Anyons and Matrix Product Operator Algebras

Dominic Williamson

Verstraete Group

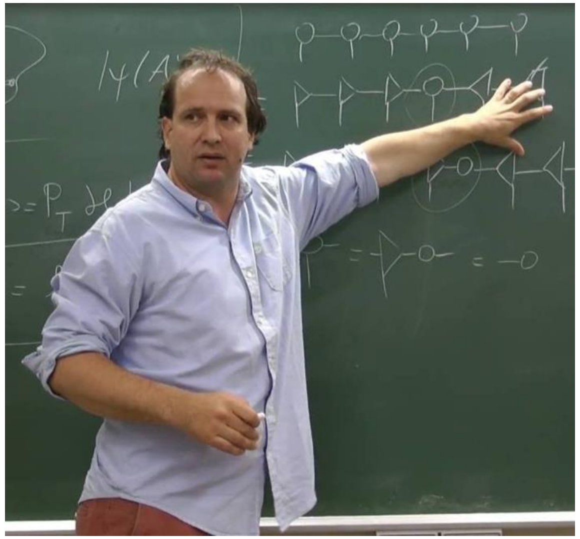
University of Vienna

Nick Bultinck

Michael Marien

Burak Sahinoglu

Jutho Haegeman



Frank Verstraete

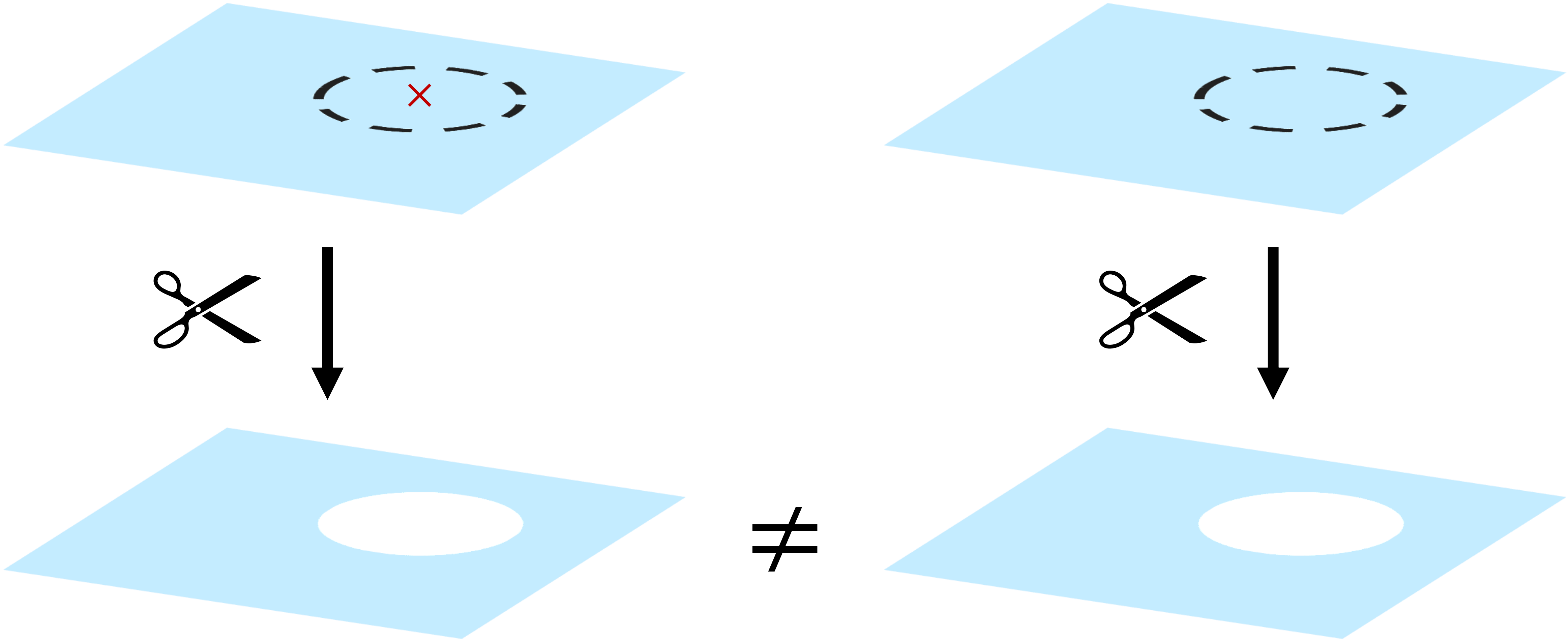
What states of matter are there?

$$[\psi] := \{\psi' \mid \psi' = U\psi, \exists U \text{ a local unitary}\}$$

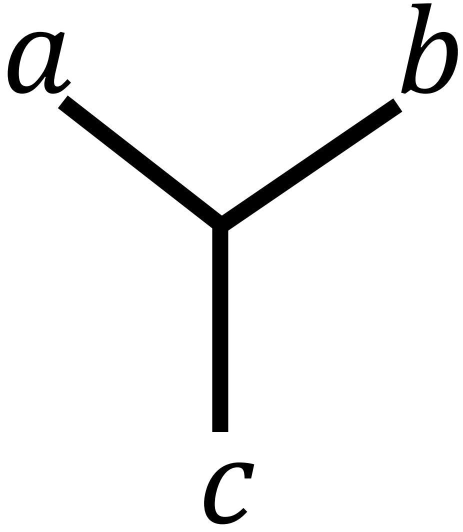
Anyons / Superselection sectors

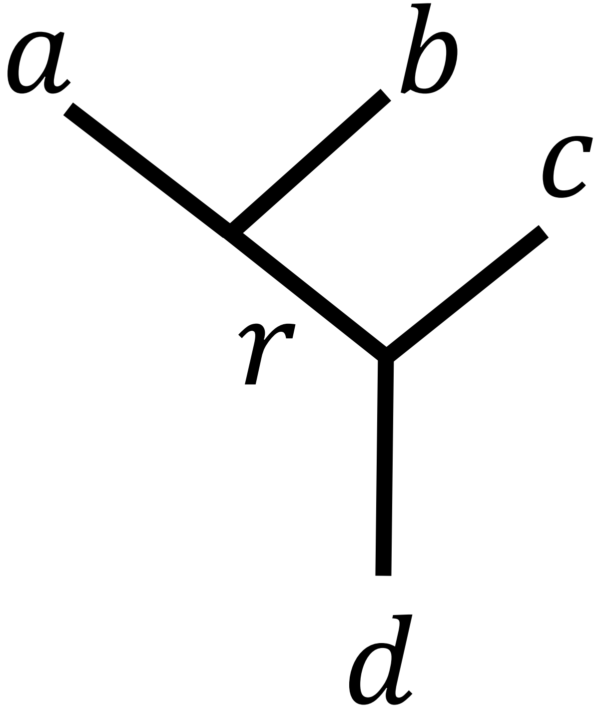
(Modular Tensor Categories)

