Symmetry protected topological order at nonzero temperature

Sam Roberts[1], Beni Yoshida[2],
Aleksander Kubica[3], Stephen Bartlett[1]

[1]University of Sydney, [2]Perimeter institute, [3]Caltech

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THE UNIVERSITY OF SYDNEY

ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

Motivation: phases of matter

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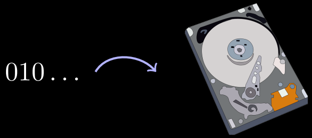

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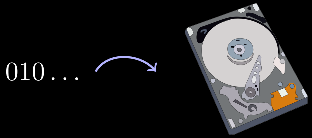

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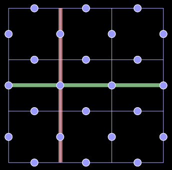

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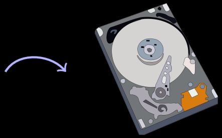

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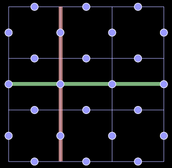

- Finding models with topological order at $T > 0$ is an important problem

The question

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2. First result: thermal instability of a class of SPT models

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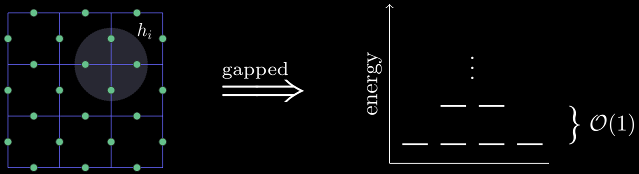
1. Introduction: what are (symmetry protected) topological phases?
2. First result: thermal instability of a class of SPT models
3. Second result: existence of thermal SPT order

What are topological phases of matter?

- Gapped Hamiltonian $H = \sum_i h_i$ with (geometrically) local terms



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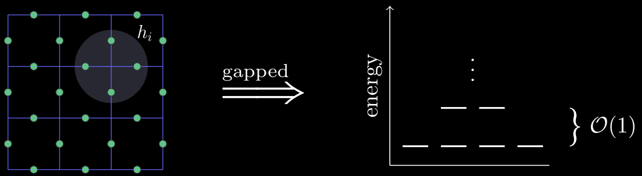

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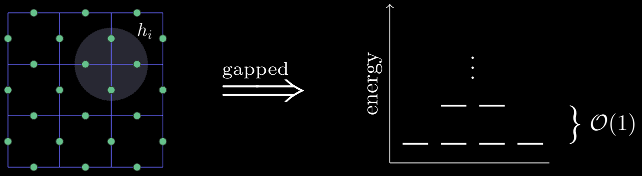

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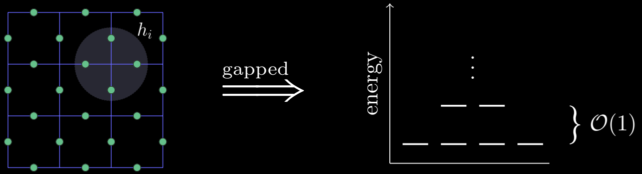

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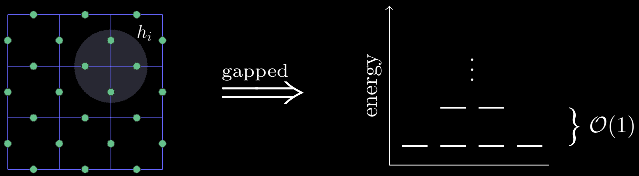

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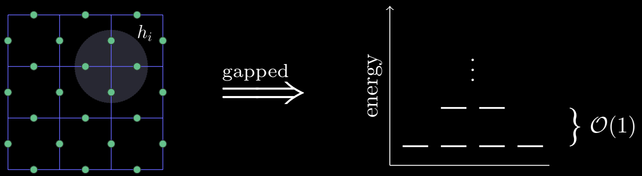

