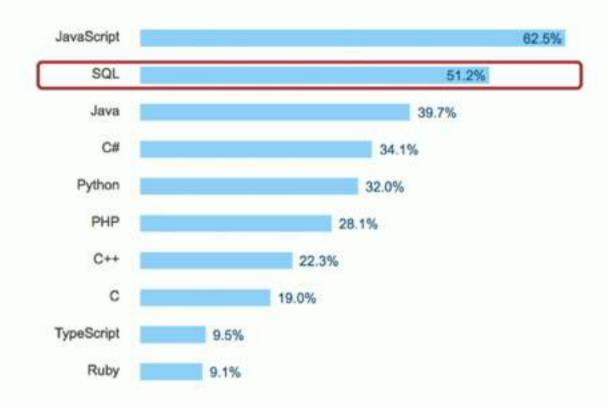
Automated Reasoning of Database Queries

Shumo Chu, University of Washington

with Konstantin Weitz, Chenglong Wang, Daniel Li, Brendan Murphy, Jared Roesch, Alvin Cheung, Dan Suciu



- SQL: a language supported by all relational databases
- Restricted abstraction enabling powerful optimizations
- 30 years of research results in many optimizations based on semantic equivalent rewrites



https://insights.stackoverflow.com/survey/2017

Lack tools that can reason about SQL equivalences

Automated Solver for SQL: Q1 = Q2?



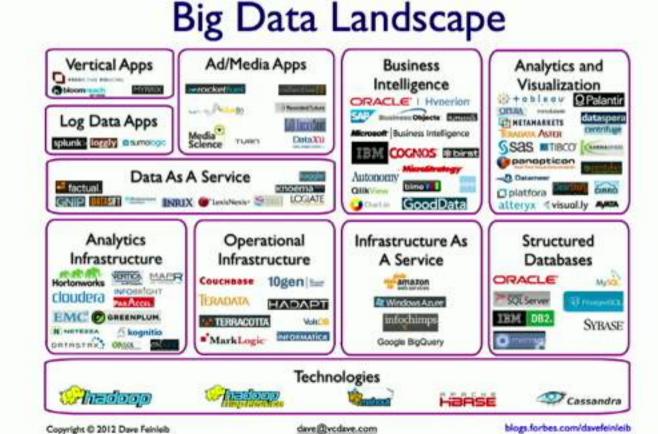
∀ possible input, Q1 = Q2?



∀ possible input, Q1 = Q2?



Correctness of query rewrite

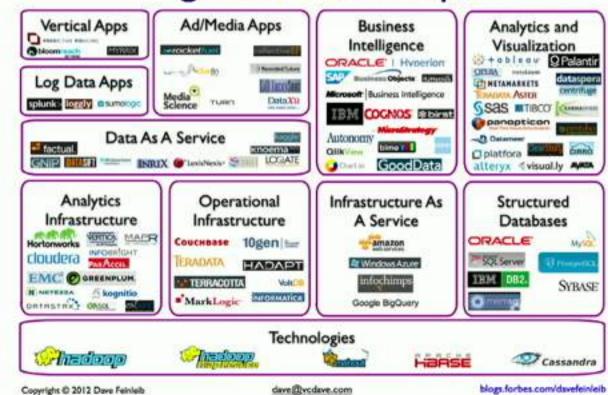


∀ possible input, Q1 = Q2?



- Correctness of query rewrite
- Semantics layer of data systems



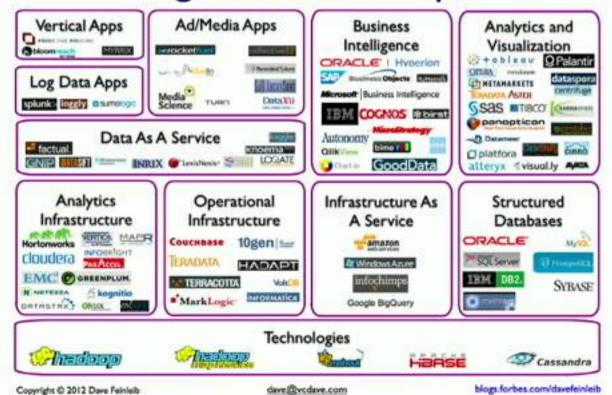


∀ possible input, Q1 = Q2?