$$[\rho] := \{\rho' \mid \rho' = U\rho U^{-1}, \exists U\}$$

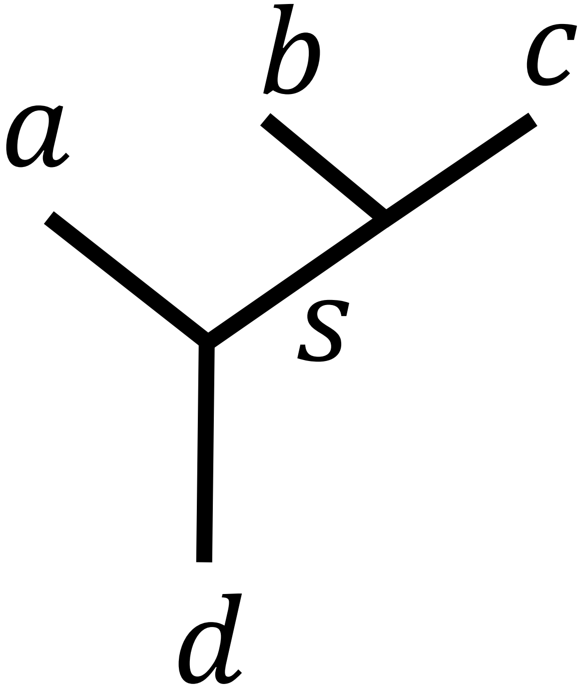


$$a \times b = c + d + \cdots$$
$$= \sum N_{ab}^{c}\, c$$

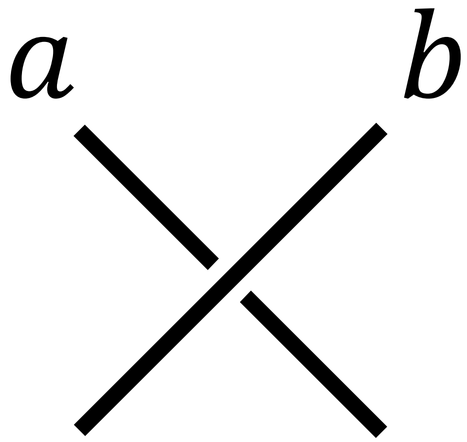


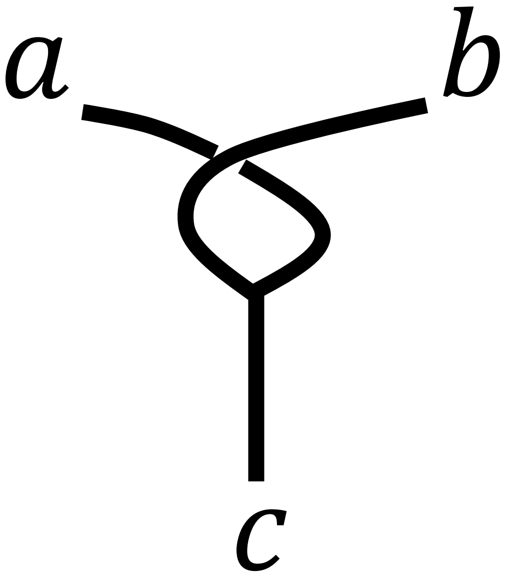


$$= F_{d;rs}^{abc}$$

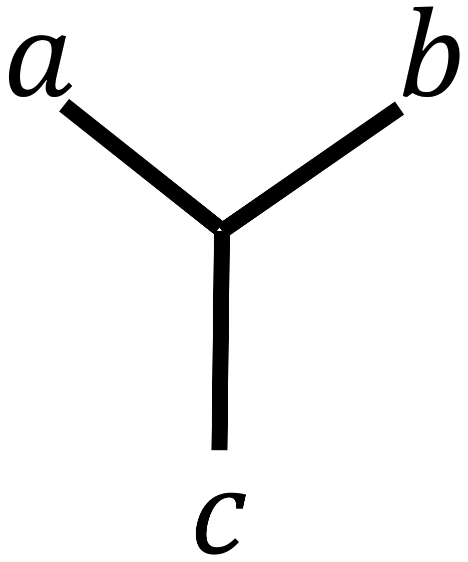


$$R^{ab} =$$

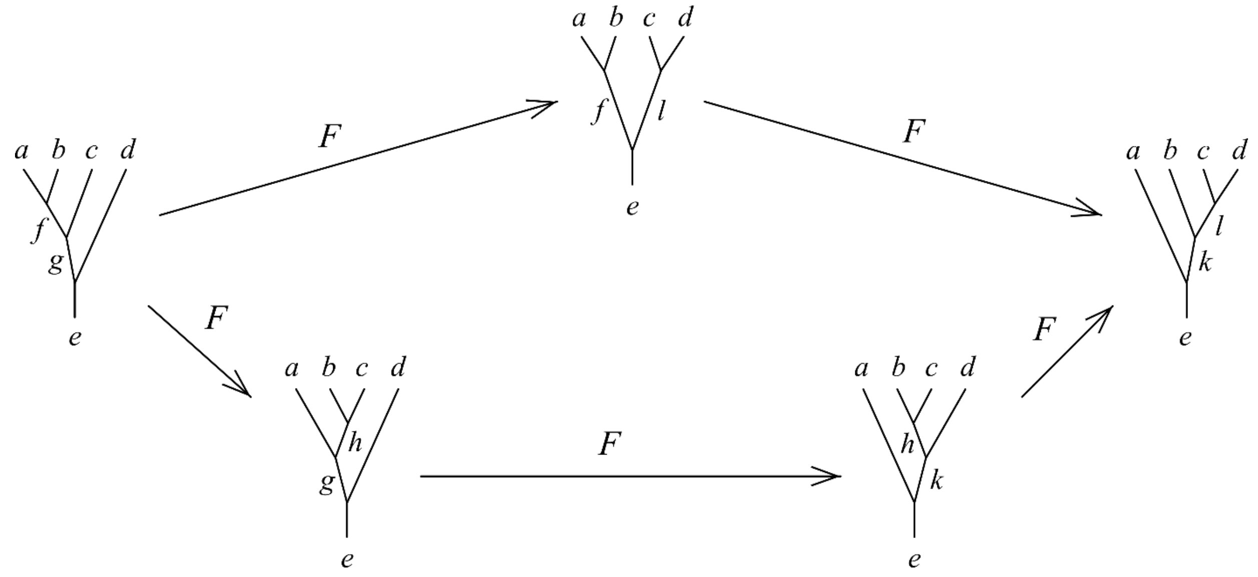




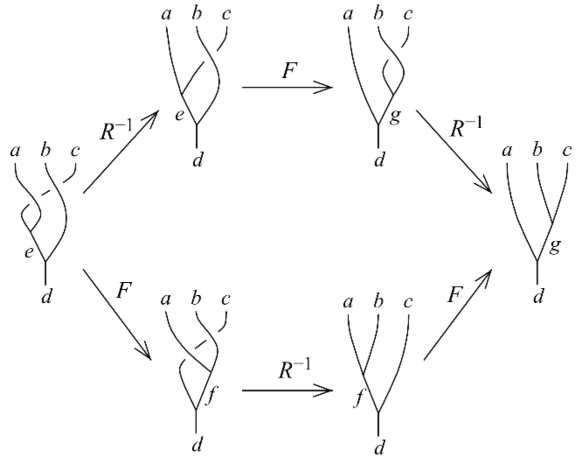
$$= R_c^{ab}$$

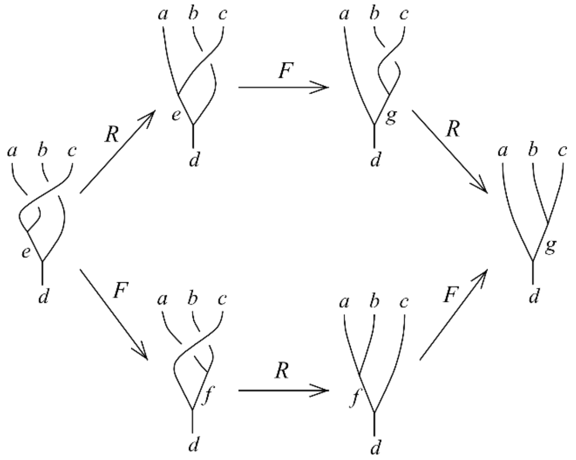


Pentagon

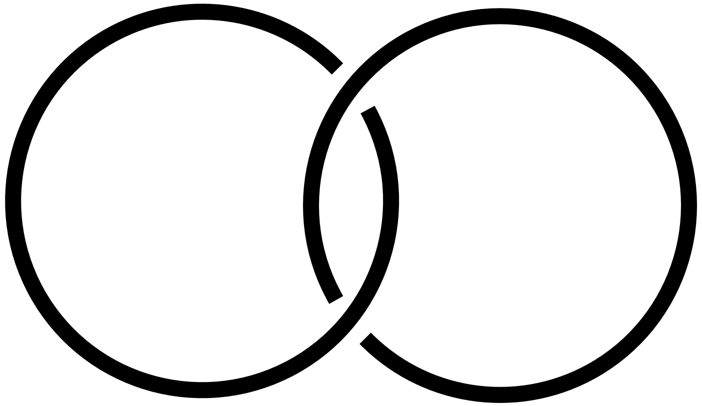


Hexagons



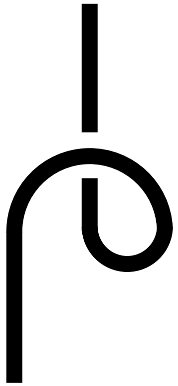


$S_{ab} =$ a



b

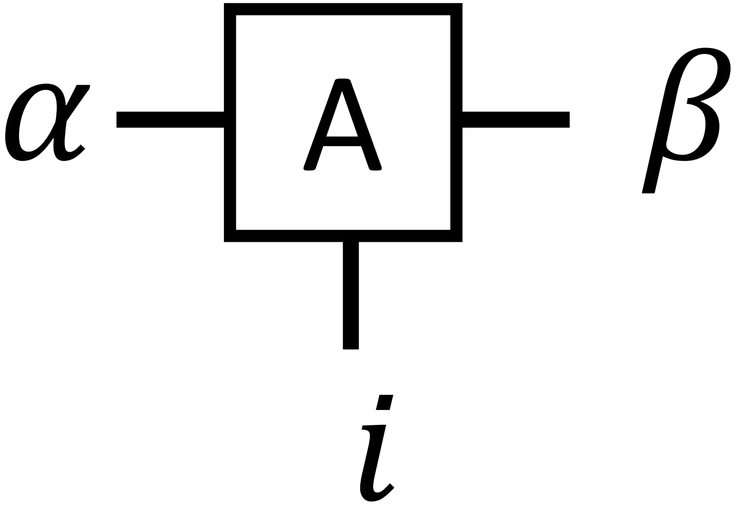
a



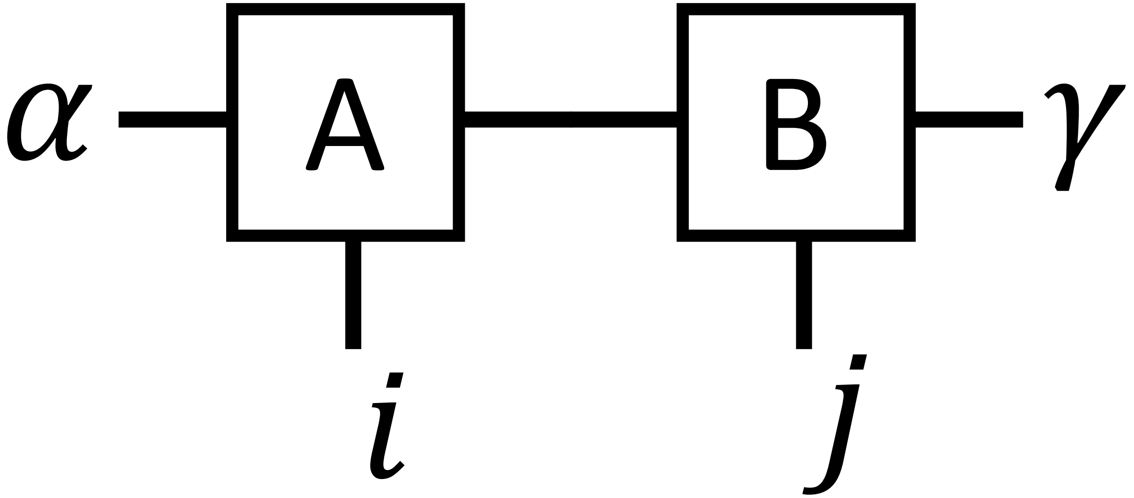
$= e^{i2\pi h_a}$

a

Tensor Networks

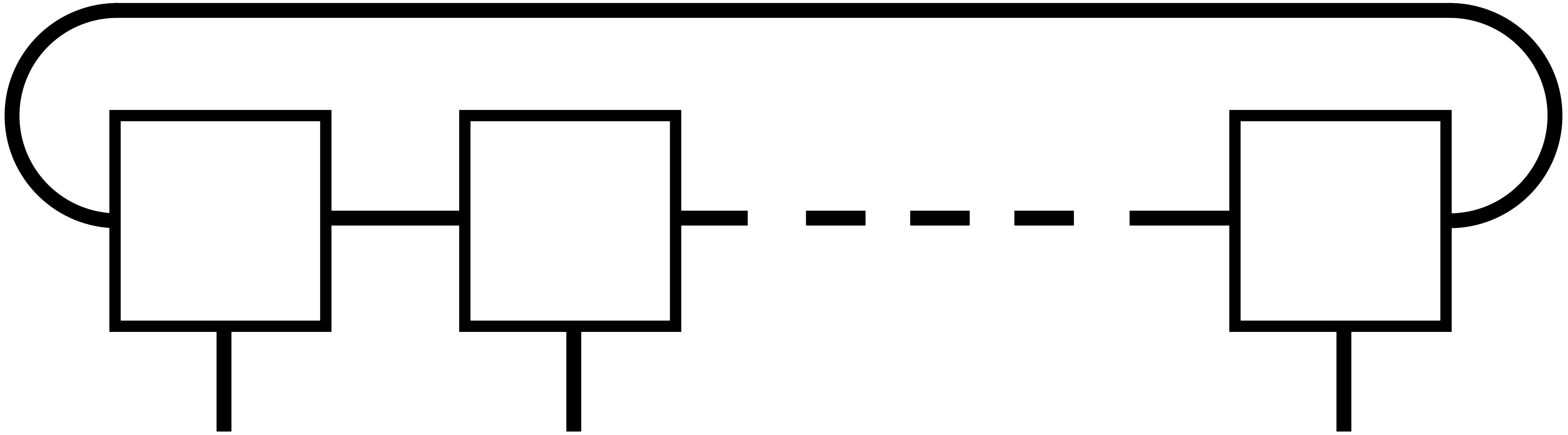


$$= \left(A^i\right)_{\alpha\beta} \in \mathbb{C}$$



$$= \sum_{\beta} \left(A^{i}\right)_{\alpha\beta} \left(B^{j}\right)_{\beta\gamma}$$

$$|MPS_N(A)\rangle = \sum_{i_1 \ldots i_N} Tr\left[A^{i_1} \ldots A^{i_N}\right]|i_1 \ldots i_N\rangle$$

$=$ 

Normal form

$$A^i \sim \begin{pmatrix} A_0^i & M_{01}^i & \cdots \\ 0 & A_1^i & \\ \vdots & & \ddots \end{pmatrix}$$

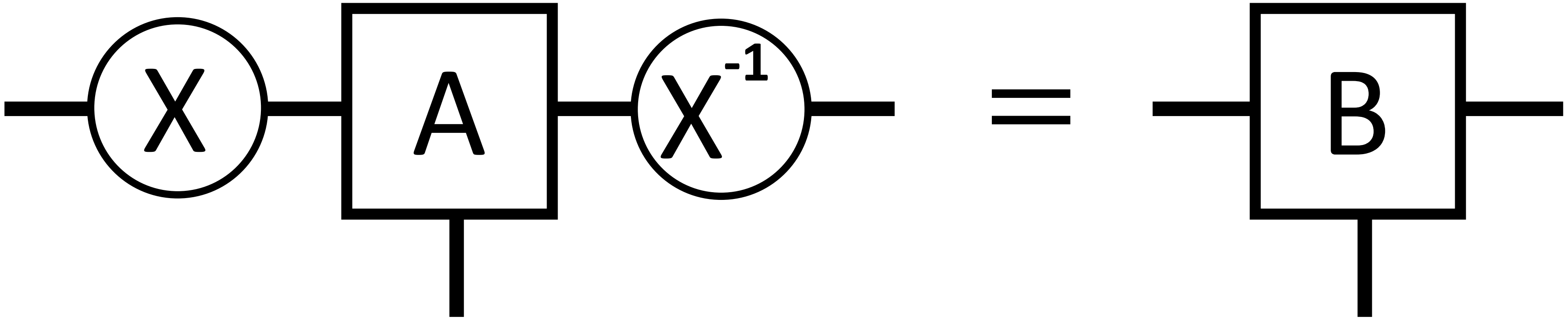
s.t. $\{A_k^i\}_i$ generates an irred. algebra

$$|MPS_N(A)\rangle = \sum_k |MPS_N(A_k)\rangle$$

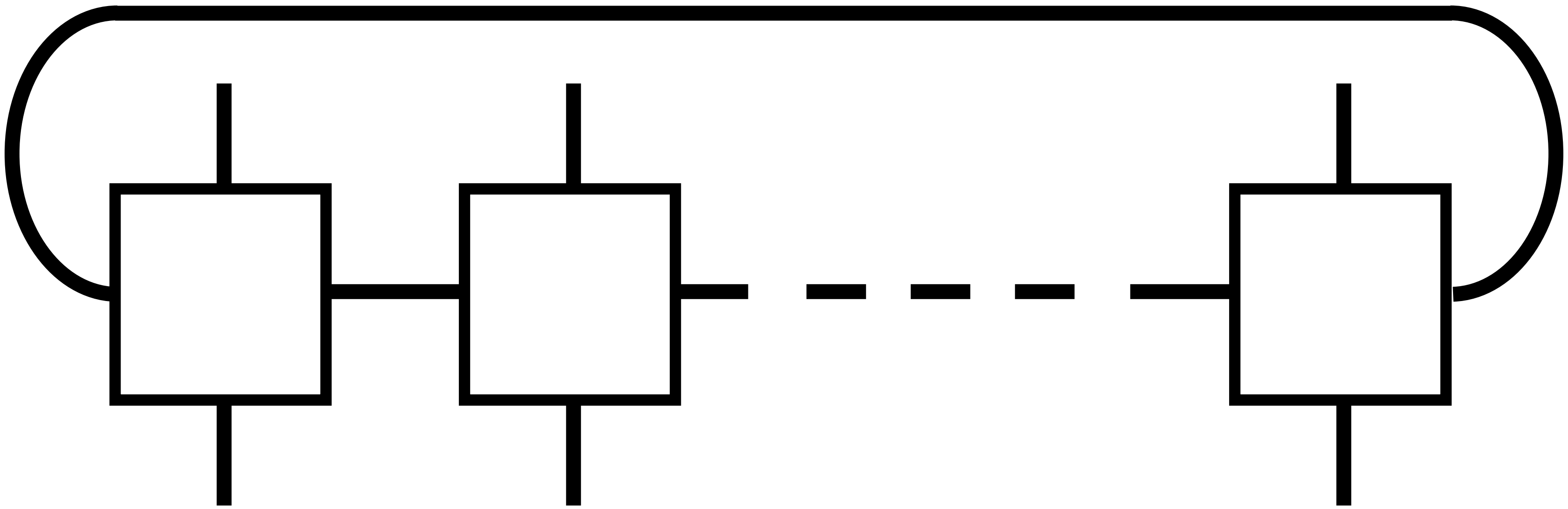
Fundamental Thm. of MPS

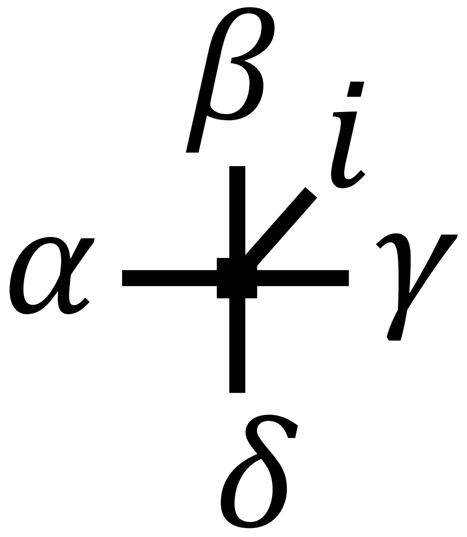
$$|MPS_N(A)\rangle = |MPS_N(B)\rangle \;\; \forall N$$

$\Rightarrow \exists\, X$ s.t.



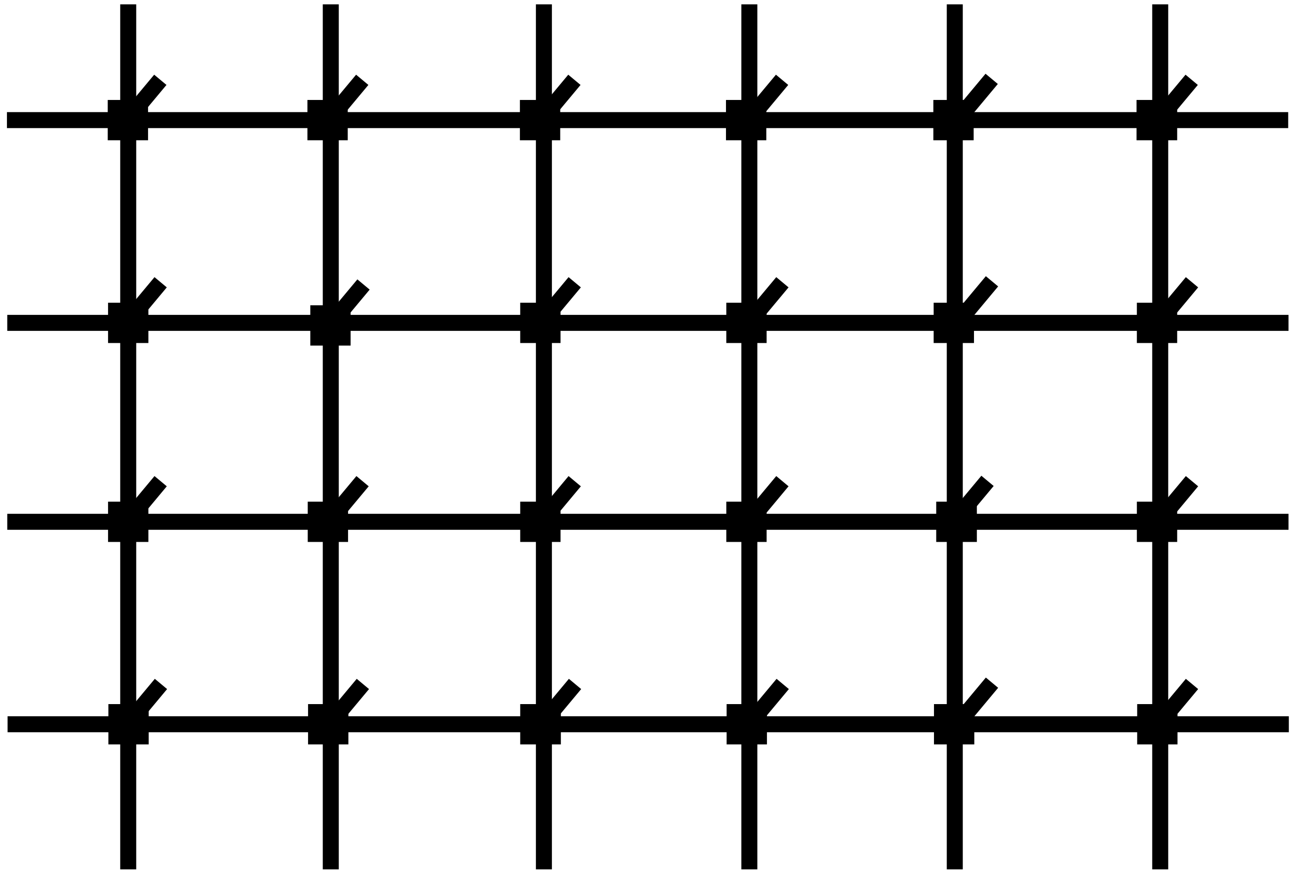
$$MPO_N(A) = \sum_{\substack{i_1 \ldots i_N \\ j_1 \ldots j_N}} Tr\left[A^{i_1 j_1} \ldots A^{i_N j_N}\right] |i_1 \ldots i_N\rangle\langle j_1 \ldots j_N|$$

