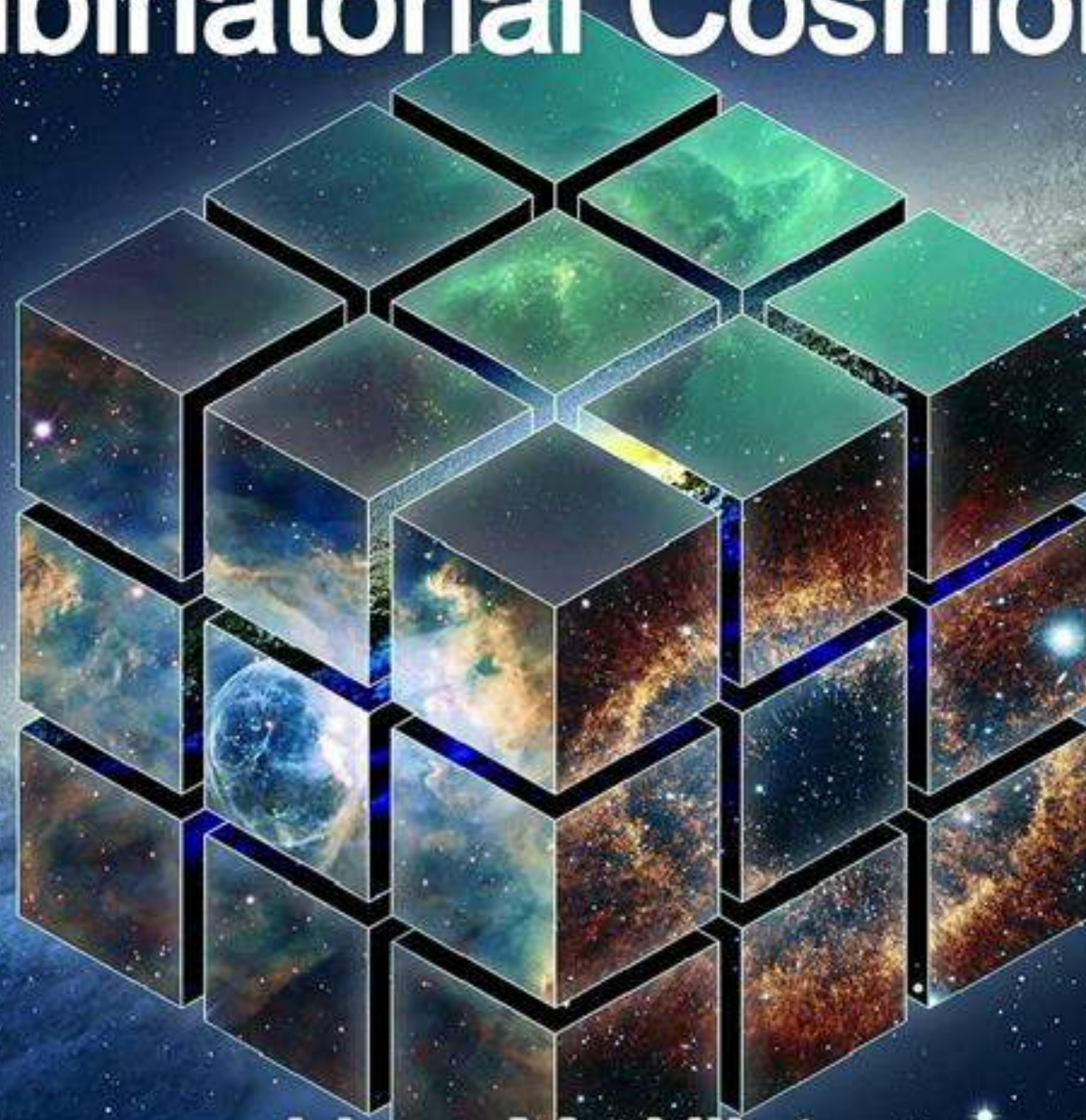


Combinatorial Cosmology



Liam McAllister
Cornell

Physics Meets ML, Microsoft Research, April 26, 2019

Summary

This talk is about realistic solutions of string theory.

I will explain why the solutions allowed by cosmological measurements are fully specified by finite lists of integers: quantized parameters.

To understand what string theory predicts, we need to connect these integers to observables.

I will formulate this task as a computational problem, and as a target for ML.