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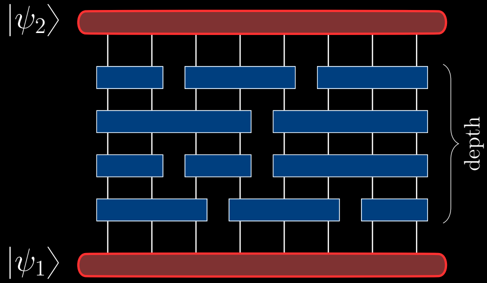
  3. Robust to local errors
  4. Often ground space degeneracy depends on boundary conditions (e.g. genus of surface)

Ground states of topologically ordered systems

Def: $|\psi_1\rangle \sim |\psi_2\rangle$ (belong to same phase) iff they are related by a constant depth unitary circuit



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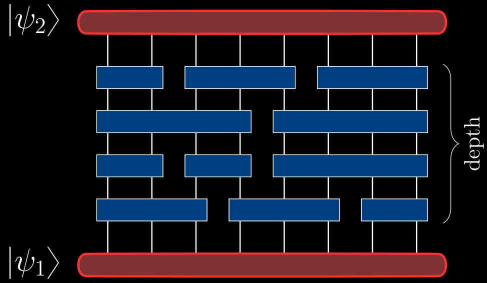

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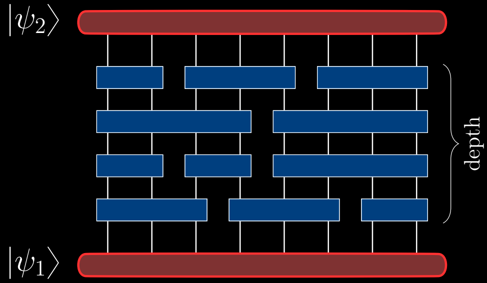

- Trivial phase = equivalence class of a product state
- Topologically ordered $\implies$ not equivalent to a product state.

Symmetry protected topological order: states

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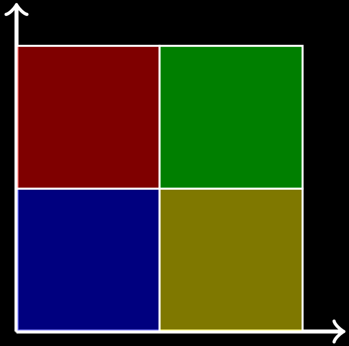

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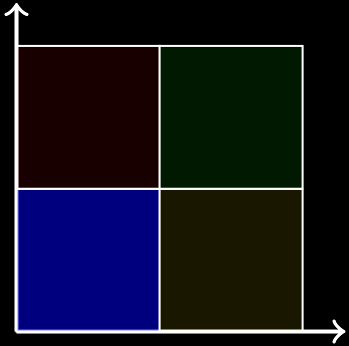

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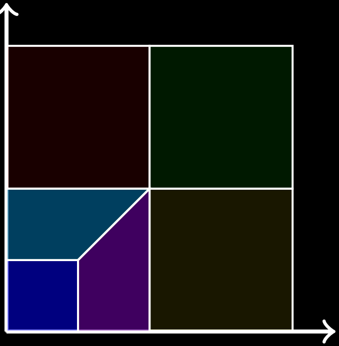

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Example: SPT order in 1D

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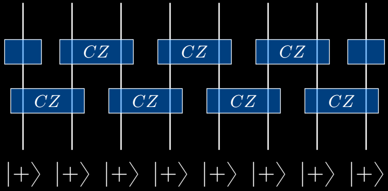

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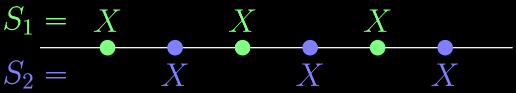

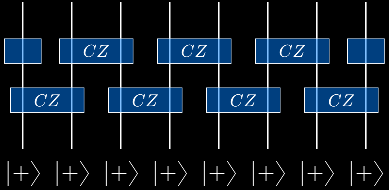

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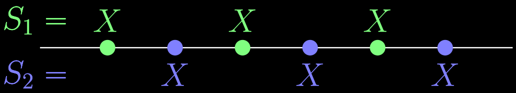