- Correctness of query rewrite
- Semantics layer of data systems
- Auto grading on SQL assignments

Big Data Landscape



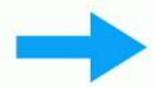
Challenges

- Checking equivalences of two FO sentences over finite models is undecidable ☺
- Rich language features
 - Aggregation and grouping
 - Index and integrity constraints
 - Correlated subqueries



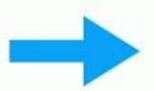
Boris A. Trakhtenbrot

Undecidability ≠ No Proof



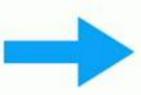
Interactive Theorem Prover that validates mechanized proofs

Undecidability ≠ No Proof



Interactive Theorem Prover that validates mechanized proofs

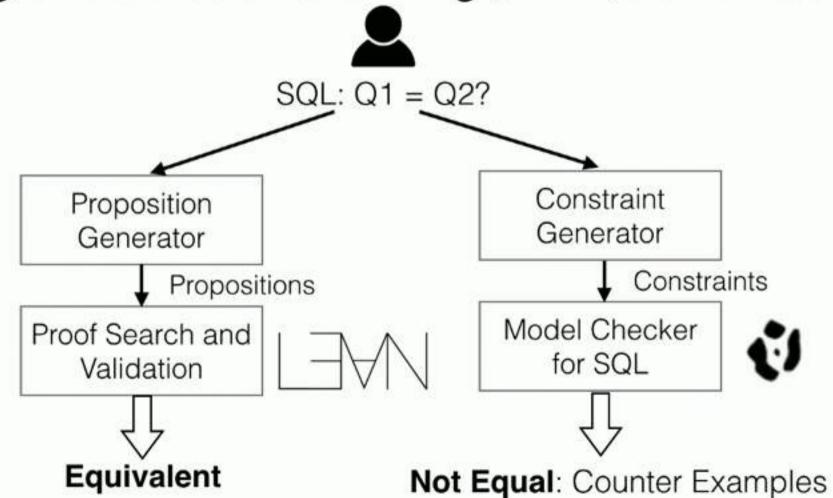
The model of an inequivalence is usually not large



Constraint Solver for Model Checking

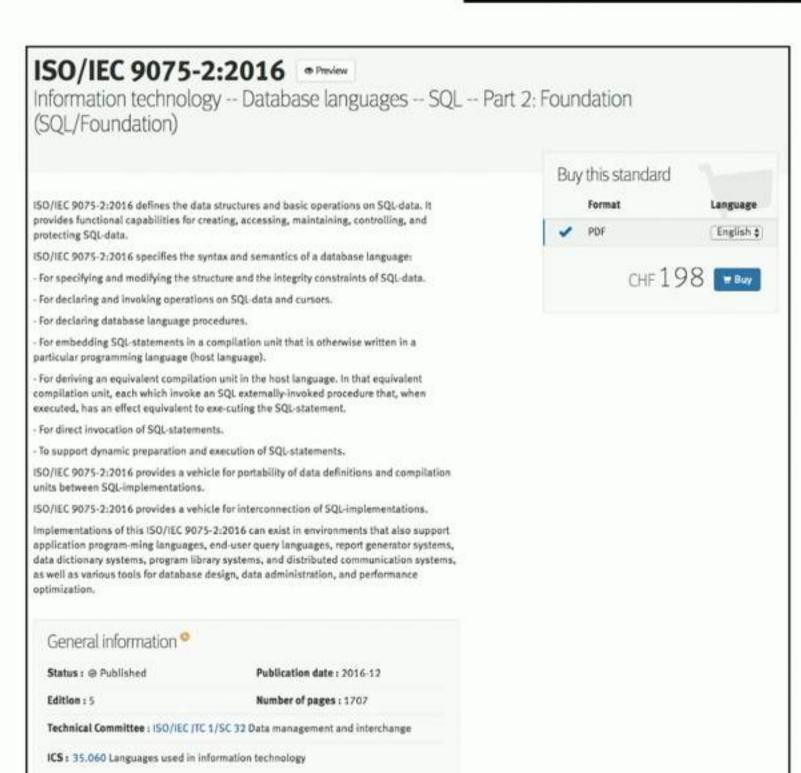
Construct a SQL Solver

 Cosette: An (almost) automated solver for SQL by combining interactive theorem proving and constraints solving (PLDI 17, SIGMOD17, CIDR17, VLDB18)



The Search for Formal SQL Semantics

What is SQL?



- More than 1,700 pages natural language description
- Insufficient for a rigorous SQL semantics

What is SQL?

- Data model: relations
 - A relation is an unordered collection of tuples
- Schema: set of attributes, each tuple in a relation must have same schema
- SQL query: $Q(R_1, R_2, ..., R_k) = R$

Relational Algebra

Relational Algebra (x, σ, Π, υ, -)

- x: e.g. Courses x Student = SELECT * FROM Course, Student
- σ : e.g. $\sigma_{len > 100}(Movie) = SELECT * FROM Movie WHERE len > 100$
- Π :e.g. $\Pi_{first}(Name) = SELECT \ first \ FROM \ Name$
- U: e.g. $R \cup S = (SELECT * FROM R)$ UNION ALL (SELECT * FROM S)

<u>Relational Algebra</u>

```
Relational Algebra (×, σ, Π, υ, -) UCQ
```

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Relational Algebra

- More operators needed
 - δ : e.g. $\delta(R) = SELECT DISTINCT * FROM R$
 - γ: e.g. γ_{name, count(*)}Person = SELECT name, count(*) FROM Person GROUP BY name
- Constraints: keys, foreign keys ...
- Lacking "meta-theory"

Relational Algebra

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- Constraints: keys, foreign keys ...
- · Lacking "meta-theory"



Leaking Abstraction: What is the semantics of RA?