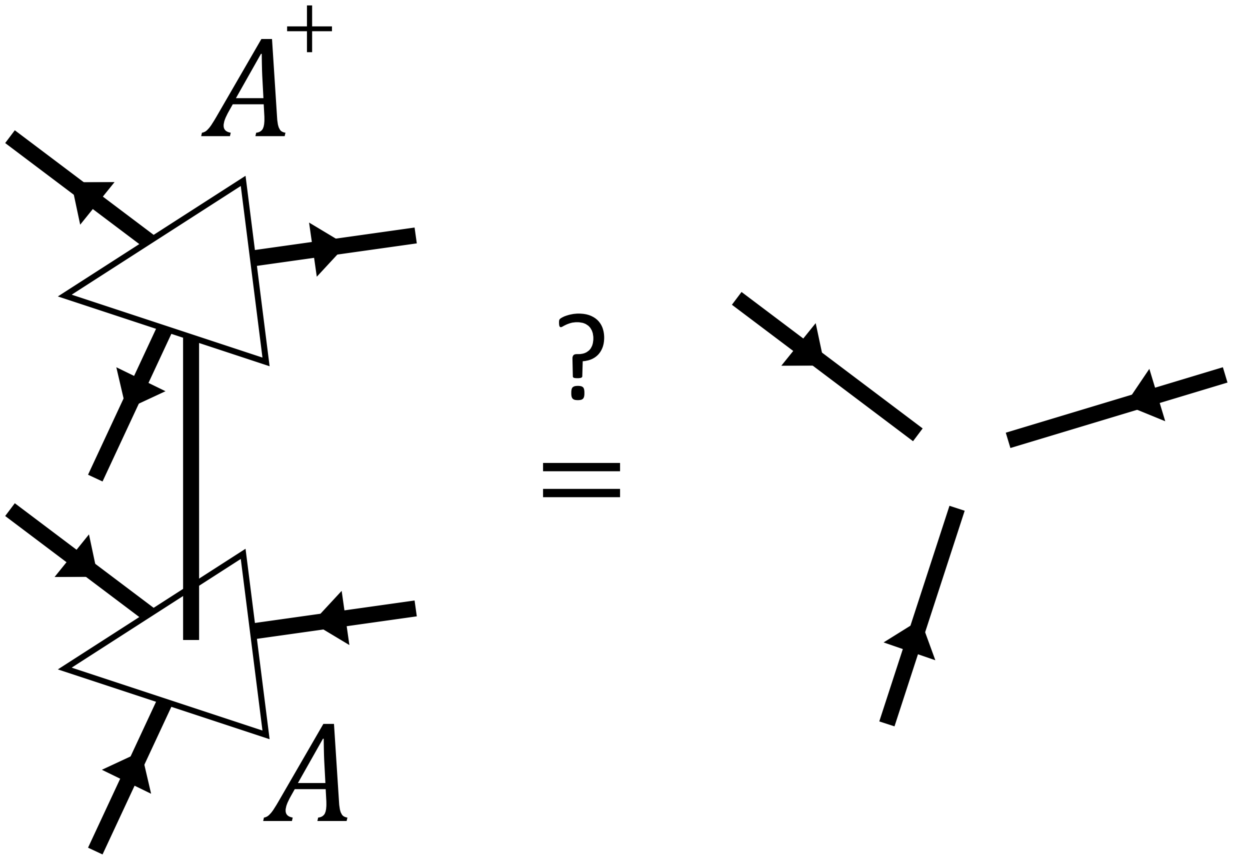
$=$ 

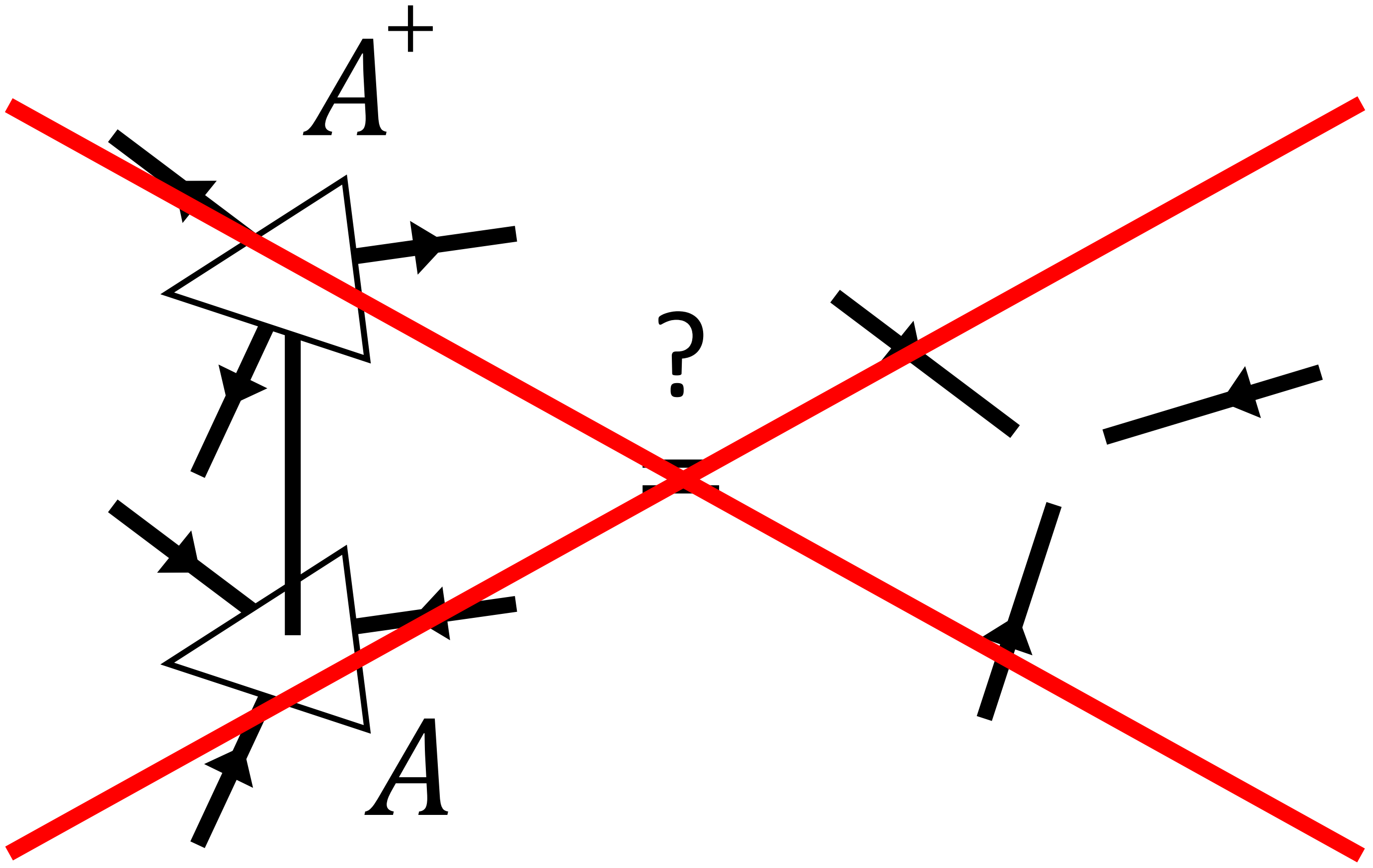


$$= \left(A^{i}\right)_{\alpha\beta\gamma\delta}$$

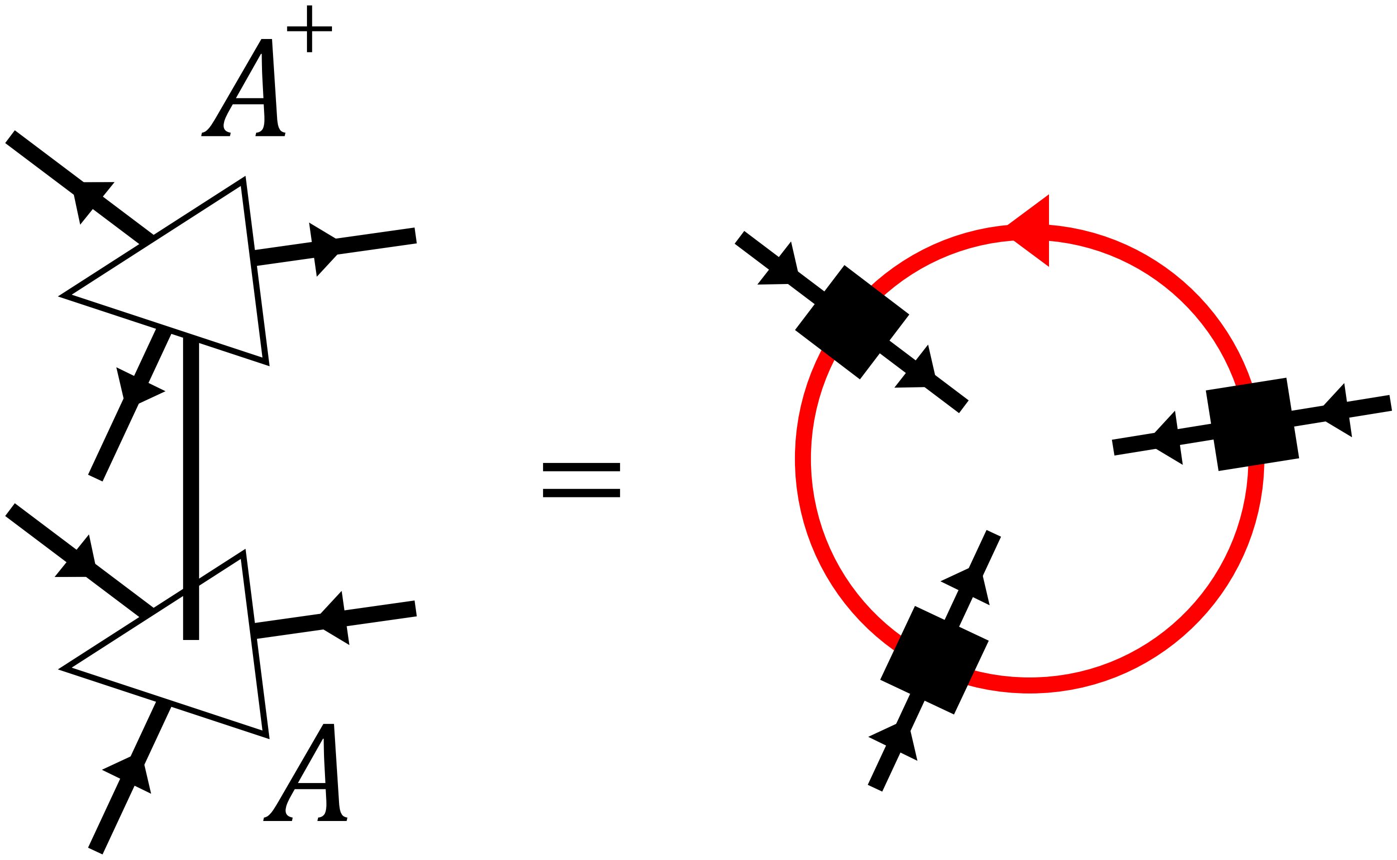
$$|PEPS(A)\rangle =$$

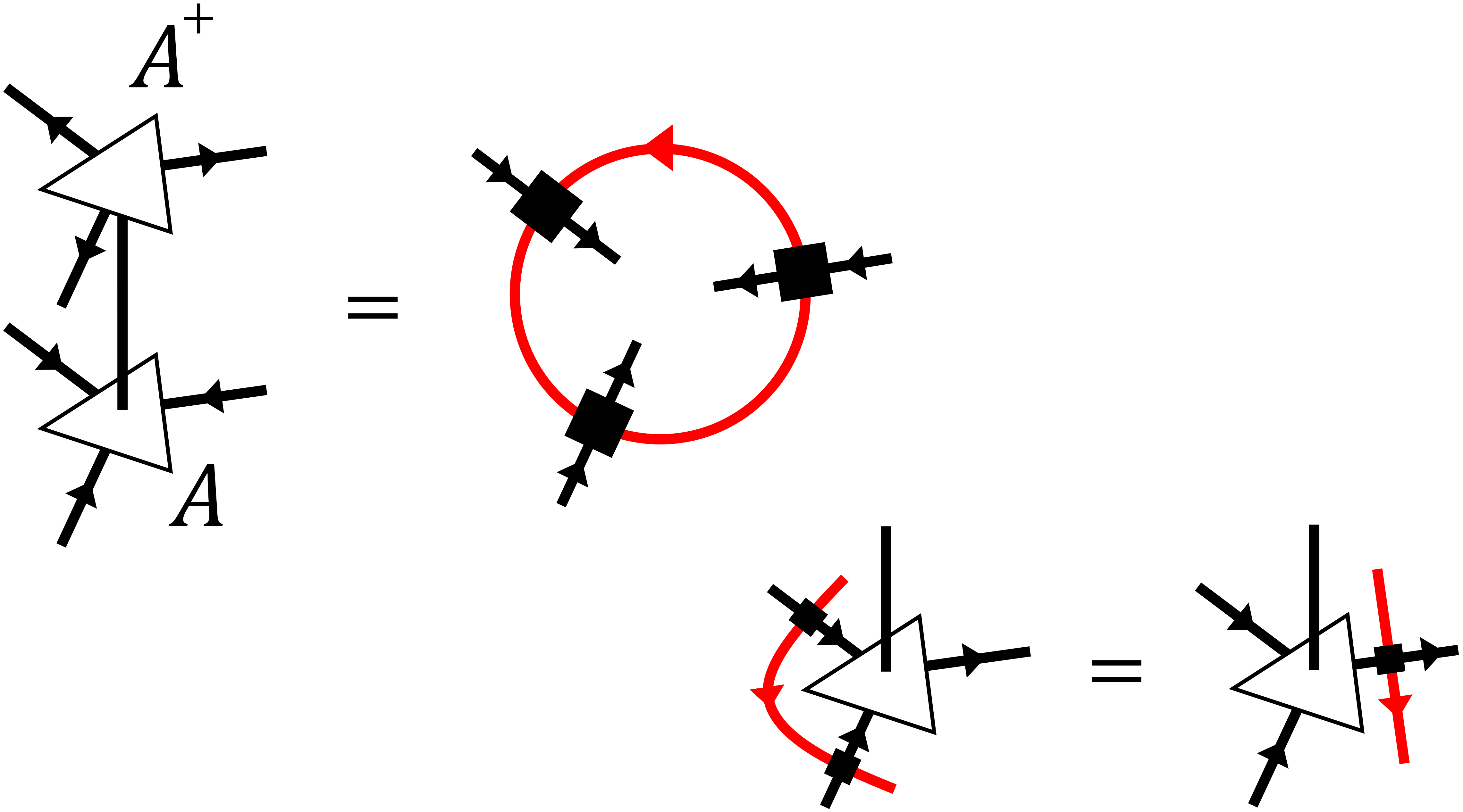



A^+
?
=
A



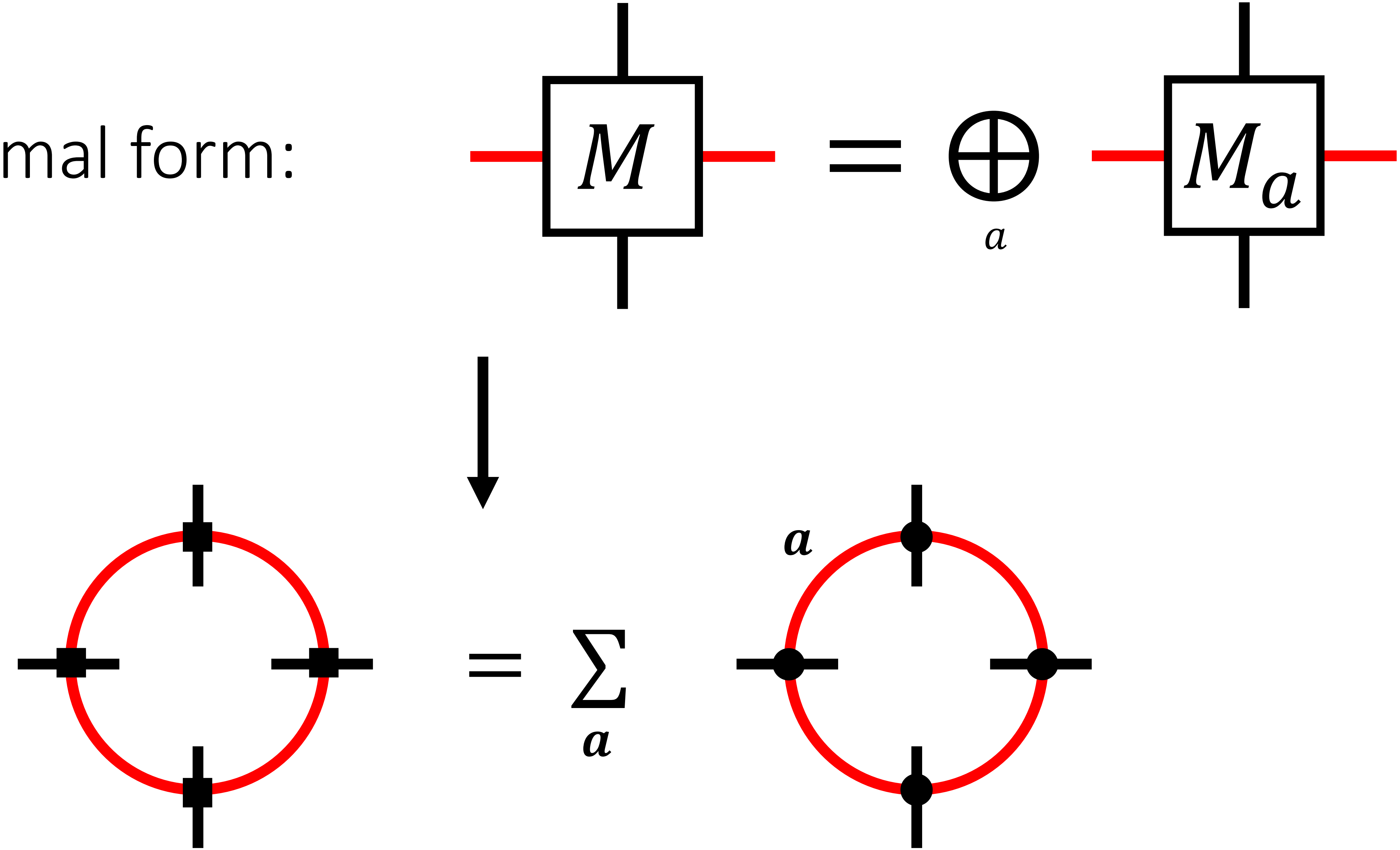
A^+
?
=
A


A^+