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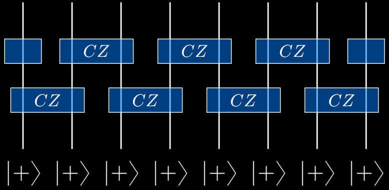

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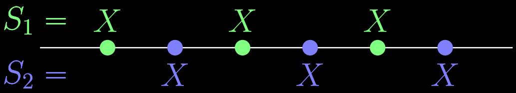

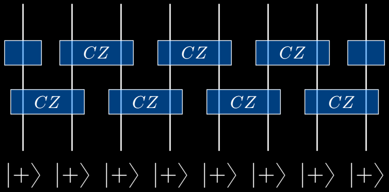

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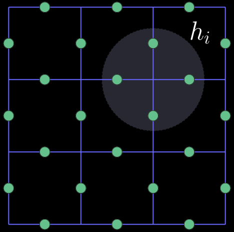
> **Def** $|\psi\rangle$ is SPT ordered if no symmetric constant depth circuit can map it to a product state, unless the symmetry is broken

Generalized SPT models in d-dimensions

- A broad class of SPT models in d dimensions are the so-called group cohomology models of Chen-Gu-Liu-Wen 13

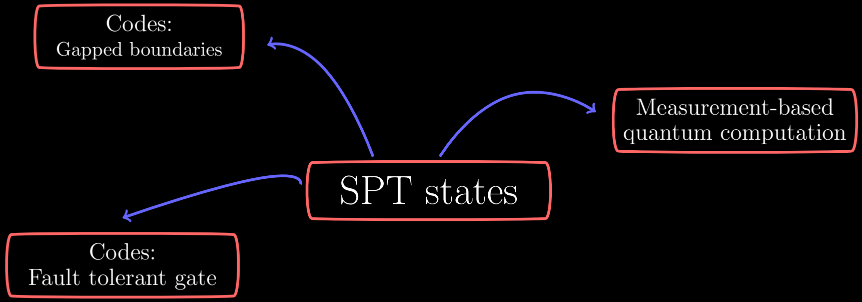
$$H = \sum_v h_v, \quad [h_v, h_w] = 0$$



- Has a global symmetry that acts *onsite*

$$S(g) = \prod_{\text{sites}} u(g), \qquad [S(g), H] = 0, \qquad g \in G$$

Applications of SPT order



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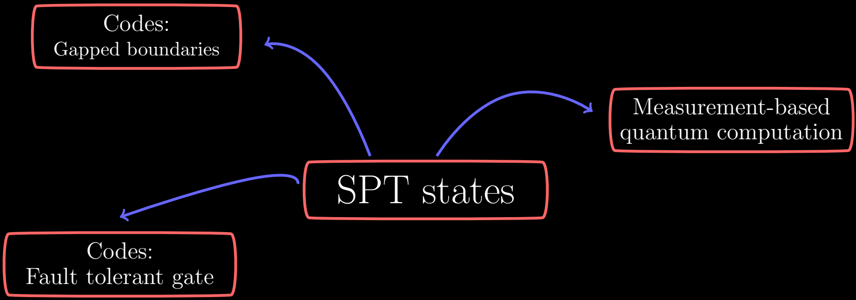

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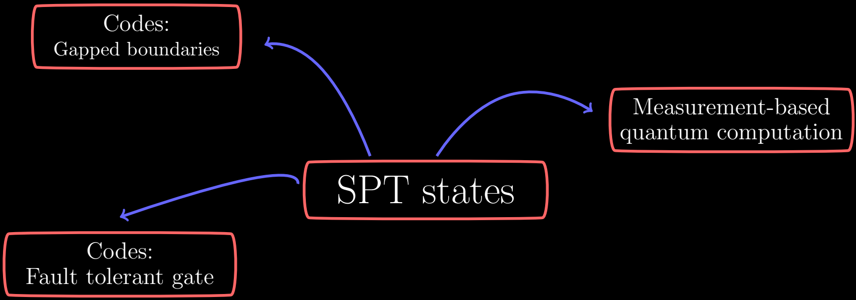