Relation as List

Existing Coq Formalization [Malecha et al., POPL 10]

- Two queries are equivalent if returning the same result for all possible input relations
- Need to reason about two lists are equivalent up to permutations

Q1 = Q2?

Induction on R:

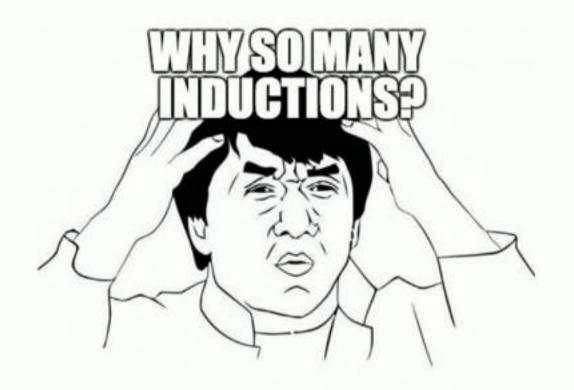
Assume Q1 == Q2 when R has N tuples Then when R is of size N+1:

. . .

Induction on S:

Assume Q1 == Q2 when S has N tuples Then when S is of size N+1:

. . .



Q1 = Q2?

Induction on R:

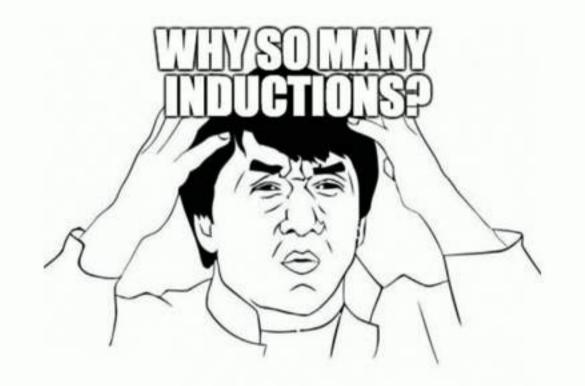
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Induction on S:

Assume Q1 == Q2 when S has N tuples Then when S is of size N+1:

. . .



400 Line of Coq for a simple rewrite, very limited rewrites has been proven

Univalent SQL Semantics

[Chu et al., PLDI 17]

Relation as Function

R:Relation → [R]:Tuple → N

Relation as Function

```
R:Relation \rightarrow [R]:Tuple \rightarrow N
   b: Predicate \rightarrow [b]:Tuple \rightarrow {0, 1}
SELECT * FROM R \longrightarrow Q(t) = [R] t × [b] t
     WHERE b
 R UNION ALL S \rightarrow Q(t) = [R] t + [S] t
```

Q1 = SELECT *
FROM (R UNION ALL S)
WHERE b

Q2 = (SELECT * FROM R WHERE b)

UNION ALL

(SELECT * FROM S WHERE b)



$$Q1(t) = ([R](t) + [S](t)) \times [b](t)$$



Q2(t):
$$[R](t) \times [b](t) + [S](t) \times [b](t)$$



$$Q1(t) = ([R](t) + [S](t)) \times [b](t)$$

Q2(t):
$$[R](t) \times [b](t) + [S](t) \times [b](t)$$

$$Q1 = Q2$$
?

Algebraic Reasoning



[SELECT first **FROM** Name] = ?

Name

| First | Last | | First |
|---------|------------|--|---------|
| Michael | Shulman | | Michael |
| Michael | Jordan | | Michael |
| Alex | Aiken | | Alex |
| Alex | Krizhevsky | | Alex |

[SELECT first **FROM** Name] = ?

Name

| First | Last |
|---------|------------|
| Michael | Shulman |
| Michael | Jordan |
| Alex | Aiken |
| Alex | Krizhevsky |

```
[SELECT first FROM Name] = ?
```

```
Q(t') = \sum_{t: Tuple} [t.first = t'.first] \times [Name](t)
```

Issues:

- Sum can potentially be infinite
- Need to reason about equality with algebraic expressions (now with sums ...)

Problem 2: Duplicate Elimination

[SELECT DISTINCT first **FROM** Name] = ?

Proposal: A step functions over \mathbb{N} : e.g. S(0) = 0, S(x) = 1

Issues: Back to inductive proof again 🙃

Univalent Semantics

- HoTT Type instead of N
- Dependent Pair Type (Σ-Type) to represent summation
- Squash Type to represent duplicate elimination
- Bonus: Unifies types and propositions



P. Seshadri, J. Hellerstein, H. Pirahesh, T. Y. Leung,

R. Ramakrishnan, D. Srivastava, P. Stuckey, S. Sudarshan

Cost-Based Optimization for Magic: Algebra and Implementation. SIGMOD 1996

Introduction of θ -semijoin:

$$R_1 \bowtie_{\theta} R_2 \equiv R_1 \bowtie_{\theta} (R_2 \bowtie_{\theta} R_1)$$