A
=

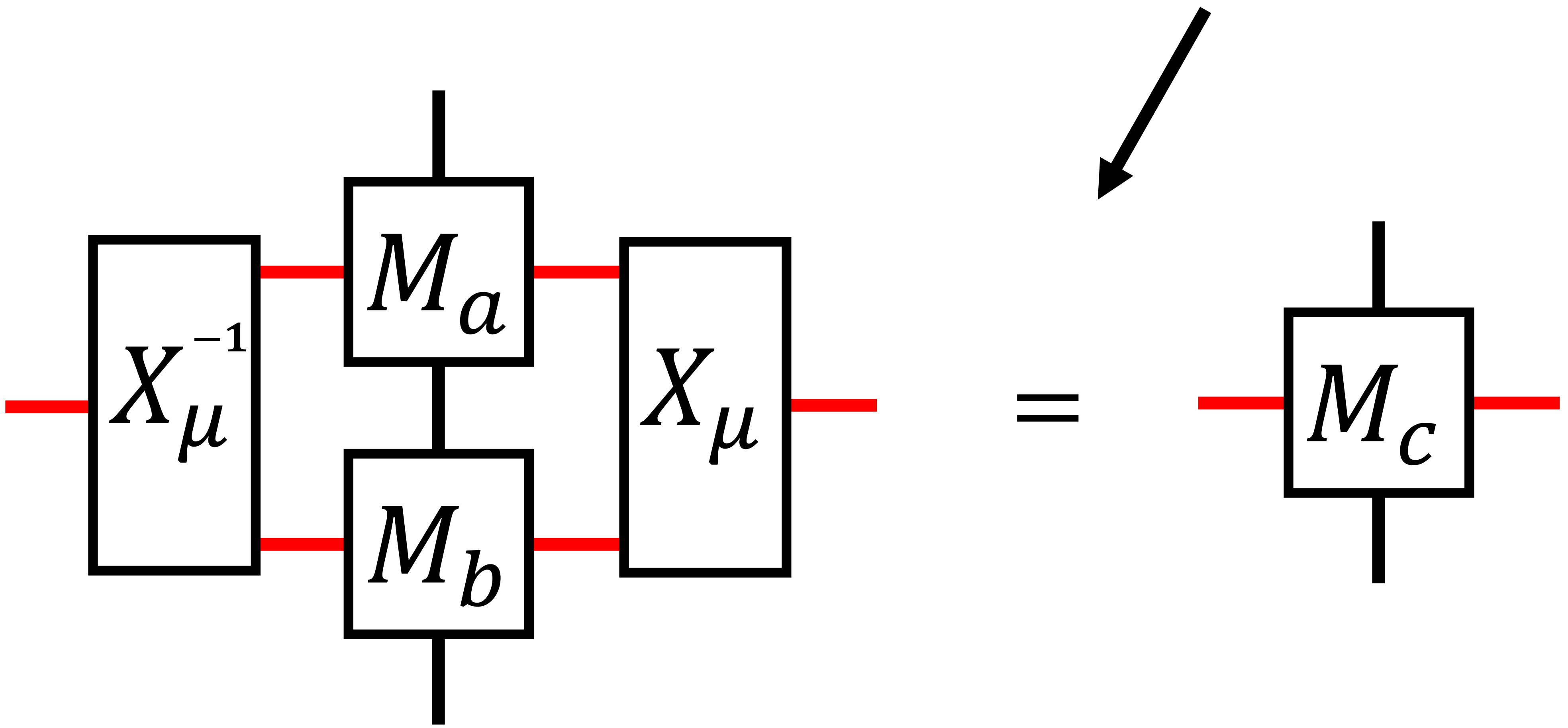

A^+
A

Matrix Product Operator Algebras

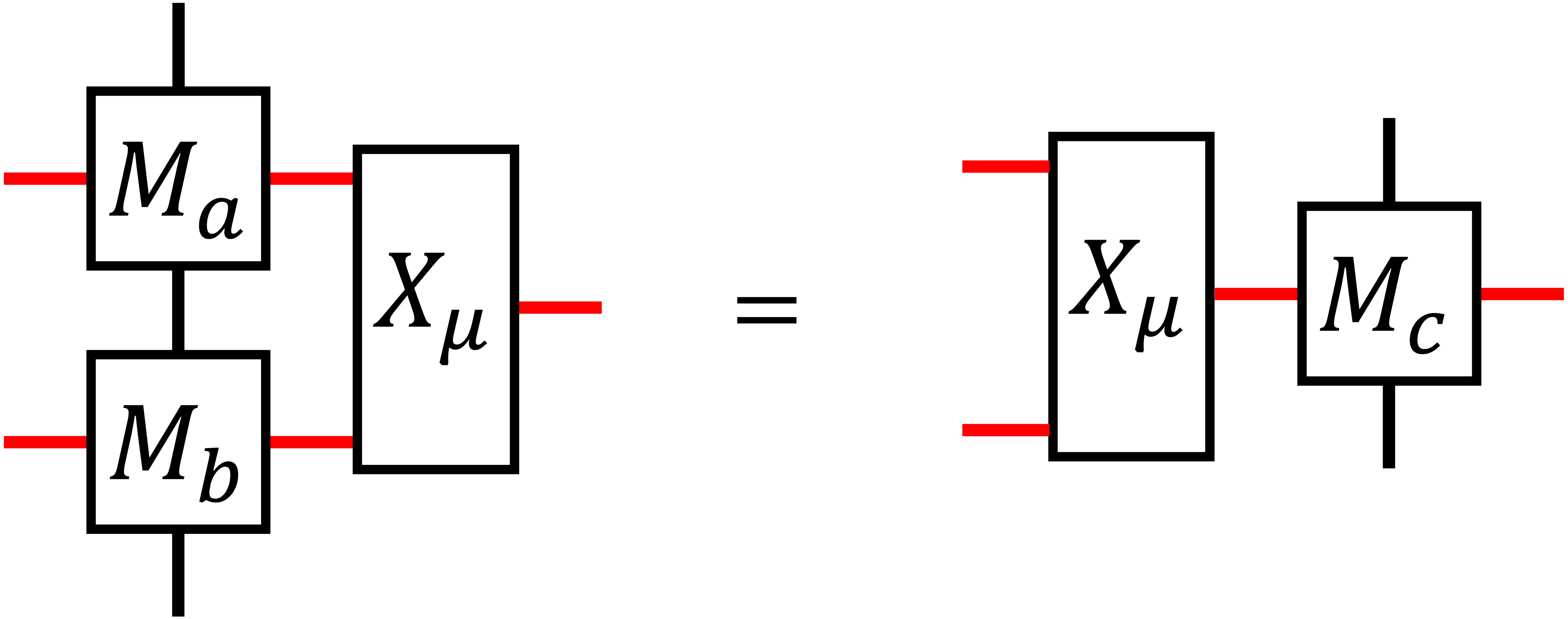
Normal form:

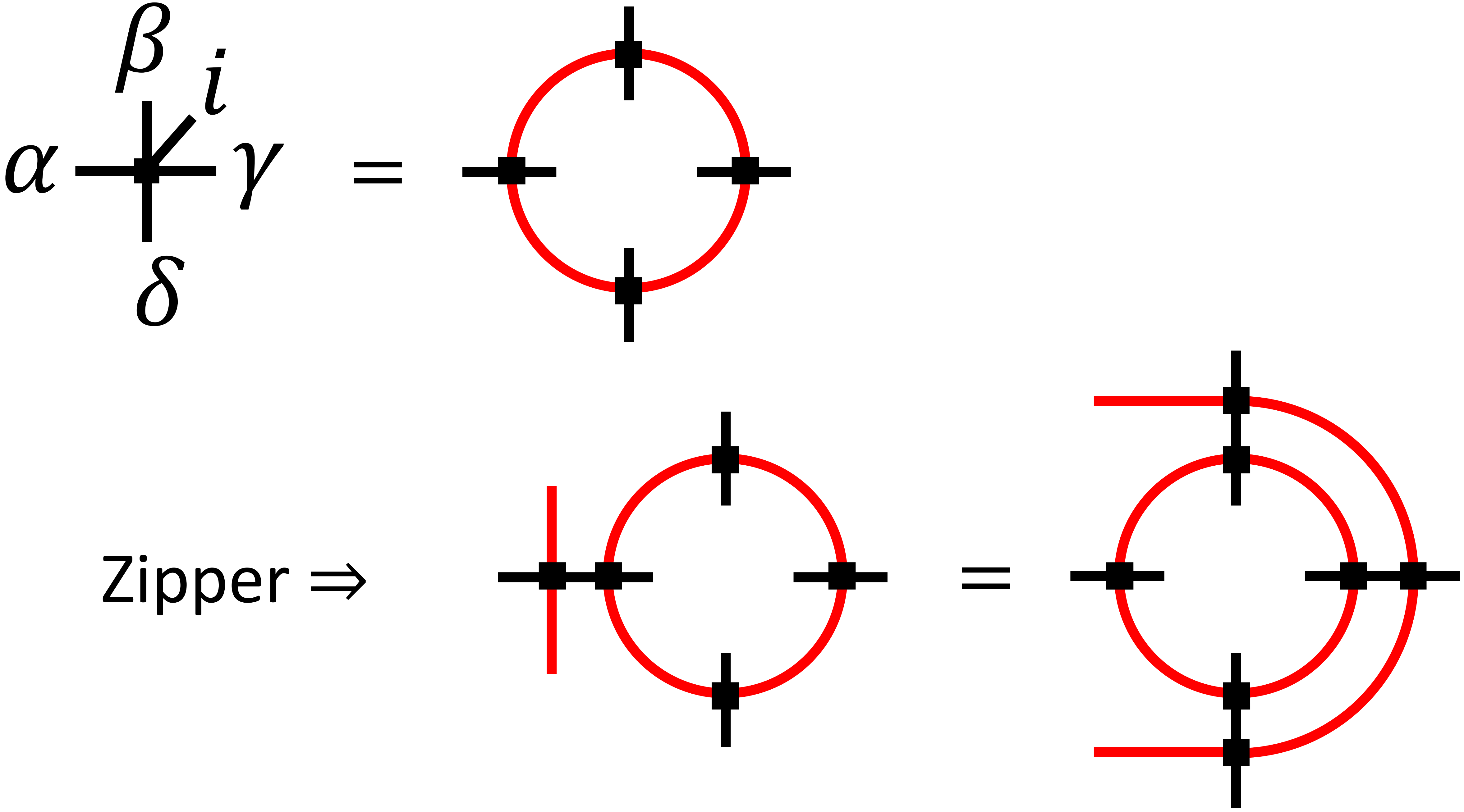


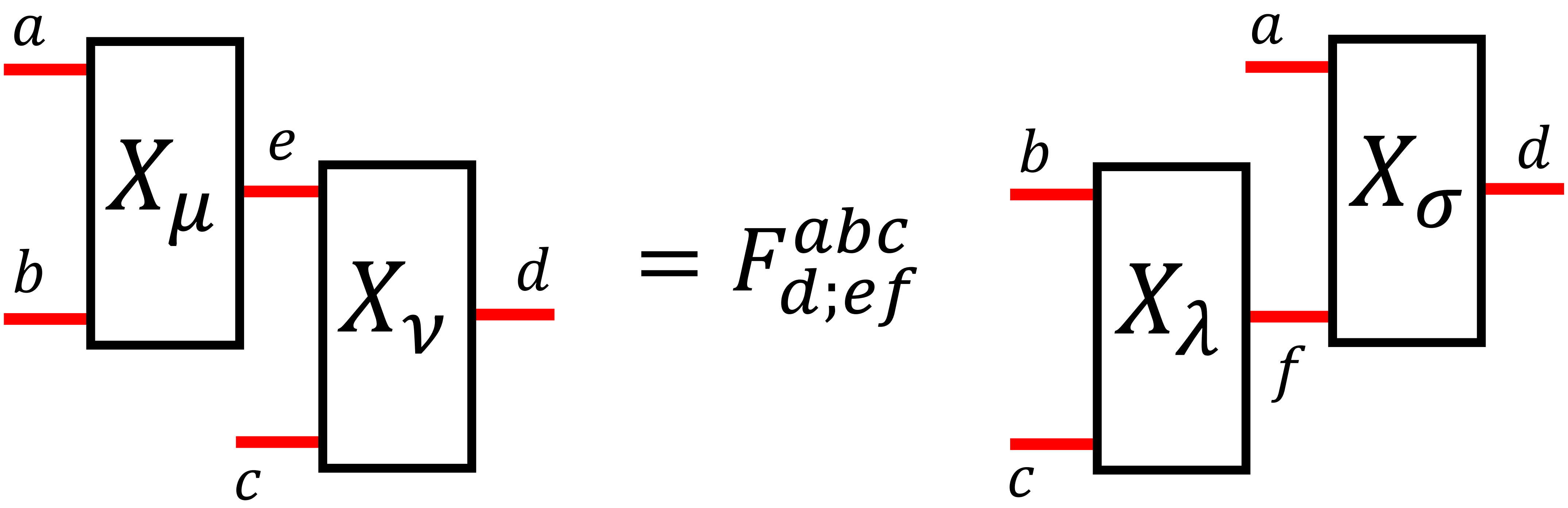
$$M(\mathrm{L})^2 = M(\mathrm{L}) \longrightarrow M_a \mathrm{M_b} = \sum_c N_{ab}^c M_c$$