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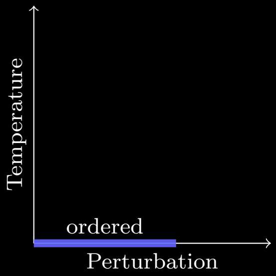
	No sym	Symmetry
$T = 0$	$2D$ toric code	$1D$ cluster
$T > 0$	$4D$ toric code	Our work

The problem

- Do any of the ground state properties of an SPT ordered system survive at nonzero temperature?



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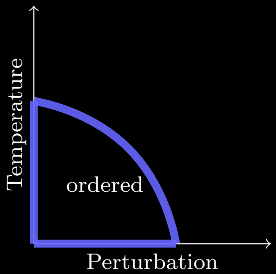

$\Longrightarrow$ Thermal resources for MBQC, stable domain walls at $T \geqslant 0, \ldots$

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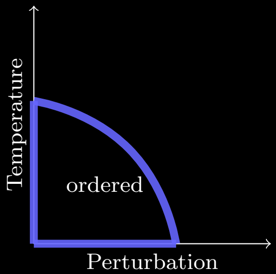

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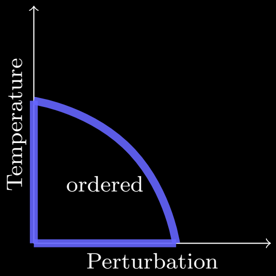

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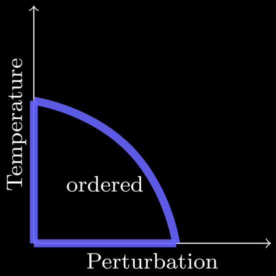

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1. We rule out thermal stability of a large class of SPT models.
2. Prove thermal SPT ordering of the 3D cluster model
   - Computational aspects of this ordering

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Def We say ρ is (r, ϵ) SPT-trivial if

$$\left\| \rho - \mathrm{Tr}_{\mathcal{H}'} \left(U \rho_{cl} U^\dagger \right) \right\|_1 < \epsilon,$$

- ρ_{cl} is the Gibbs state of a classical Hamiltonian on an enlarged space
- U is a symmetric circuit of depth r
- $\mathcal{H}'$ is the ancillary space

First result: instability of global onsite models

Result 1: Theorem: For any $T > 0$, SPT models protected by global *onsite* symmetries are not thermally robust, i.e, they are (r, ϵ) SPT-trivial for

- $r = \mathcal{O}(\log^{\frac{d+1}{d}}(L))$
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$\rightarrow$ Proof for the class of models described by group cohomology

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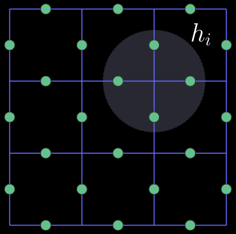

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  Hastings 11, Siva-Yoshida 16

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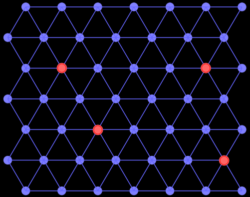
Hamiltonian $H = \sum_v h_v$ → Remove terms with probability p_β → Imperfect Hamiltonian $H(p_\beta)$

$$p_\beta = \frac{2}{1+e^{2\beta}}$$

- Ground space of $H(p_\beta)$ approximates the Gibbs state of H up to $\text{poly}^{-1}(L)$ error



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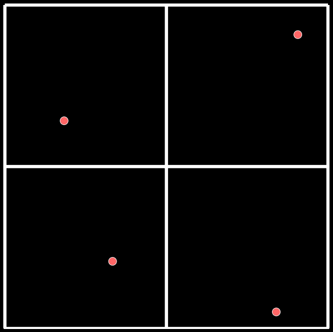

