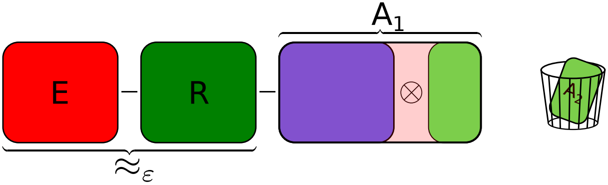

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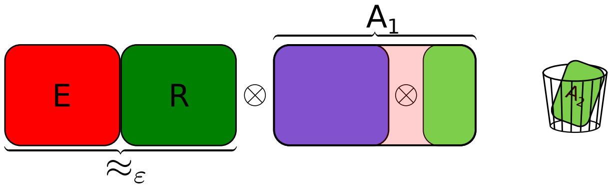

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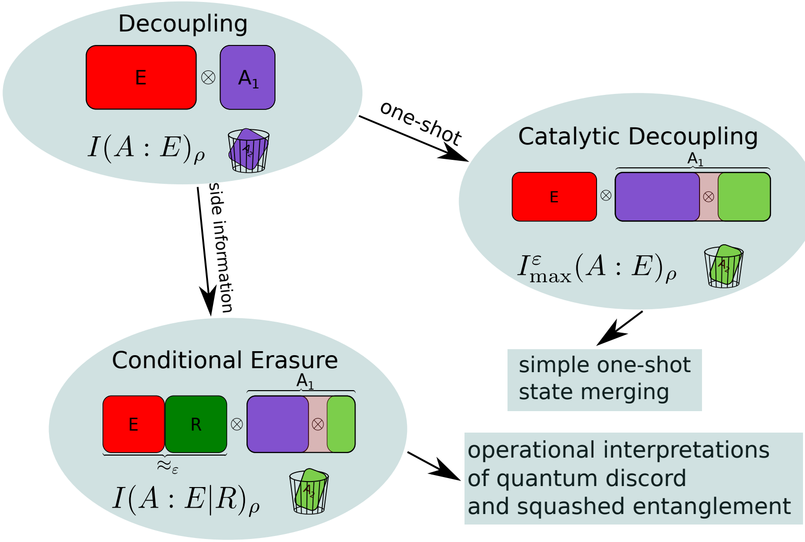
- Operational interpretation of quantum conditional mutual information!

The End


Decoupling
E
A1
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A2
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A2
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A1
E
R
$\approx_\varepsilon$
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A2
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backup slides

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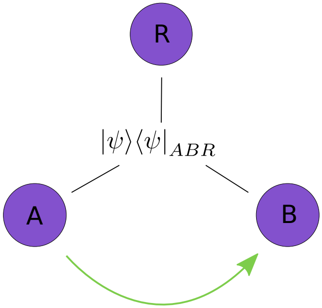

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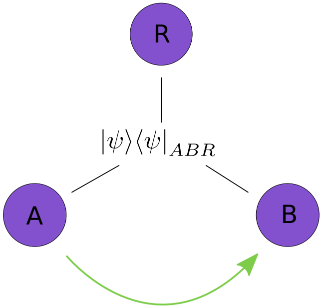

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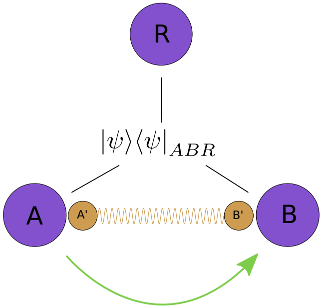

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$D(\overline{A} : B)_{\rho,\Lambda}$ is equal to the rate of noise necessary to simulate the loss of correlations incurred by $\rho^{\otimes n}$ under the action of $\Lambda^{\otimes n}$.

- Squashed entanglement: $E_{sq}(A : B)_\rho = \inf_\sigma I(A : B|E)_\sigma$, inf over all σ_{ABE} with $\mathrm{tr}_E\, \sigma_{ABE} = \rho_{AB}$

Applications

- 2-party state ρ_{AB}, measurement $\Lambda_{A \to X}$
- (unoptimized) quantum discord:
  $D(\overline{A} : B)_{\rho,\Lambda} = I(A : B)_\rho - I(X : B)_{\Lambda(\rho)}$
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$\Rightarrow$ Squashed entanglement is amount of noise necessary to make many i.i.d. copies of ρ_{AB} close to separable by operation on A and arbitrary catalytic side information E