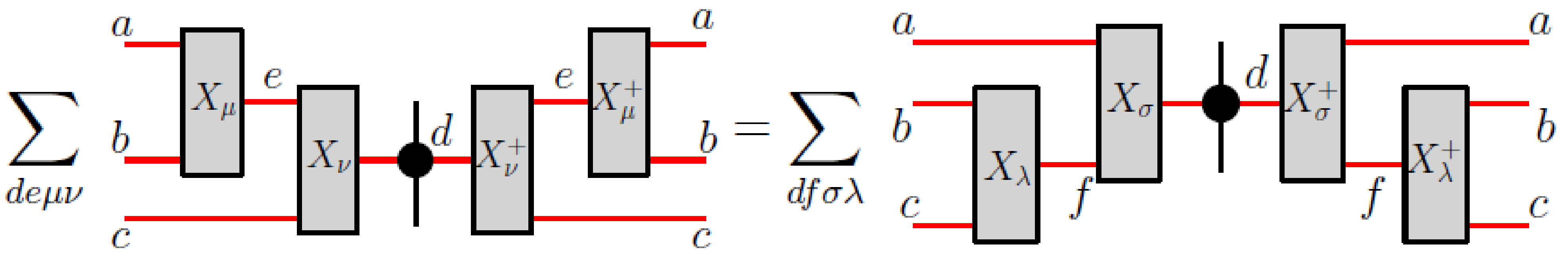


The Zipper condition

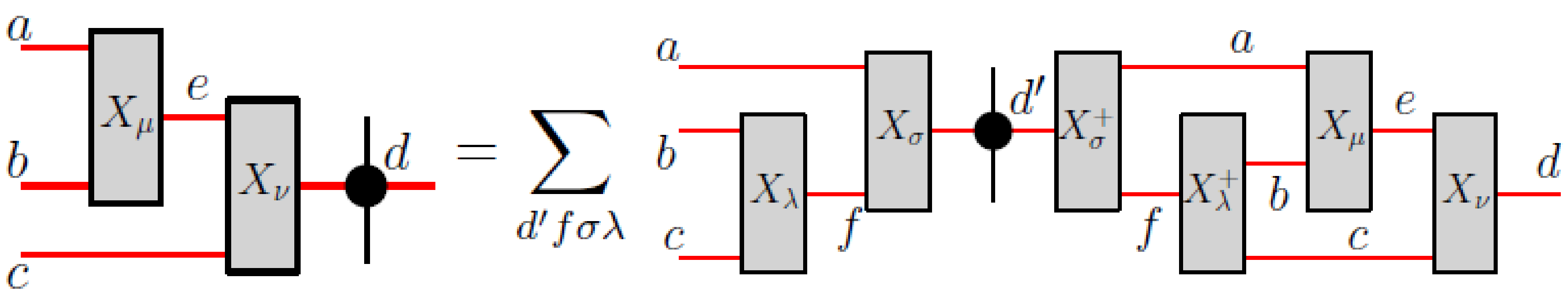



β
i
α
γ
δ
=
Zipper ⇒
=

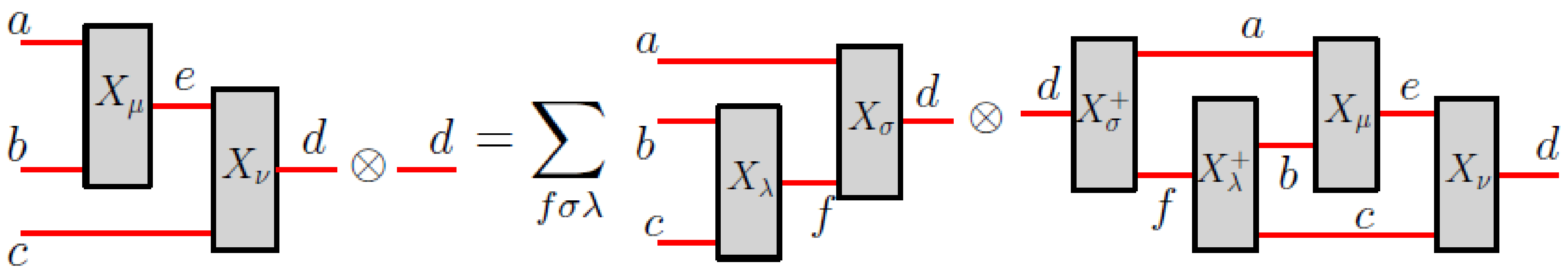

a
b
e
c
d
X_μ
X_ν
$= F^{abc}_{d;ef}$
a
b
c
d
f
X_λ
X_σ



a
b
c
e
d
X_μ
X_ν
X_ν^+
X_μ^+
$\sum_{de\mu\nu}$
=
$\sum_{df\sigma\lambda}$
f
X_λ
X_σ
X_σ^+
X_λ^+