Pushing θ-semijoin through join:

$$(R_1 \bowtie_{\theta_1} R_2) \bowtie_{\theta_2} R_3 \equiv (R_1 \bowtie_{\theta_1} R_2') \bowtie_{\theta_2} R_3$$
$$R_2' = E_2 \bowtie_{\theta_1 \land \theta_2} (R_1 \bowtie R_3)$$

Pushing θ -semijoin through aggregation:

$$_{\bar{g}}\mathcal{F}_{\bar{f}}(R_1) \ltimes_{c_1=c_2} R_2 \equiv_{\bar{g}} \mathcal{F}_{\bar{f}}(R_1 \ltimes_{c_1=c_2} R_2)$$



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Can we do better?

Axiomatic Foundations and Semi-Decision Procedures

[Chu et al., VLDB 18]

Idea 1: projection requires summation

```
[SELECT first FROM Name] =
[Q](t') = ∑t [t.name = t'.name] × [R](t)
```

Solution: new* infinitary operator ∑t

Idea 2: DISTINCT converts bags to sets

```
[SELECT DISTINCT first FROM Name] \equiv
[Q](t') = ||\sum_{t}[t.first=t'.first] \times [Name](t)||
```

Solution: add the <u>squash</u>* operator ||...|

Intuition: $\|0\| = 0$ and $\|x\| = 1$ otherwise

Idea 3: non-monotone operators e.g. EXCEPT, NOT EXIST

Solution: add a not (...) operator

Intuition: not(0) = 1, not(x) = 0 otherwise

<u>U-Semirings</u>

Definition: An **Unbounded-semiring** is a structure (**U**, **0**, **1**, +, ×, $\|...\|$, **not**, $(\sum_{D})_{D \in \mathfrak{D}}$) satisfying a list of axioms

U-Semirings

Comm. Semiring

Definition: An **Unbounded-semiring** is a structure (**U, 0, 1, +, ×,** ||...||, **not,** (\sum_{D}) satisfying a list of axioms

Example: Axiomization of Sum

$$\sum_{t} (f_1(t) + f_2(t)) = \sum_{t} f_1(t) + \sum_{t} f_2(t)$$

$$\sum_{t_1} \sum_{t_2} f(t_1, t_2) = \sum_{t_2} \sum_{t_1} f(t_1, t_2)$$

$$x \times \sum_{t} f(t) = \sum_{t} x \times f(t)$$

$$\|\sum_{t} f(t)\| = \|\sum_{t} \|f(t)\|\|$$

Distributivity

Commutativity

Associativity

Idempoten on Squash

<u>Axiomization of Integrity Constraints</u>

The key constraint on R.k is the identity:

$$[t_k=t'_k]\times R(t)\times R(t') = [t=t']\times R(t)$$

<u>Axiomization of Integrity Constraints</u>

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Axiomization of Integrity Constraints

The key constraint on R.k is the identity:

$$[t_k=t'_k] \times R(t) \times R(t') = [t=t'] \times R(t)$$

The foreign key constraint from S.fk to R.k is:

$$S(t') = S(t') \times \sum_{t} R(t) \times [t_k=t'_fk]$$

Proving Equivalences with ICs

itemno is a key of itm

Proving Equivalences with ICs

```
\begin{array}{lll} Q_1(t) &=& \sum_{t_1,t_2}[t_1.\mathsf{np}=t.\mathsf{np}] \times [t_2.\mathsf{type}=t.\mathsf{type}] \times \\ & [t_2.\mathsf{itemno}=t.\mathsf{itemno}] \times [t_1.\mathsf{itn}=t_2.\mathsf{itemno}] \times \\ & || \sum_{t'}[t'.\mathsf{itemno}=t_1.\mathsf{itn}] \times [t'.\mathsf{np}=t_1.np] \times \\ & [t'.\mathsf{np}>1000] \times \mathsf{price}(t') \mid| \times \mathsf{itm}(t_2) \end{array}
```

```
\begin{array}{lll} Q_2(t) &=& \| \sum_{t_1,t_2} [t_2.\mathsf{type} = t.\mathsf{type}] \times [t_2.\mathsf{itemno} = t.\mathsf{itemno}] \times \\ & [t_1.\mathsf{itemno} = t.\mathsf{itemno}] \times [t_2.\mathsf{itemno} = t_1.\mathsf{itemno}] \times \\ & [t_1.\mathsf{np} = t.np] \times [t_1.\mathsf{np} > 1000] \times \mathsf{price}(t_1) \times \mathsf{itm}(t_2) \parallel \end{array}
```