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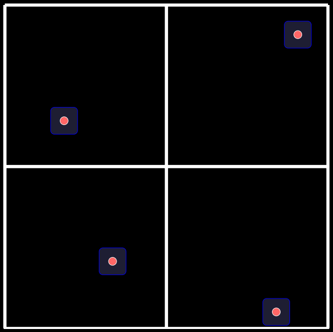

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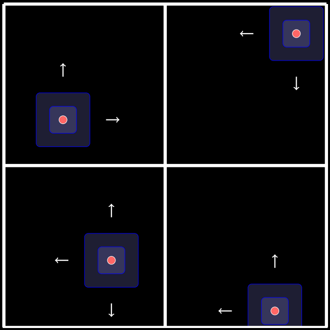

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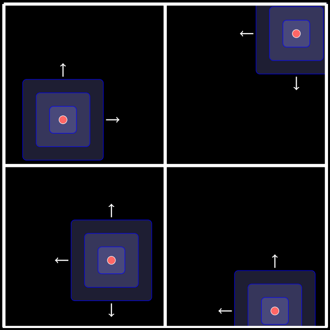

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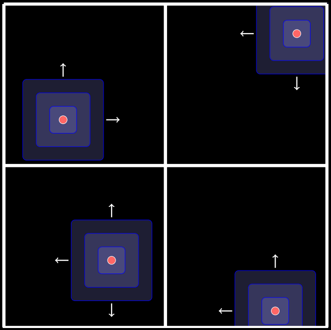

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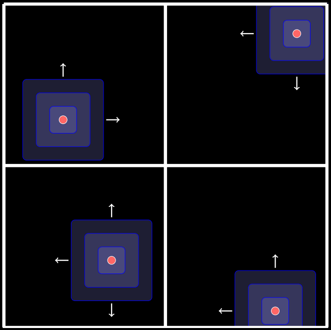

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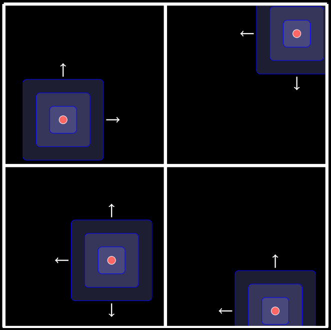
First result: Thermal triviality

- Second technical tool: local disentangler



- Can construct a symmetric disentangler near each missing term, e.g. for qubits

$$\mathcal{D}_v : h_v \mapsto X_v$$

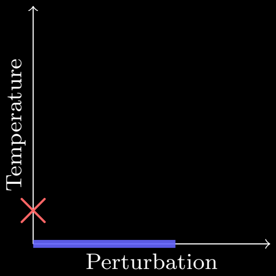
...and continue:

$$\mathcal{D} : h_{v'} \mapsto X_{v'}$$

- High probability of a missing term in each $\log^{\frac{1}{2}}(L) \times \log^{\frac{1}{2}}(L)$ region
- $\mathcal{O}(\log(L))$ spins to disentangle with gates of range $\leqslant \mathcal{O}(\log^{\frac{1}{2}}(L))$
- This gives a low-depth preparation of the Gibbs ensemble.

Instability of SPT models

- The group cohomology models completely classify SPT phases that are protected by global onsite symmetries in $d = 1, 2$



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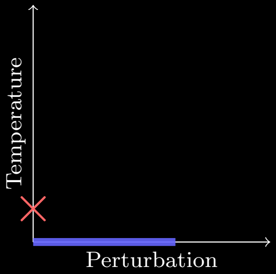

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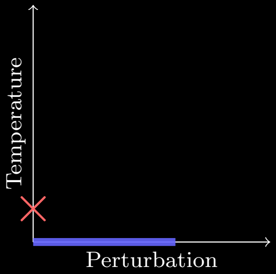

- Instability of the associated computational structures at $T > 0$?
- Beyond group cohomology?