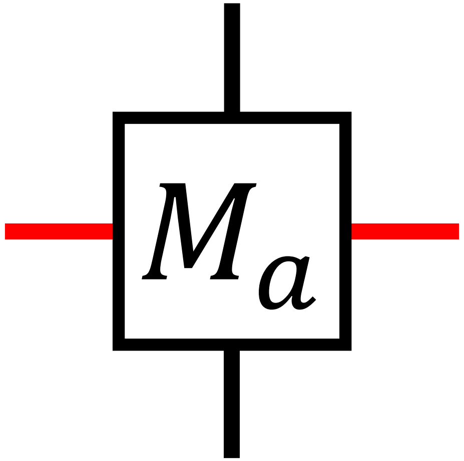


a
b
c
e
d
X_μ
X_ν
=
$\sum_{d' f\sigma\lambda}$
f
d'
X_λ
X_σ
X_σ^+
X_λ^+

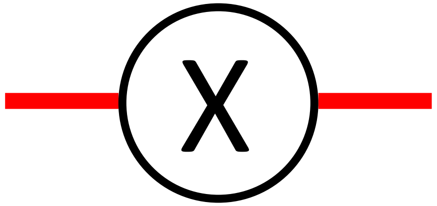


a
b
c
e
d
X_μ
X_ν
$\otimes$
=
$\sum_{f\sigma\lambda}$
f
X_λ
X_σ
X_σ^+
X_λ^+

Fusion category $\mathcal{C}$



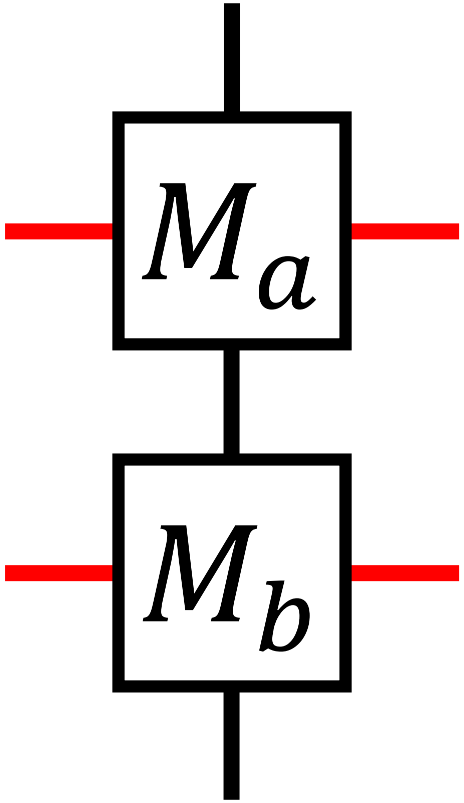
Objects

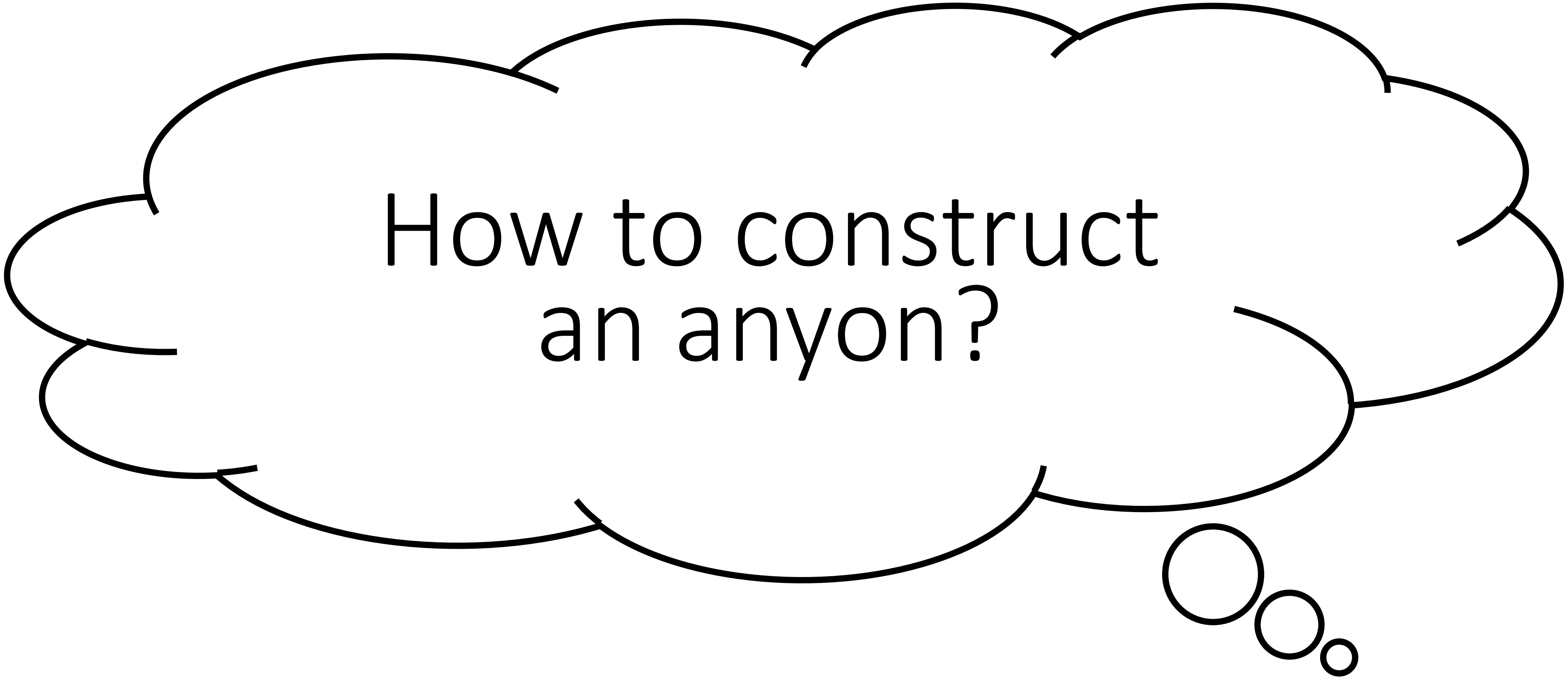


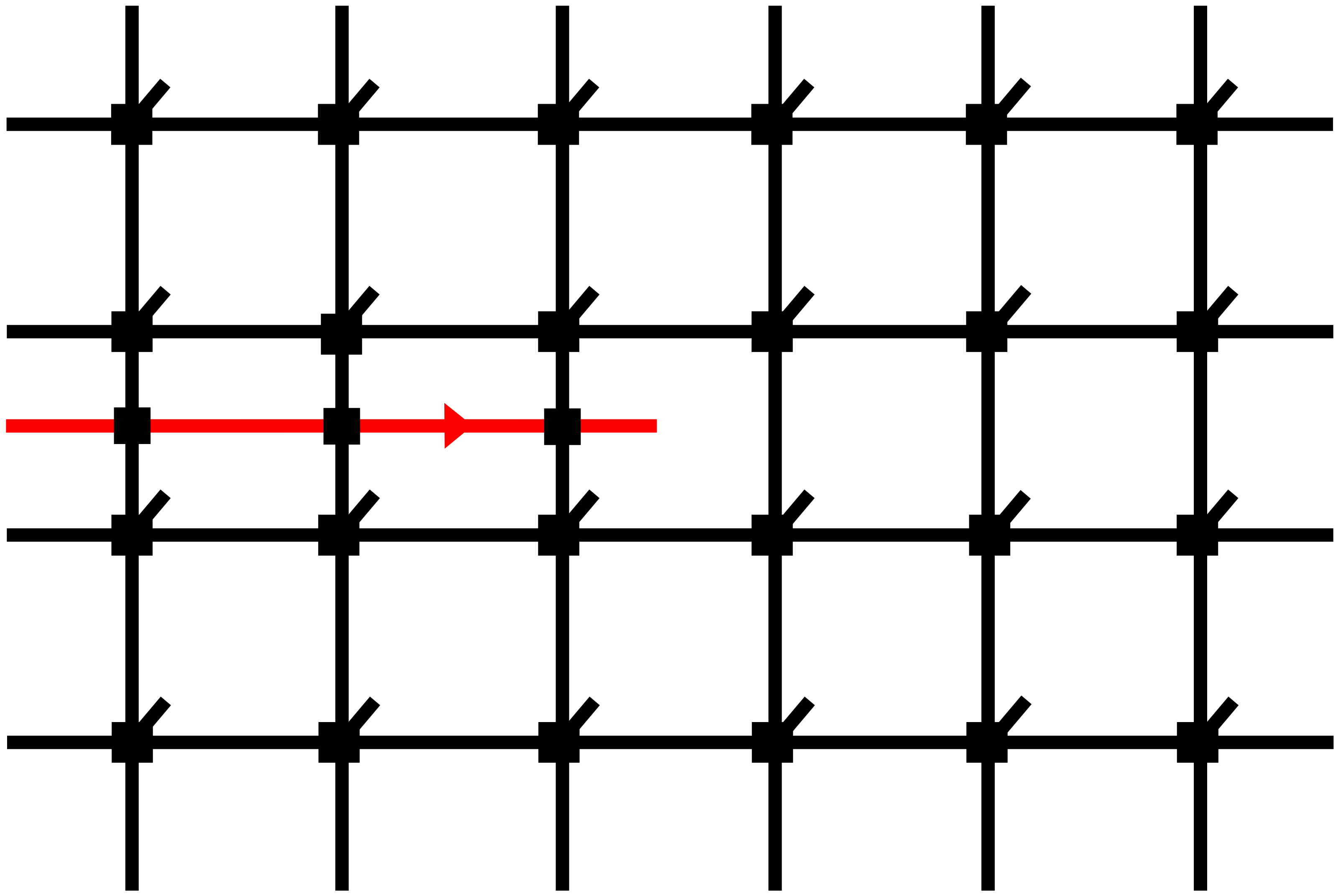
morphisms

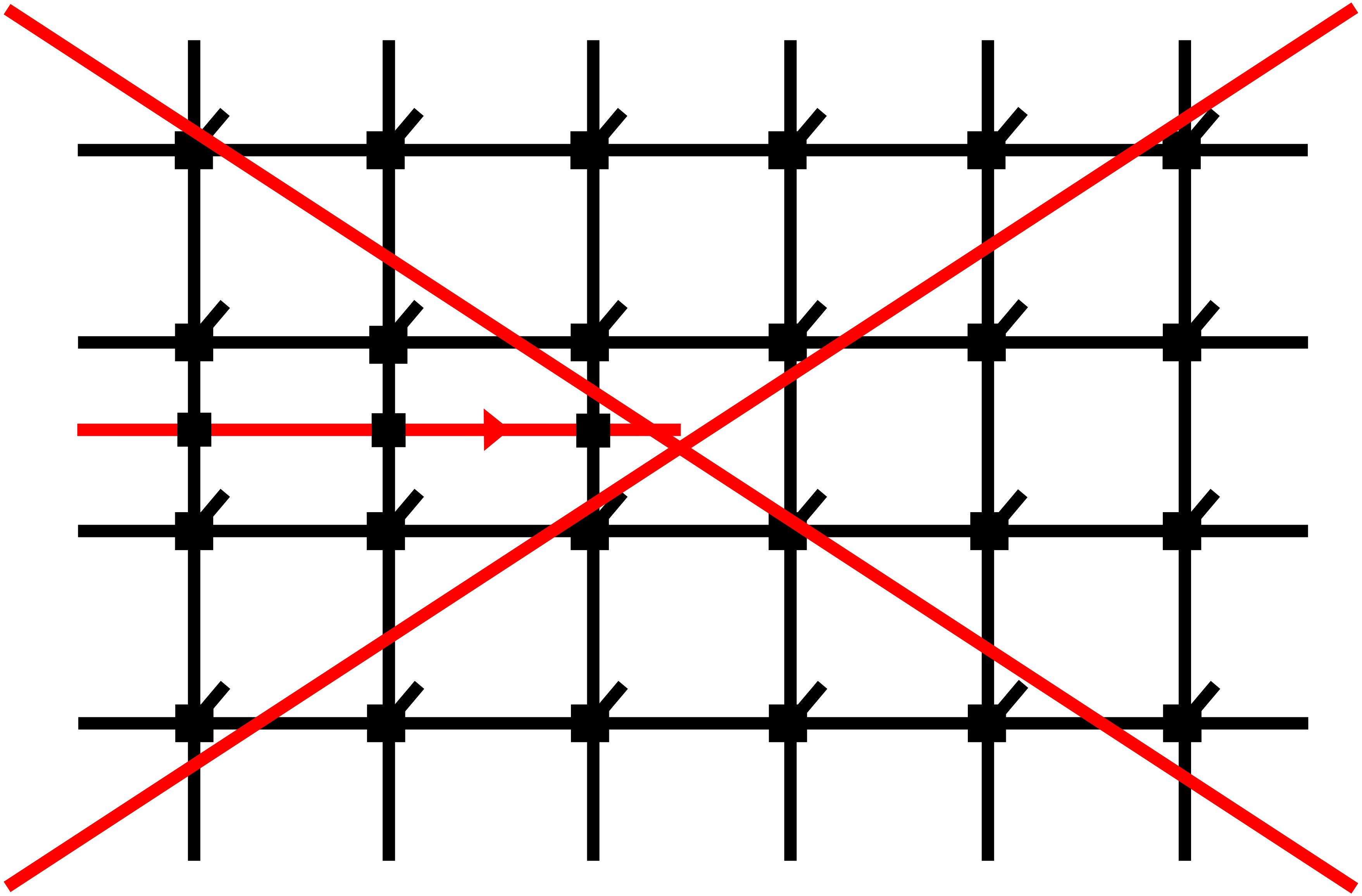
Tensor product

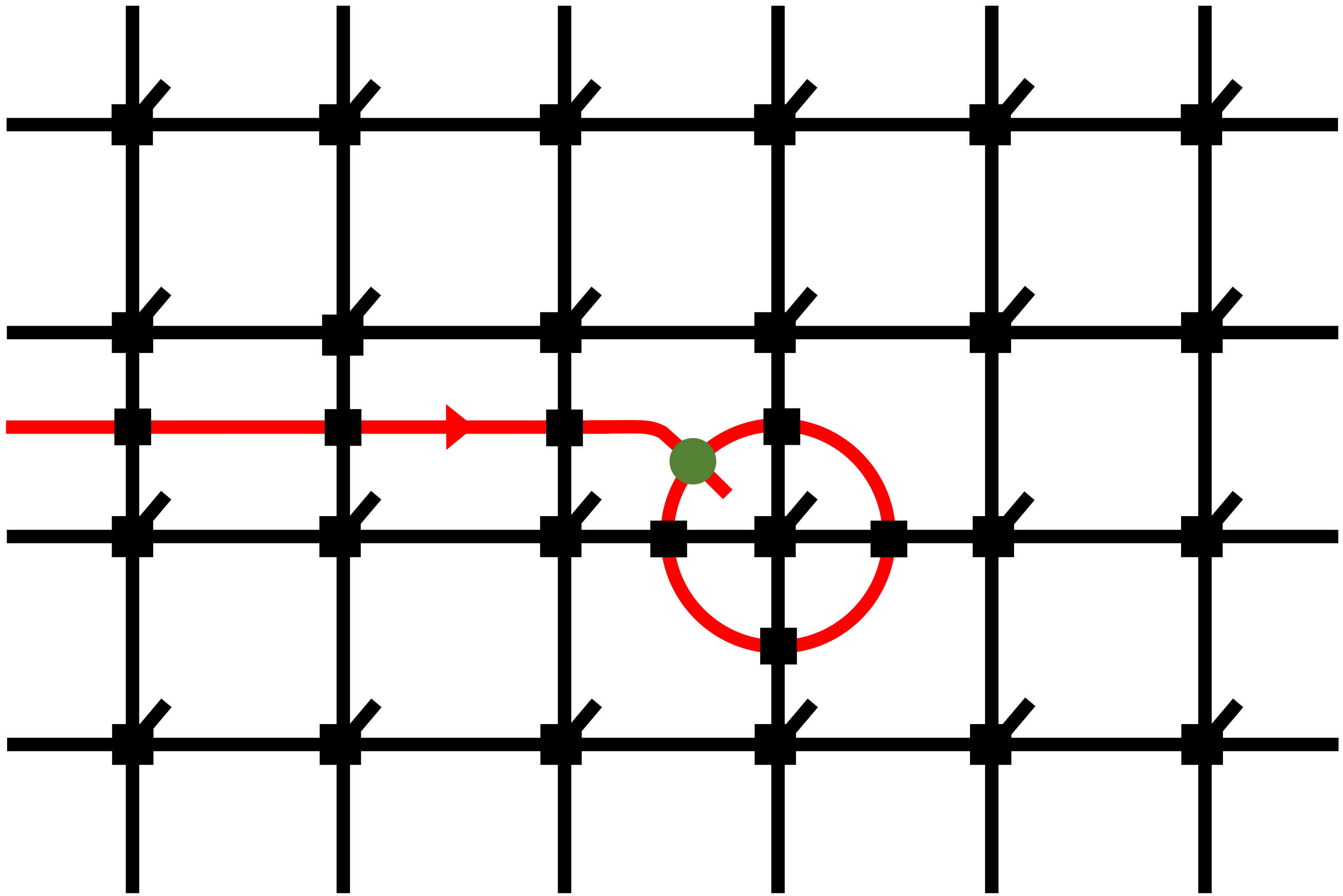
$a \otimes b$

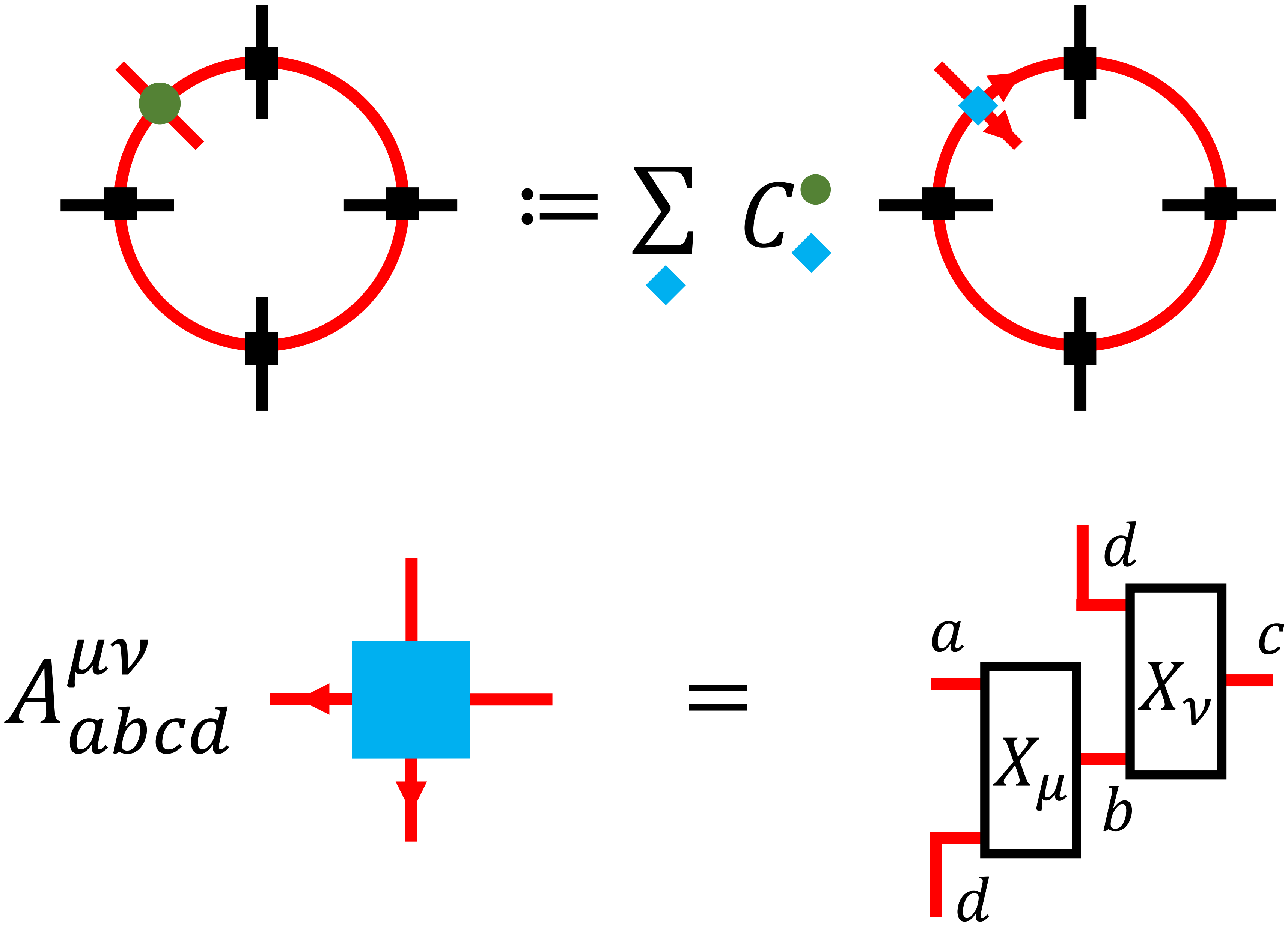


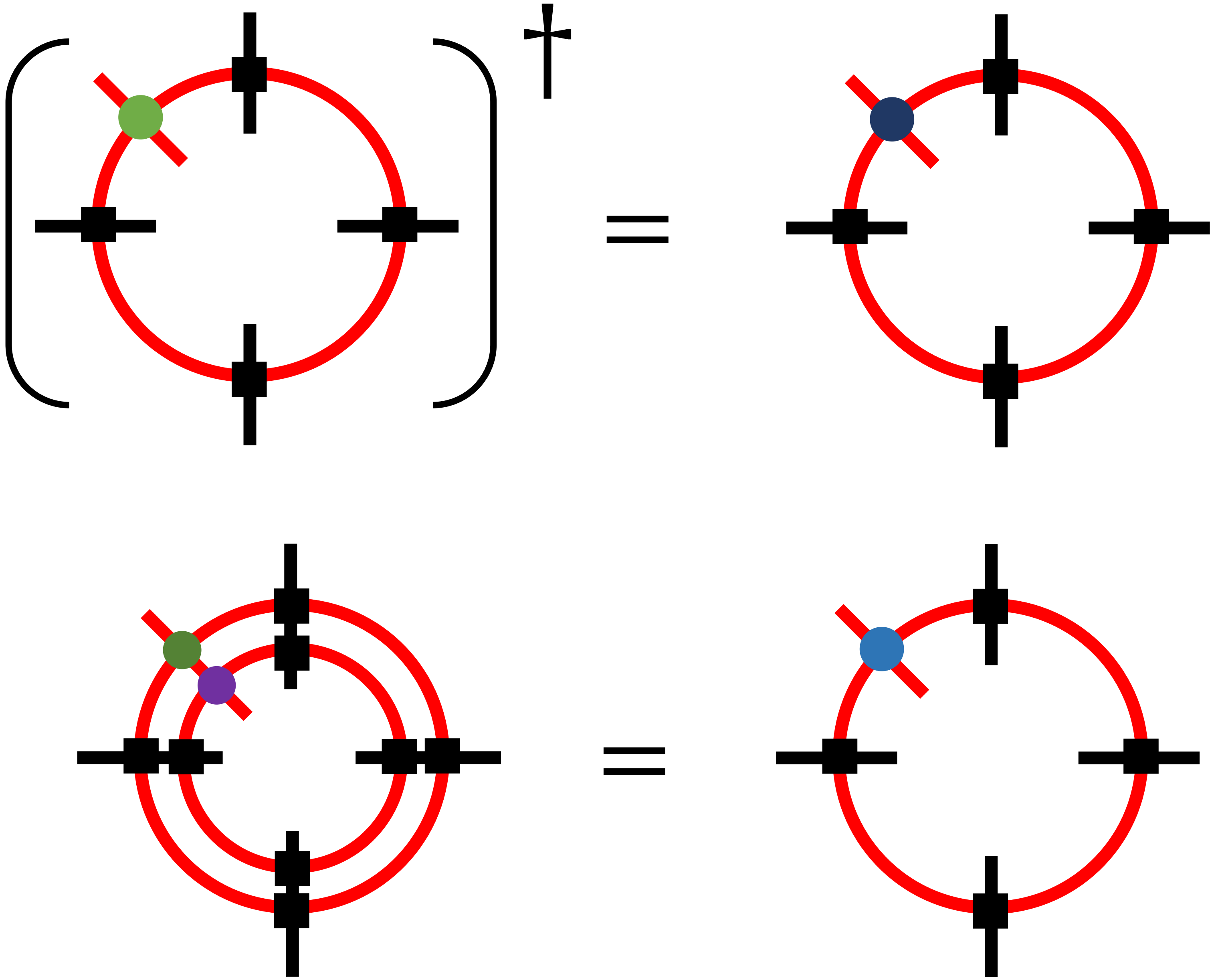

How to construct
an anyon?