Proving Equivalences with ICs

```
\begin{array}{lll} Q_1(t) &=& \sum_{t_1,t_2}[t_1.\mathsf{np}=t.\mathsf{np}] \times [t_2.\mathsf{type}=t.\mathsf{type}] \times \\ &=& [t_2.\mathsf{itemno}=t.\mathsf{itemno}] \times [t_1.\mathsf{itn}=t_2.\mathsf{itemno}] \times \\ &\parallel \sum_{t'}[t'.\mathsf{itemno}=t_1.\mathsf{itn}] \times [t'.\mathsf{np}=t_1.np] \times \\ &=& [t'.\mathsf{np}>1000] \times \mathsf{price}(t') \parallel \times \mathsf{itm}(t_2) \end{array}
```

```
\begin{array}{ll}Q_1(t) &=& \parallel \sum_{t_2,t'} [t_2.\mathsf{type} = t.\mathsf{type}] \times [t_2.\mathsf{itemno} = t.\mathsf{itemno}] \times \\ & [t'.\mathsf{itemno} = t.\mathsf{itemno}] \times [t'.\mathsf{np} = t.np] \times \\ & [t'.\mathsf{np} > 1000] \times \mathsf{price}(t') \times \mathsf{itm}(t_2) \parallel \end{array}
```

Rewrite Using U-Semiring Axioms

```
\begin{array}{lll} Q_2(t) &=& \|\sum_{t_1,t_2}[t_2.\mathsf{type}=t.\mathsf{type}] \times [t_2.\mathsf{itemno}=t.\mathsf{itemno}] \times \\ & [t_1.\mathsf{itemno}=t.\mathsf{itemno}] \times [t_2.\mathsf{itemno}=t_1.\mathsf{itemno}] \times \\ & [t_1.\mathsf{np}=t.np] \times [t_1.\mathsf{np}>1000] \times \mathsf{price}(t_1) \times \mathsf{itm}(t_2) \ \| \end{array}
```

<u>Algorithm: Main Idea</u>

Check $[Q_1] = [Q_2]$ by generalizing two known cases:

- UCQ under set semantics:
 - Check for homomorphisms Q1 ↔ Q2
- UCQ under bag semantics:
 - Check for isomorphisms Q1 → Q2
- "Chase" the axiomatization of constraints

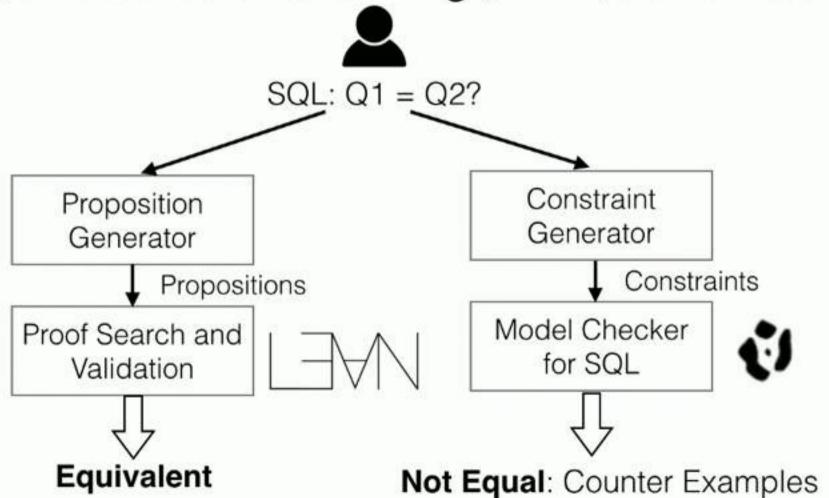
What about inequivalent queries?

Finding Counterexamples using Constraint Solver

[Chu et al., CIDR17]

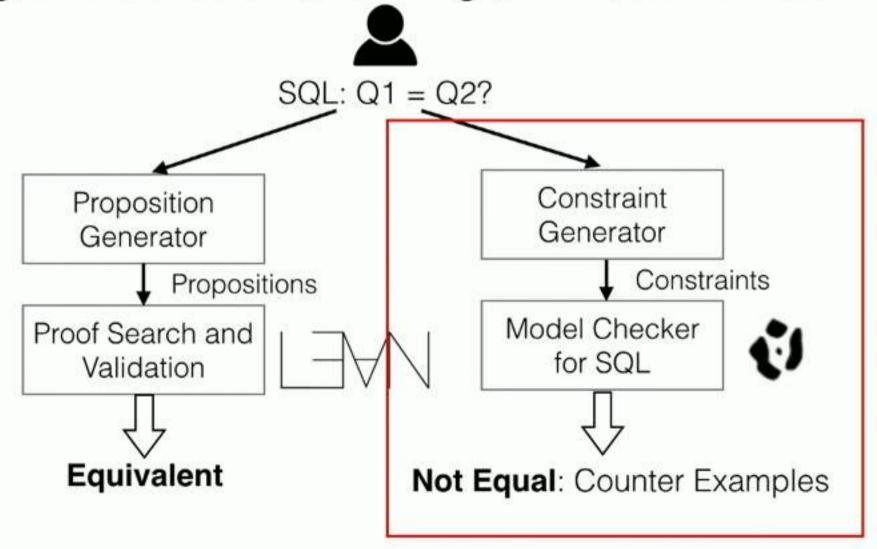
Cosette Architecture

 Cosette: An (almost) automated solver for SQL by combining interactive theorem proving and constraints solving (PLDI 17, SIGMOD17, CIDR17, VLDB18)