Second result

- The existence of thermally stable SPT order

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Result 2: The Raussendorf-Bravyi-Harrington (RBH) cluster model in 3D belongs to a thermally stable SPT phase for $0 \leqslant T < T_c$

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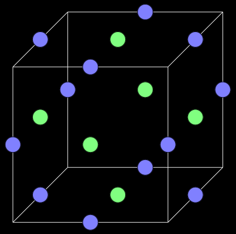
- Underpins the fault-tolerant, topological measurement based scheme of Raussendorf-Harrington-Goyal 06



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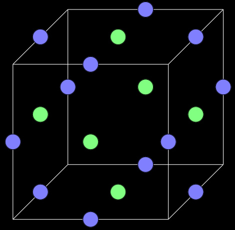

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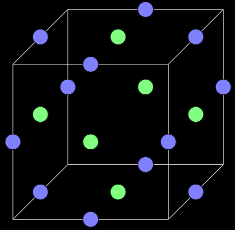

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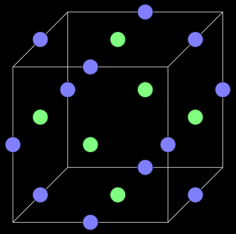

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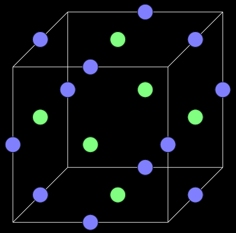

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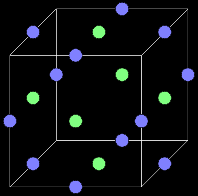
  $\Longrightarrow$ Lets explore in the context of SPT phases!

The Raussendorf-Bravyi-Harrington (RBH) model

- Cubic lattice with qubits on edges and faces - RBH 05

$$H_{\mathcal{C}} = -\sum_{u} K_u$$



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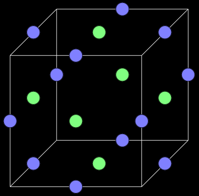

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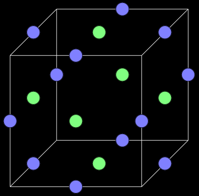

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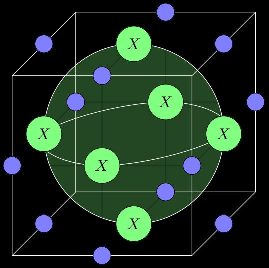
- Constant depth preparation: $\left|\psi_{\mathcal{C}}\right\rangle = \prod_{\langle u,w\rangle} CZ_{u,w} \left|+\right\rangle^N$

Generalized symmetries

- Generalized symmetry: $\mathbb{Z}_2 \times \mathbb{Z}_2$ 1-form symmetry.

$$S_{\mathcal{M}}(g) = \prod_{u \in \mathcal{M}} X_u, \qquad \mathcal{M} \text{ a 2-dim surface}$$



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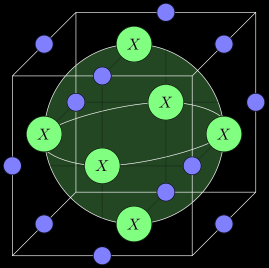

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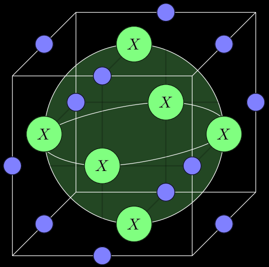

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- A symmetry for each sublattice
- Operators naturally arise in error correction for the topological MBQC scheme

Thermal SPT in the RBH model

Result 2: There exists a temperature T_c such that the Gibbs state of the RBH model is SPT ordered under this 1-form symmetry for $0 \leqslant T < T_c$.