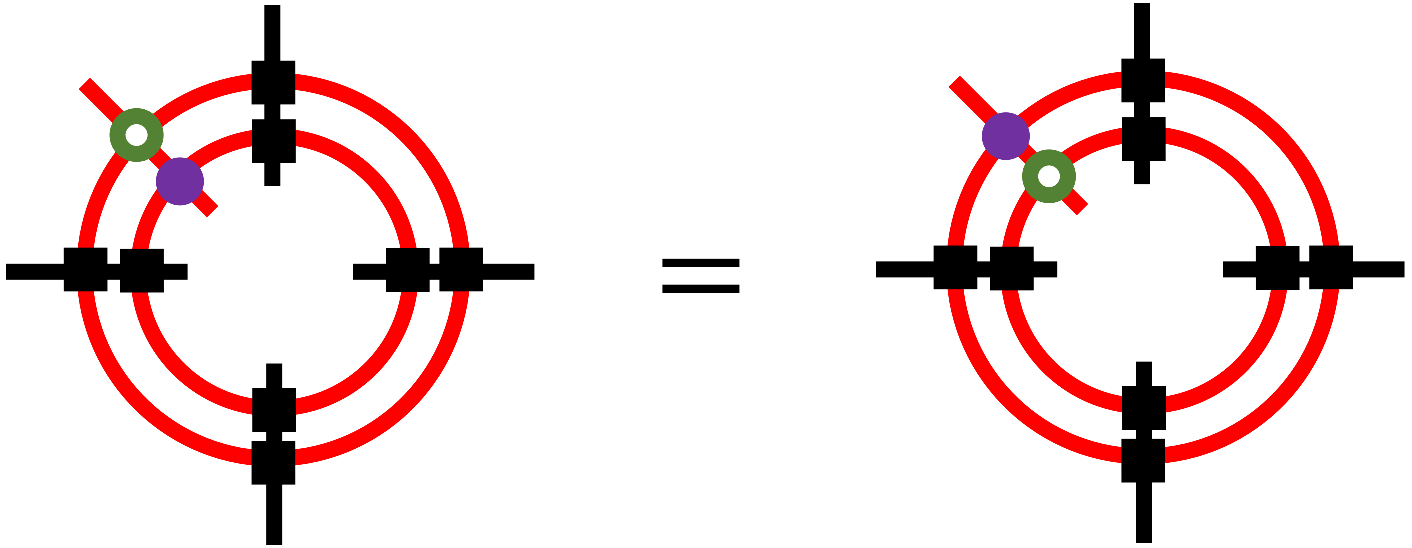



:= Σ C
A
μν
abcd
=
d
a
c
X
ν
X
μ
b
d

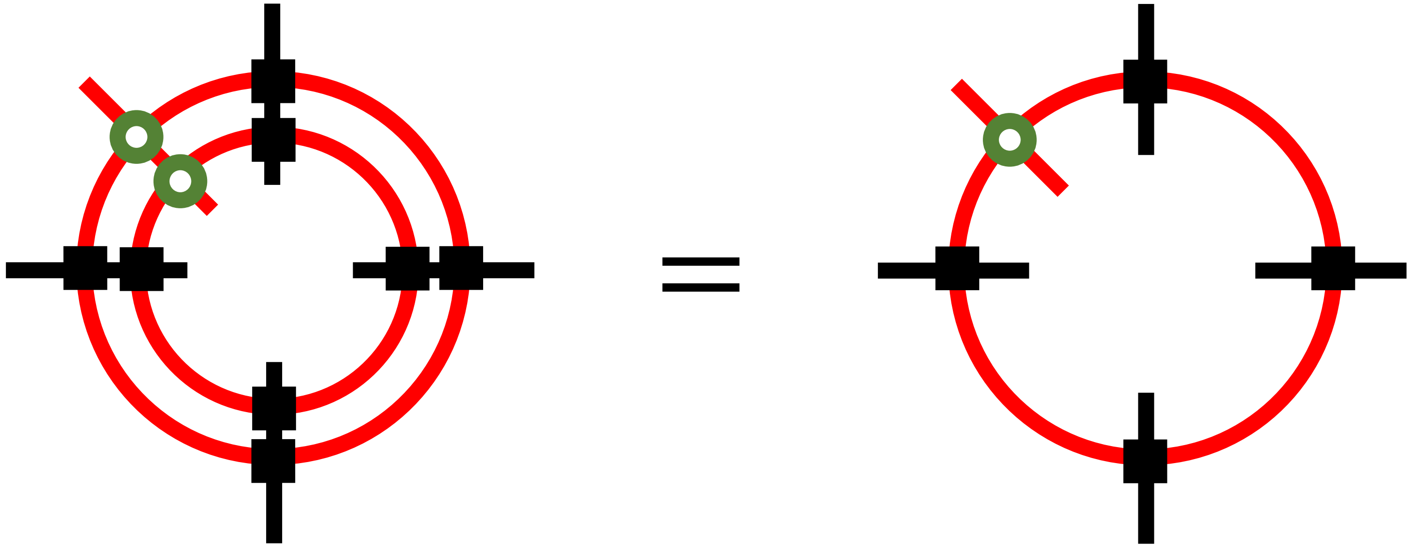


*

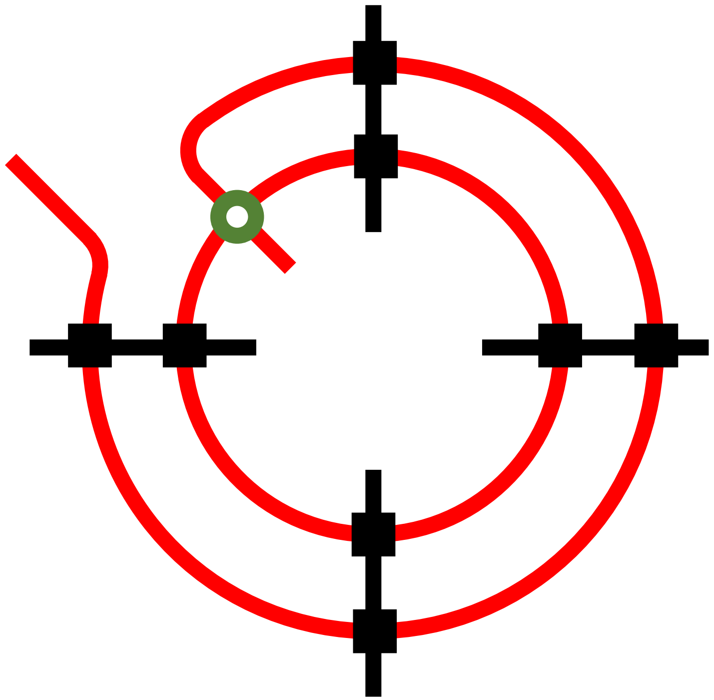
Algebra



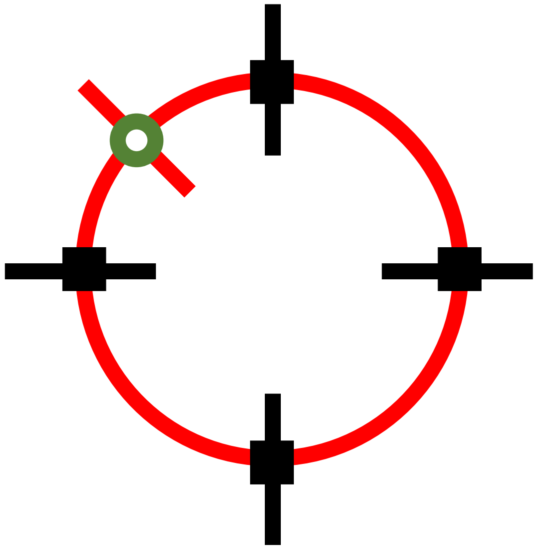
Central

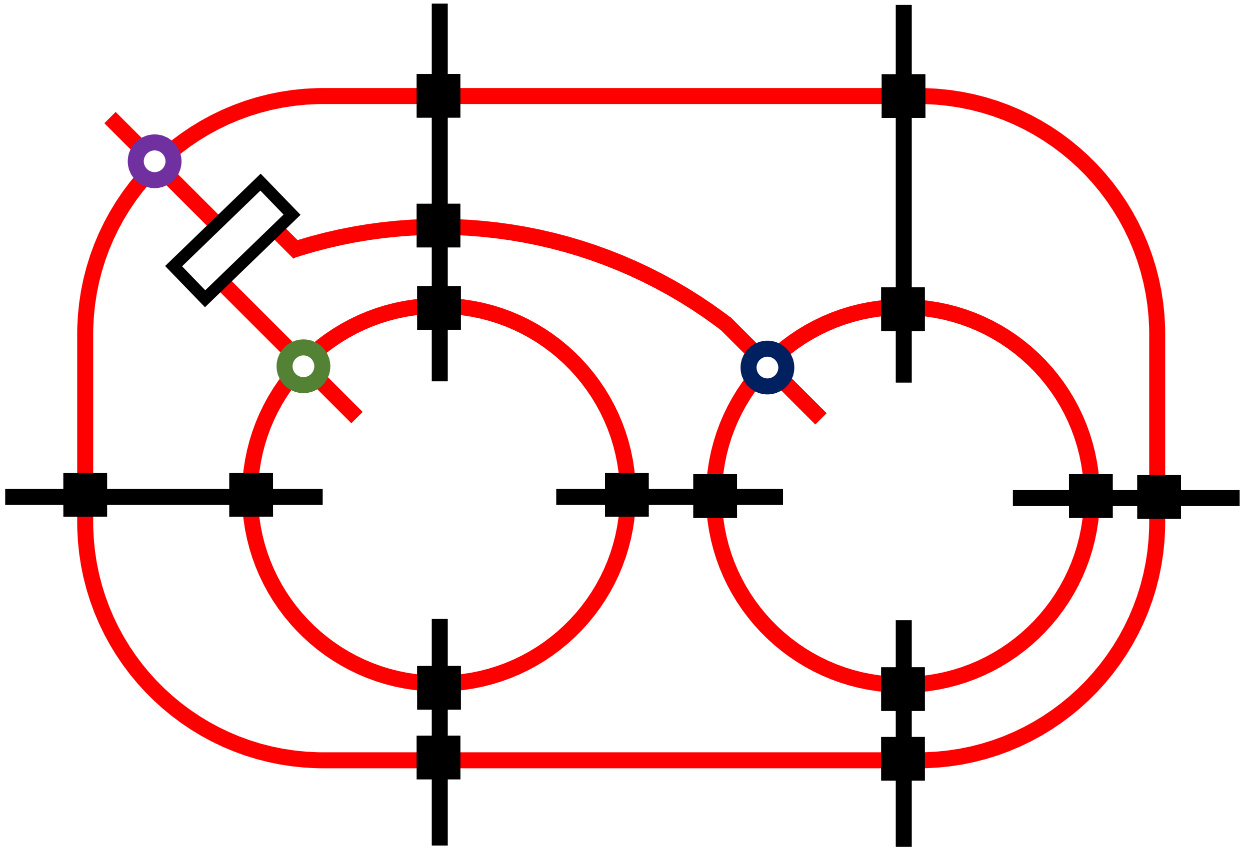


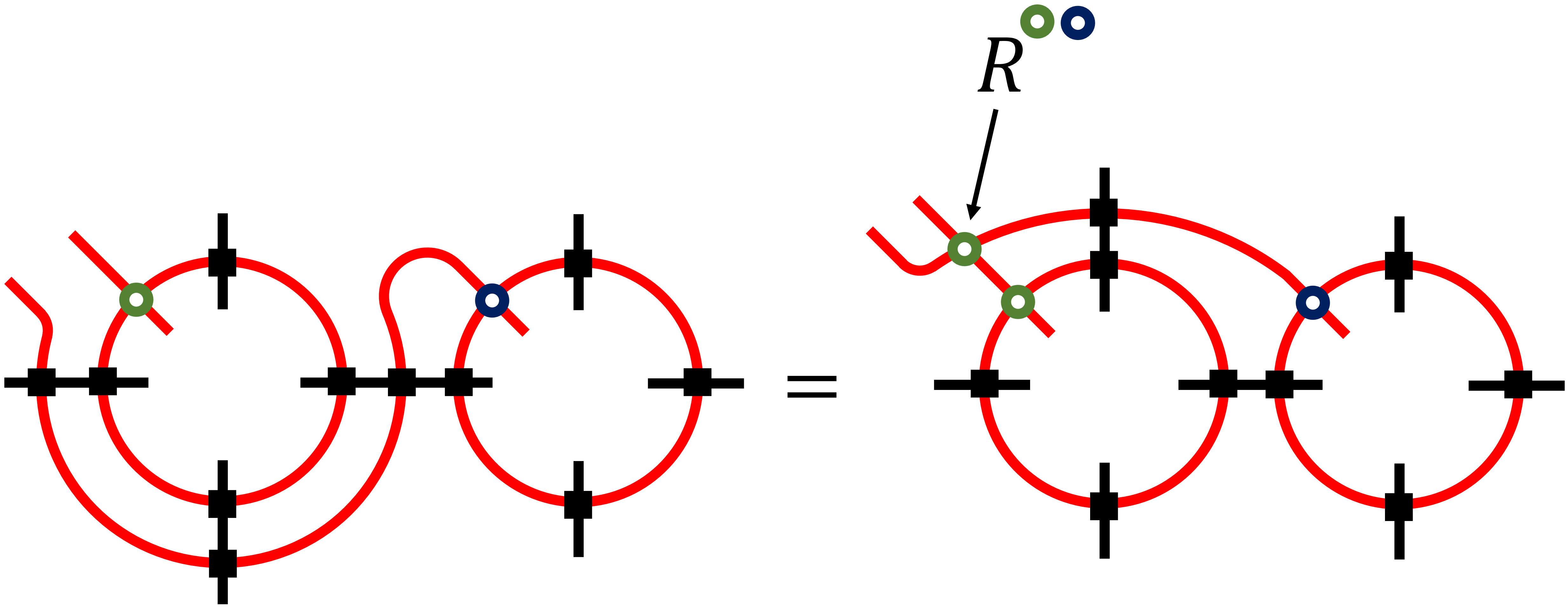
Idempotent



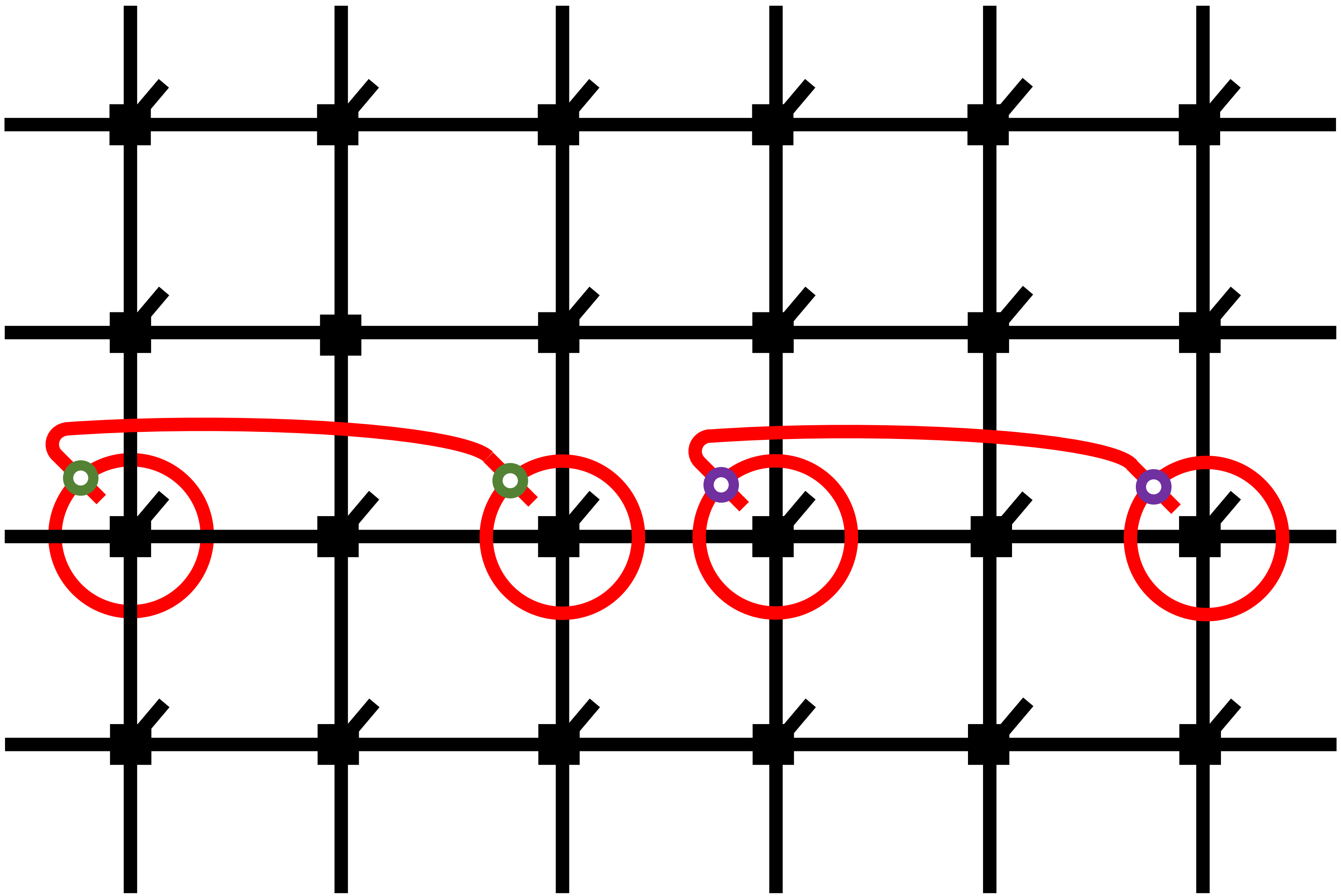
$$= e^{i2\pi h_i}$$

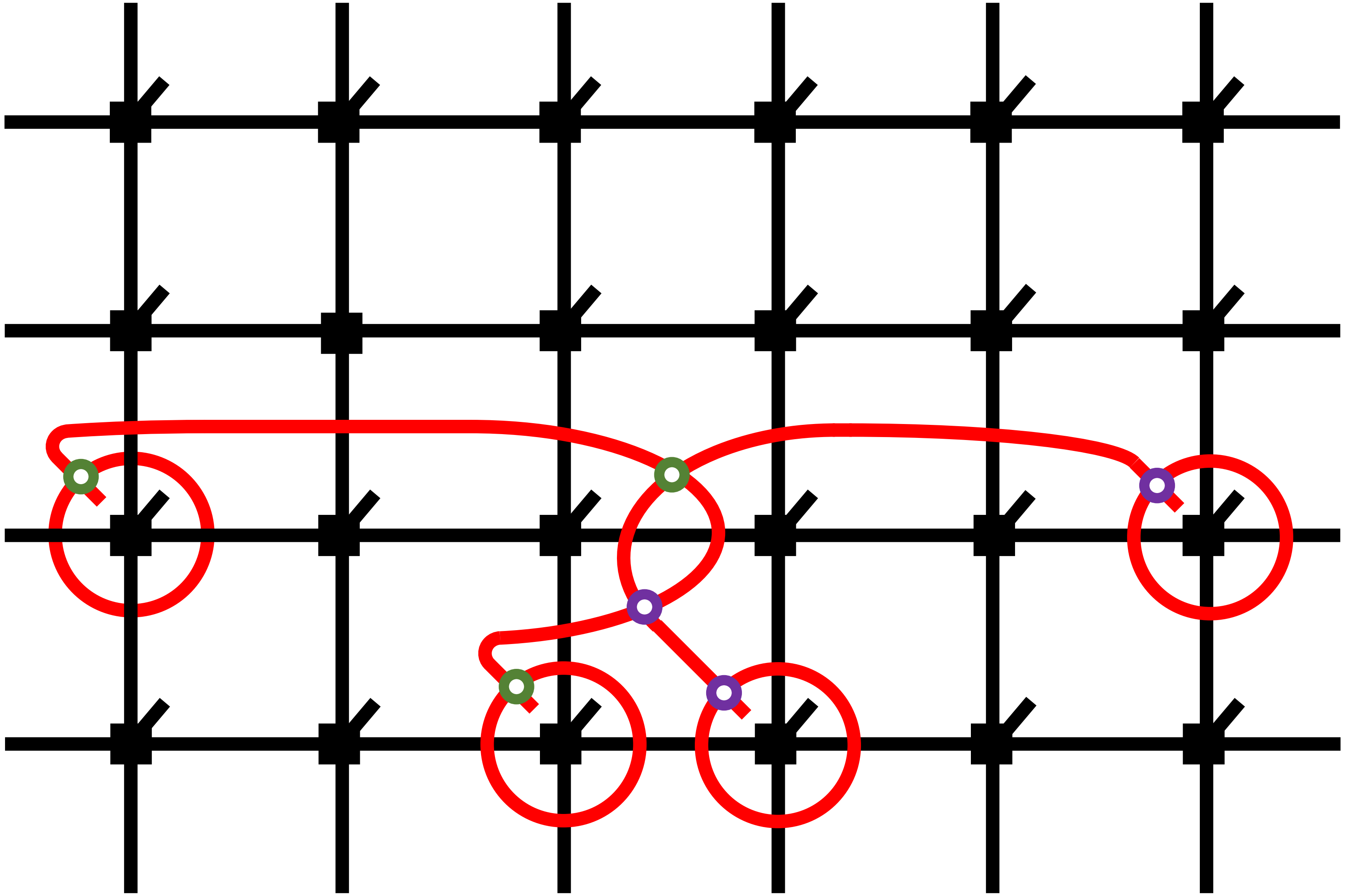


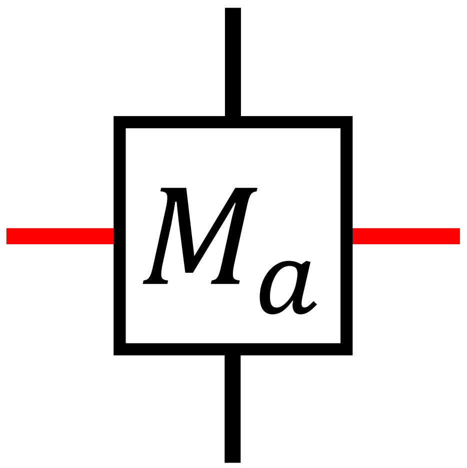




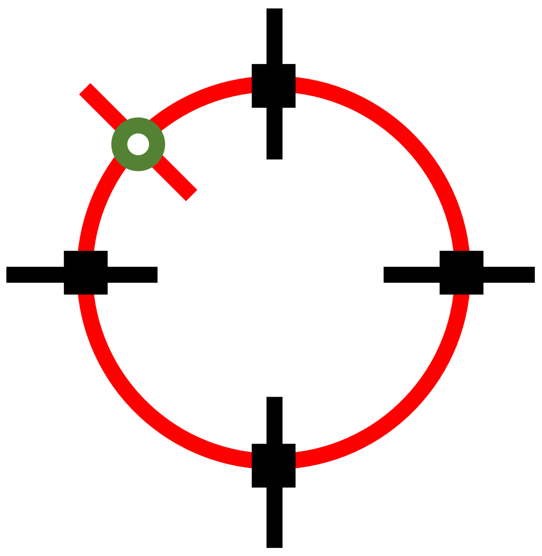
R





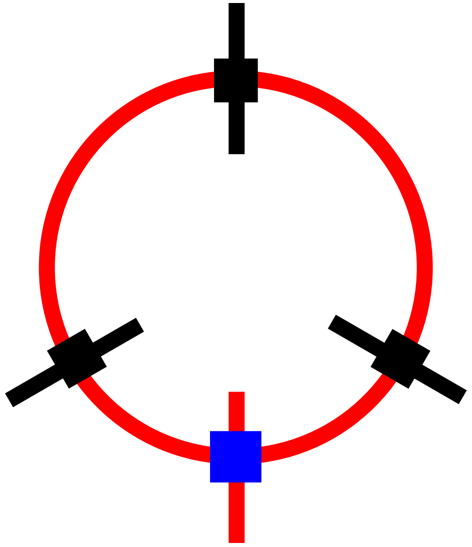


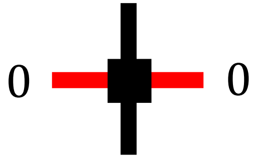
$: \mathcal{C}$

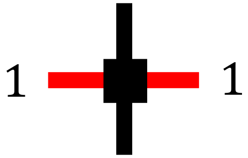


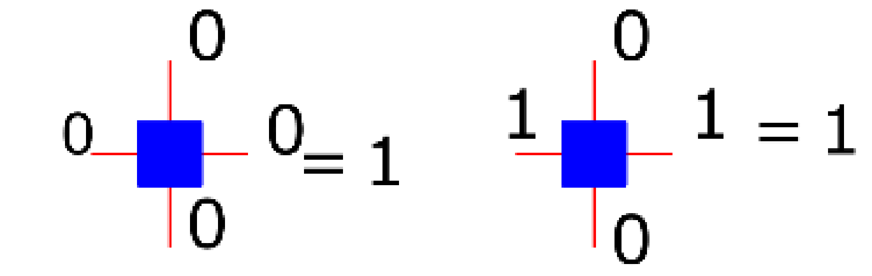
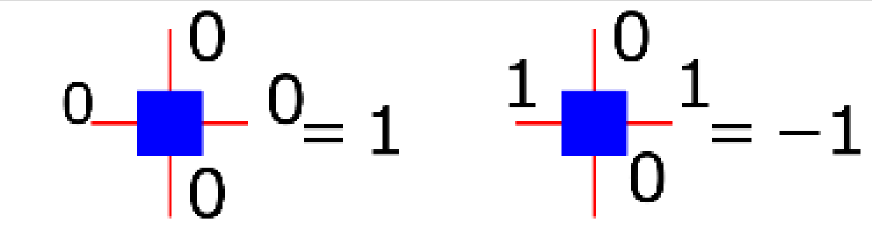
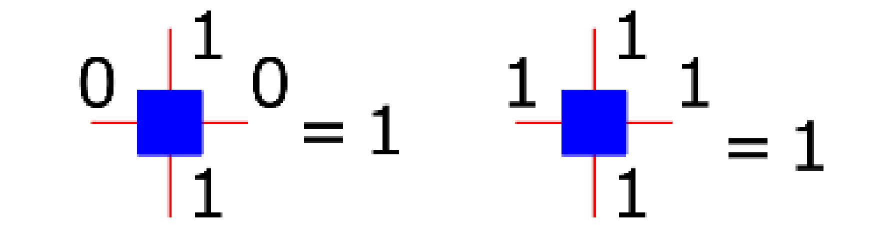
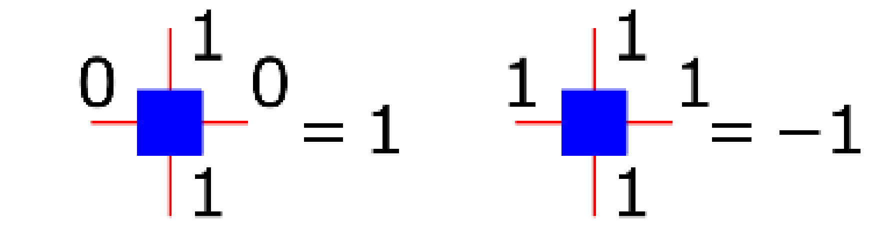
$: Z(\mathcal{C})$

Toric Code



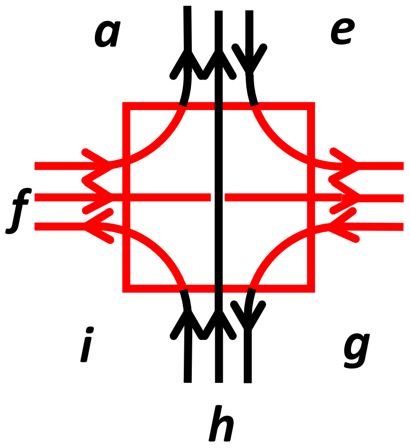
 $= I$

 $= X$

1	
e	
m	
em	

Double Fibonacci

$$\tau \times \tau = 1 + \tau$$



$$= F^{haf}_{g;ei}$$

$$A_{1111}, A_{\tau\tau\tau 1}, A_{1\tau 1\tau}, A_{1\tau\tau\tau}, A_{\tau 1\tau\tau}, A_{\tau\tau 1\tau}, A_{\tau\tau\tau\tau}$$