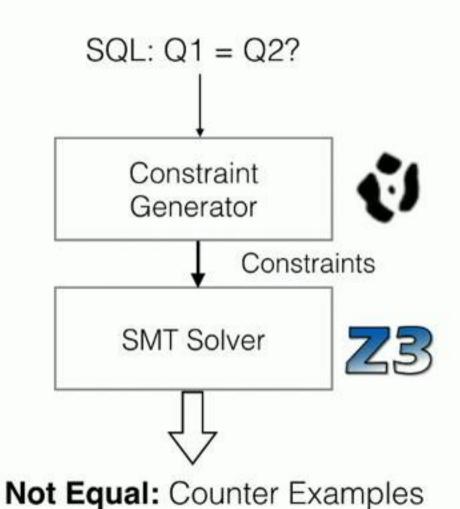


Cosette Architecture

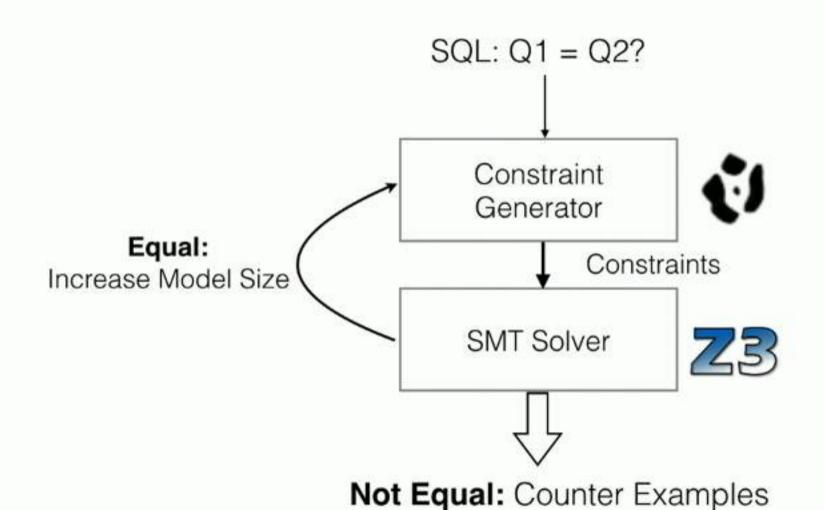
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Finding Counter Examples with SMT Solver



Finding Counter Examples with SMT Solver



Encoding SQL in Solvers

A tuple as a list

Tuple := List <Integer>

Encoding SQL in Solvers

A tuple as a list

```
Tuple := List <Integer>
```

A relation as tuples tagged with multiplicity

A SQL query as constraints over symbolic values

Encoding SQL in Solvers

A tuple as a list

```
sv
Tuple := List <Integer>
```

A relation as tuples tagged with multiplicity

```
sv
Relation := List <Pair<Tuple, Integer>>
```