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Thermal SPT in the RBH model

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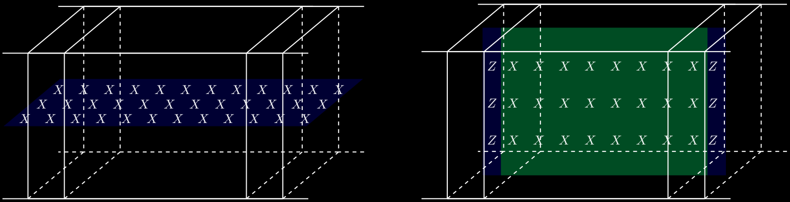
- Two ways of proving this:
  1. Explicit order parameters
     $\implies$ Measurement based quantum computation and error correction
  2. Gauging the model
     $\implies$ Domain wall in quantum error correcting code

Sheet order parameter

- Sheet order parameters: symmetry operators with ‘twisted boundaries’



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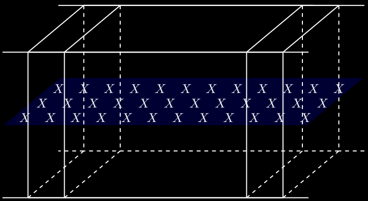

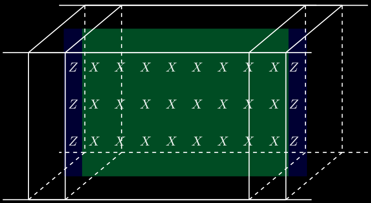

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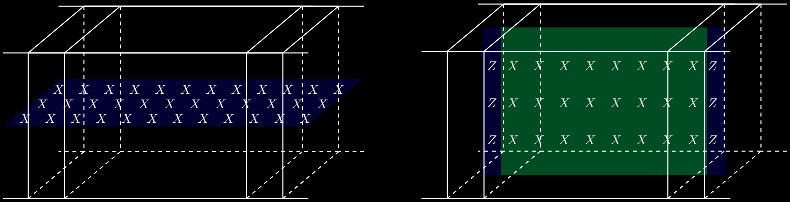

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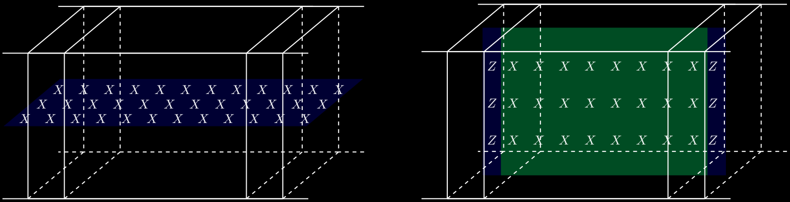


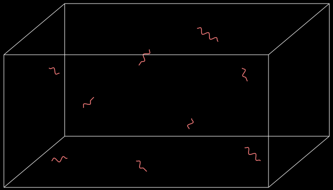
- If ρ_{triv} is (r, ϵ)-trivial with $r < L/2$, then the expectation value of these membrane operators is *small*
- Compare with

$$\langle \overline{XX} \rangle + \langle \overline{ZZ} \rangle \leqslant 1$$

for product states

Sheet order parameter

- Error correction can maximize the expectation value of the RBH thermal state with these membranes



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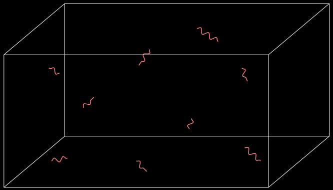

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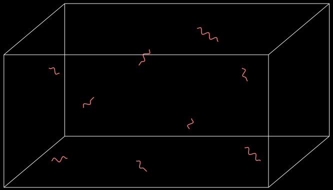

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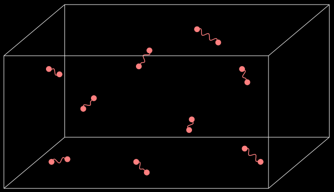

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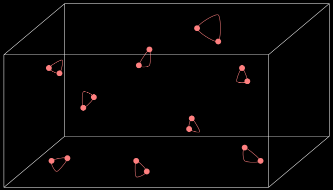