$$(1,1) = \frac{1}{\sqrt{5}}\left(\frac{1}{\phi}A_{1111} + \sqrt{\phi}A_{1\tau 1\tau}\right)$$

$$(1,\bar{\tau}) = \frac{1}{\sqrt{5}}\left(\frac{1}{\phi}A_{\tau\tau\tau 1} + \frac{1}{\sqrt{\phi}}e^{-\frac{4\pi i}{5}}A_{\tau 1\tau\tau} + e^{\frac{3\pi i}{5}}A_{\tau\tau\tau\tau}\right)$$

$$h_{(1,\bar{\tau})} = -4/5$$

$$(\tau,1) = \frac{1}{\sqrt{5}}\left(\frac{1}{\phi}A_{\tau\tau\tau 1} + \frac{1}{\sqrt{\phi}}e^{\frac{4\pi i}{5}}A_{\tau 1\tau\tau} + e^{-\frac{3\pi i}{5}}A_{\tau\tau\tau\tau}\right)$$

$$h_{(\tau,1)} = 4/5$$

$$(\tau,\bar{\tau}) = \frac{1}{\sqrt{5}}\left(\phi A_{1111} + A_{\tau\tau\tau 1} - \sqrt{\phi}A_{1\tau 1\tau} + \sqrt{\phi}A_{\tau 1\tau\tau} + \frac{1}{\phi}A_{\tau\tau\tau\tau}\right)$$

Future Directions

- Fermions
- SET & transversal gates
- Topological Quantum Computation
- Domain Walls
- ≥ 3 Dimensions

- Tensor networks with topological order are not generic

- By studying their entanglement structure we extract a fusion algebra

- From this we can construct a tube algebra to extract (all) information about the topological order in the system