A SQL query as constraints over symbolic values

Performance Optimizations

- Maximize concrete evaluation/ Minimize Symbolic Execution
- Symmetry breaking

Cosette

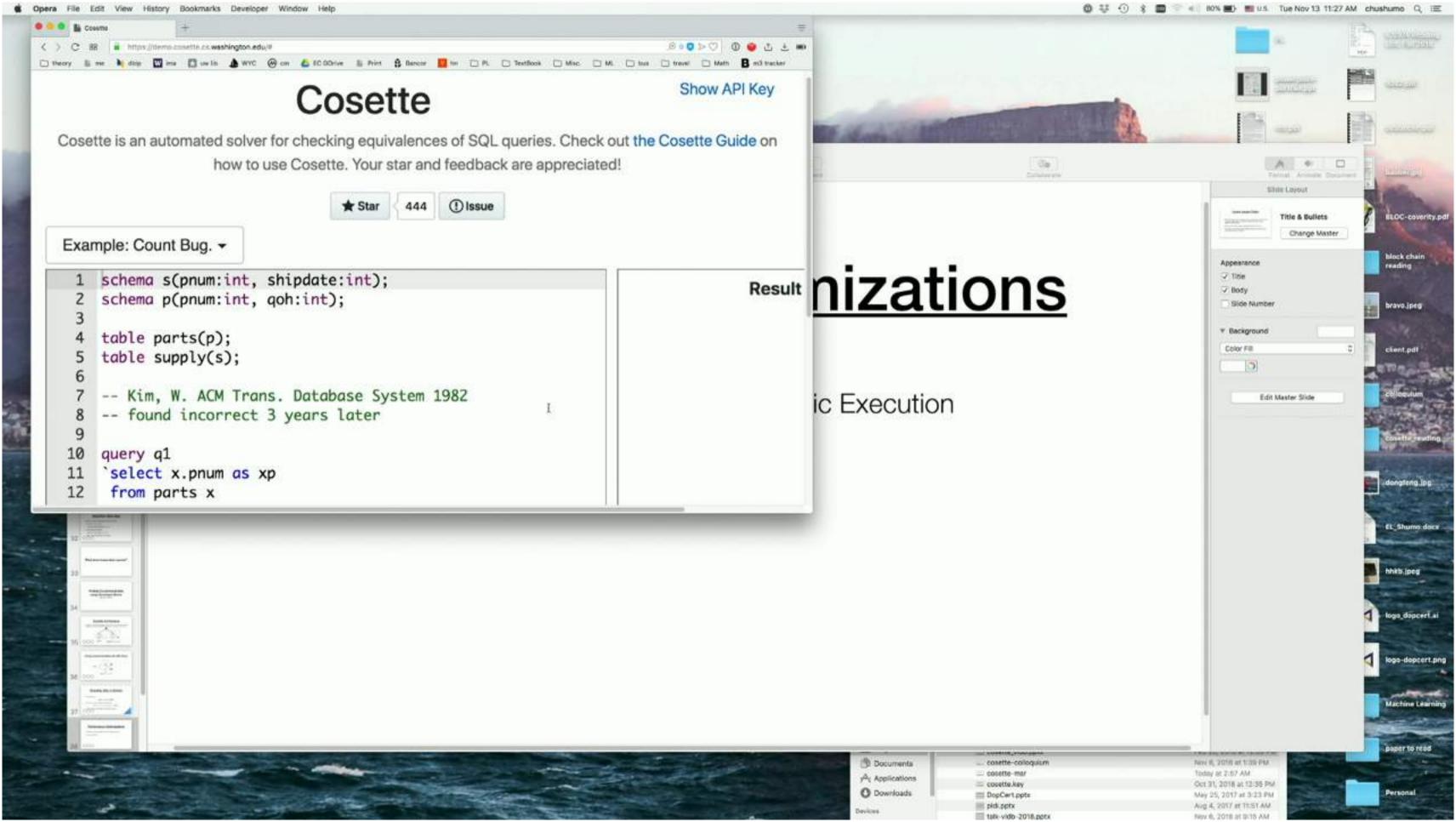
Cosette is an automated solver for checking equivalences of SQL queries. Check out the Cosette Guide on how to use Cosette. Your star and feedback are appreciated!

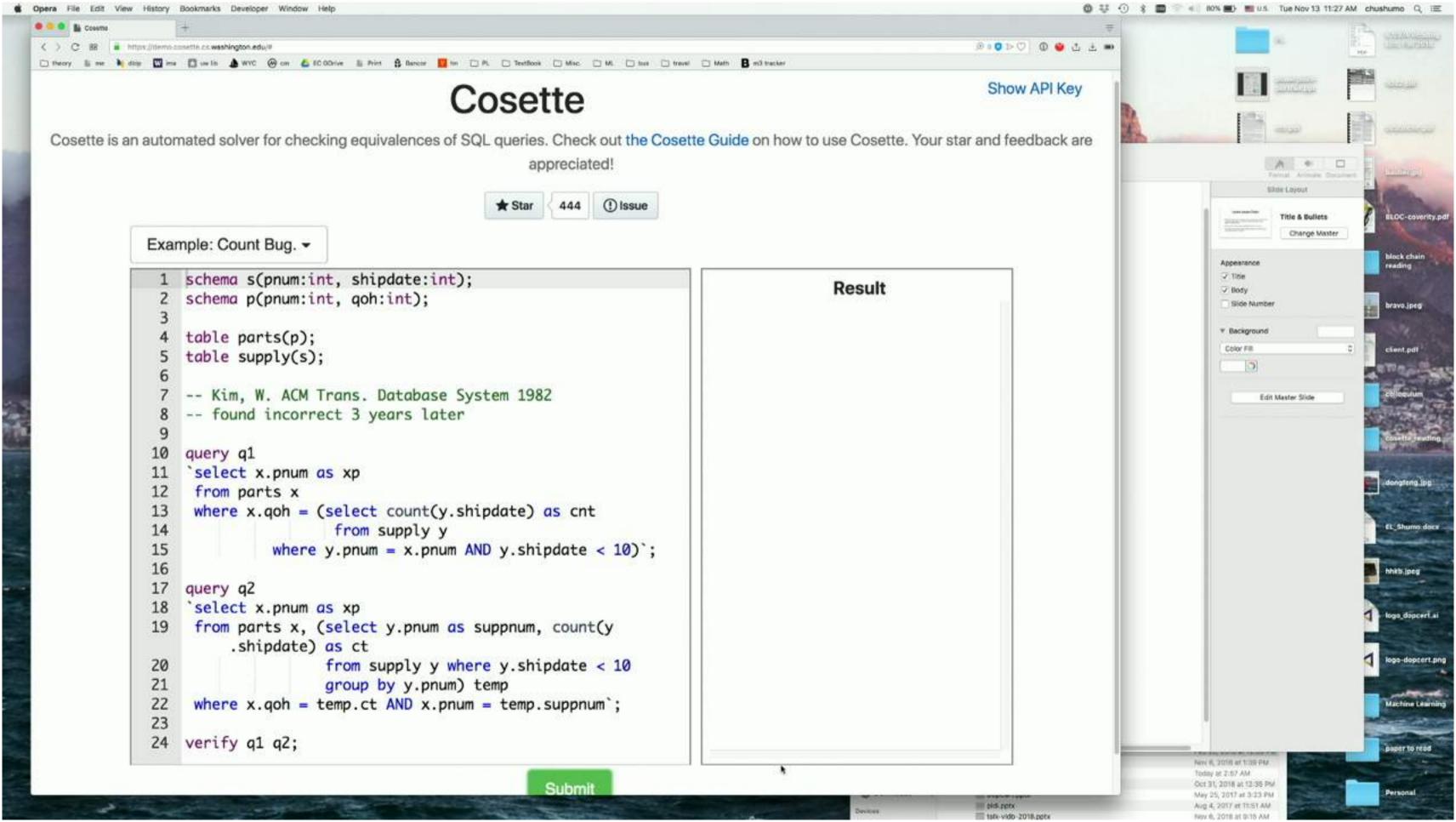
```
★ Star 444 ① Issue
```

```
Example: Count Bug. -
 1 schema s(pnum:int, shipdate:int);
   schema p(pnum:int, qoh:int);
   table parts(p);
   table supply(s);
    -- Kim, W. ACM Trans. Database System 1982
    -- found incorrect 3 years later
   query q1
11 `select x.pnum as xp
   from parts x
    where x.qoh = (select count(y.shipdate) as cnt
13
14
                    from supply y
             where y.pnum = x.pnum AND y.shipdate < 10);
15
16
17
   query q2
    `select x.pnum as xp
    from parts x, (select y.pnum as suppnum, count(y.shipdate) as
                   from supply y where y.shipdate < 10
20
21
                   group by y.pnum) temp
22
    where x.qoh = temp.ct AND x.pnum = temp.suppnum';
23
24 verify q1 q2;
```

```
Result
```