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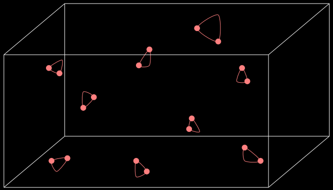

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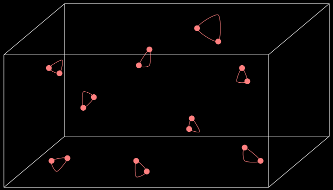

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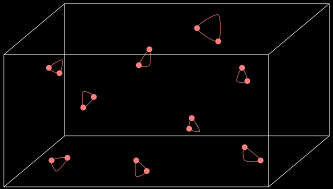

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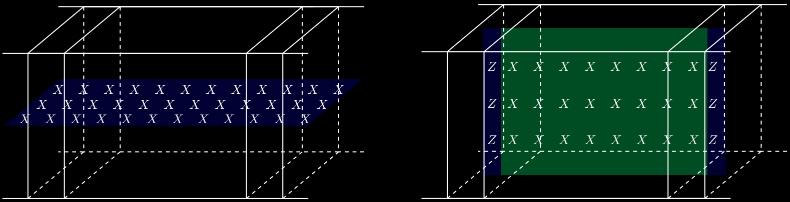
- Closed loops that are boundaries commute with membrane operators!
- This protocol succeeds below T_c due to string tension of excitations

Operational features

- Operationally: sheet order parameter quantifies the ability to distil maximally entangled pairs (encoded in toric codes) using single qubit measurements



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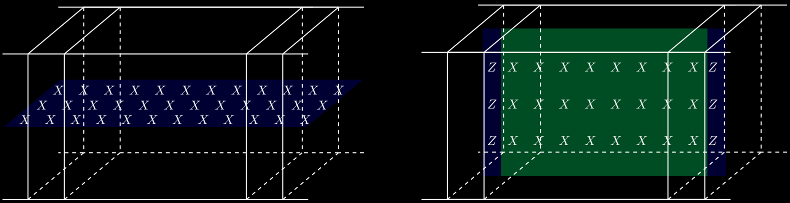

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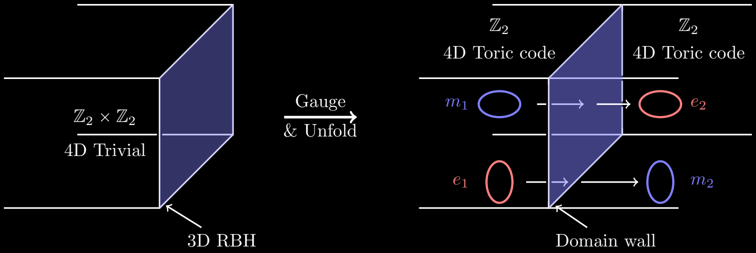
- Definition of SPT is protocol independent as one can use optimal decoder i.e. maximum likelihood decoding

Briefly: Generalized gauging

- Can define a 4D system with boundary, that is 1-form symmetric

$$H = H^{4D}_{\text{bulk}} + H^{3D}_{\text{boundary}}$$



- Gauging gives 4D toric code with domain wall:
  - Exchanges 1D loop-like electric and magnetic excitations

$$e_1 \leftrightarrow m_2 \qquad m_1 \leftrightarrow e_2$$

Conclusion: in this talk

1. Thermal fragility of SPT models protected by global onsite symmetries
2. Robustness of SPT in the 3D cluster scheme
3. Computational aspects (distilling entanglement, fault tolerant gates, error correction)

$\Longrightarrow$ Usefulness of SPT for measurement based quantum computation with 1-form symmetry

- Steps toward understanding what is possible: thermally stable computational phases of matter

Further questions

1. The relationship between thermal SPT non triviality and computational power (in MBQC)

   $\Longrightarrow$ Analogous to the question of thermal topological order and its relationship to self-correcting quantum memories

2. Interesting topological defects in 3D
3. Symmetry principles for the single-shot error correction in 3D gauge color code
4. More models: interplay with transversality, symmetry enriched topological phases