Submit





- Bug: 3 real-world optimizer bugs
- XData: query and mutant pairs collected from XData, a test generation framework
- Exams: a set of questions from the undergraduate data management class
- Rules: 68 query rewrite rules from database literatures and real-world optimizers

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- Rules: 68 query rewrite rules from database literatures and real-world optimizers

Unequal SQLs

Equivalent SQLs

| Dataset | Total # | Average time taken |
|---------|---------|--------------------|
| Bugs | 3 | 8.3s |
| XData | 9 | <1s |
| Exams | 5 | 1.3s |

| Dataset | Equiv? | Total Number | Automatically Decided | | Manually proved |
|---------|--------|-----------------|-----------------------|-----------|-----------------|
| | | | No. | Avg. Time | |
| Rules | Yes | 68 | 62 | < 20 s | 6 |
| Exams | Yes | 4 | 3 | < 1 s | 1 |

| Dataset | Total # | Average time taken |
|---------|---------|--------------------|
| Bugs | 3 | 8.3s |
| XData | 9 | <1s |
| Exams | 5 | 1.3s |

Rule No. 29: 400 LOC in [Malecha et al., POPL 10] to 15 LOC in Cosette

| Dataset | Equiv? | Equiv? Total Number | Automatically Decided | | Manually proved |
|---------|---------------------------|---------------------|--------------------------|-----------|-----------------|
| | See that the heldful that | | No. | Avg. Time | manually provou |
| Rules | Yes | 68 | 62 | < 20 s | 6 |
| Exams | Yes | 4 | 3 | < 1 s | 1 |

Cosette in Action

- Full stack: solver core, web service, online demo, automated grader
- Deployed for UW 344/544 automated grading since 2017
- SIGMOD 2017 Best Demo
- Top 1 Trending Racket Project in GitHub (July, 2017)

```
/* define schema s1,
      here s1 can contain any number of attributes,
      but it has to at least contain integer attributes
      x and y */
   schema s1(x:int, a:int, ??);
                                -- define schema s2
   schema s2(a:int, ??);
   table a(s1);
                           -- define table a using schema s1
   table b(s2);
                           -- define table b using schema s1
   query q1
                           -- define query al on tables a
    'select distinct x.x as ax from a x, b y
    where x.a = y.a';
                           -- define query q2 likewise
   query q2
    'select distinct x.x as ax from a x, a y, b z
    where x.x = y.x and x.a = z.a;
19
20 verify q1 q2;
                           -- does al equal to a2?
```

Conclusions and Takeaways

Cosette: The first practical SQL solver

- A new axiomatic semantics for SQL
- Semi-decision procedure for UCQ under bag/set with ICs
- Integrated interactive theorem proving and constraints solving techniques
- Automated reasoning brought by Formal Methods + Domain Specific
 Semantics leads to more reliable, more optimized future data systems
- Website: cosette.cs.washington.edu