Plan

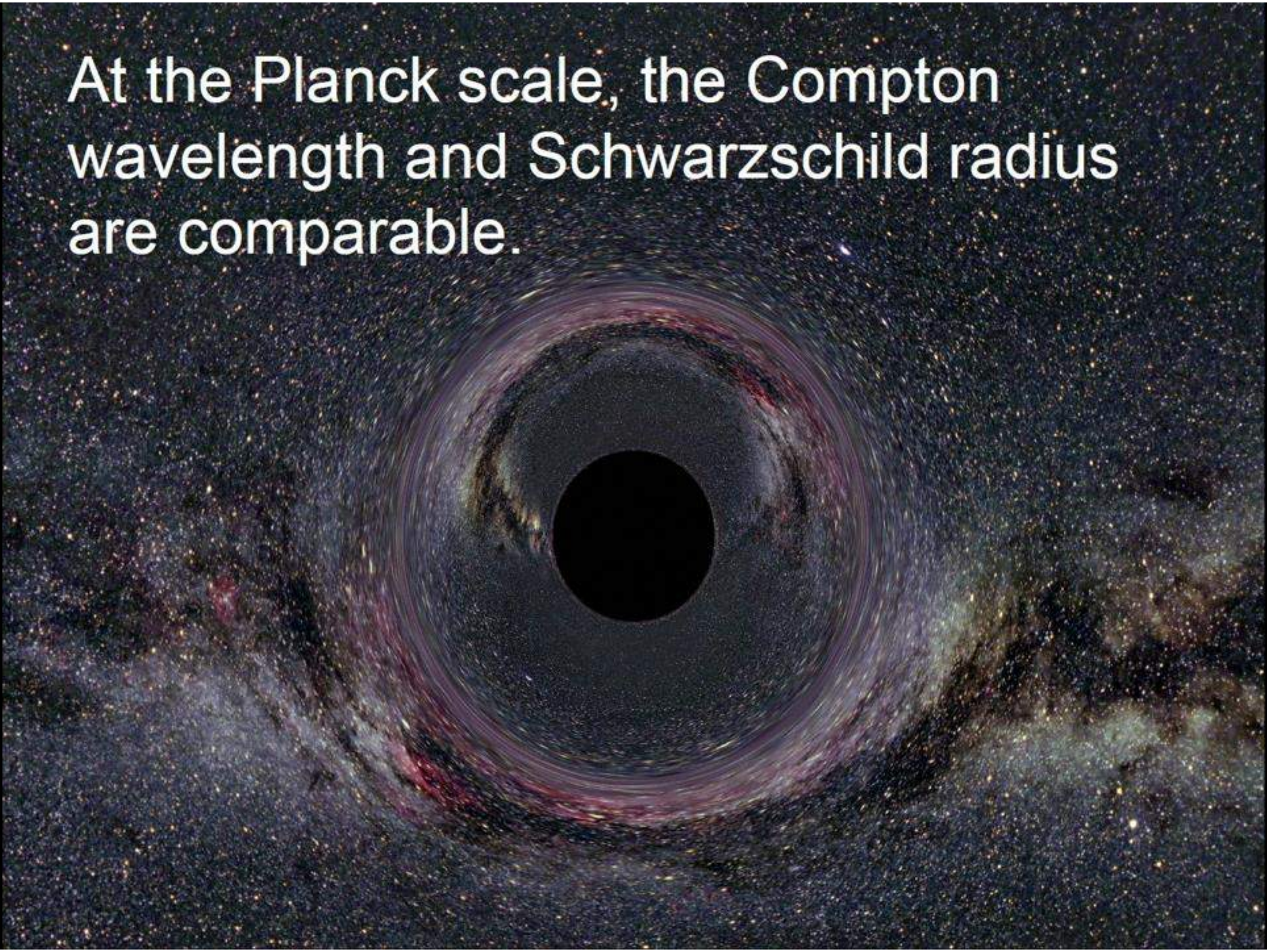
- I. Invitation to quantum gravity
- II. Quantized parameters and the string landscape
- III. Example: toric hypersurfaces
- IV. Targets for ML

Quantum gravity and the Planck scale

$$M_p = \sqrt{\frac{\hbar c}{G_N}} \approx 22\mu\text{g} \quad E_p \approx 1.2 \times 10^{18} \text{ GeV} \approx 500 \text{ kWh}$$

$$t_p \approx 5.4 \times 10^{-44} \text{ s} \quad \ell_p \approx 1.6 \times 10^{-35} \text{ m}$$

At the Planck scale, the Compton wavelength and Schwarzschild radius are comparable.



When does quantum gravity matter?

In extreme conditions:

- Black hole singularities
- Planckian-energy collisions
- Cosmological singularities
- Cosmological inflation

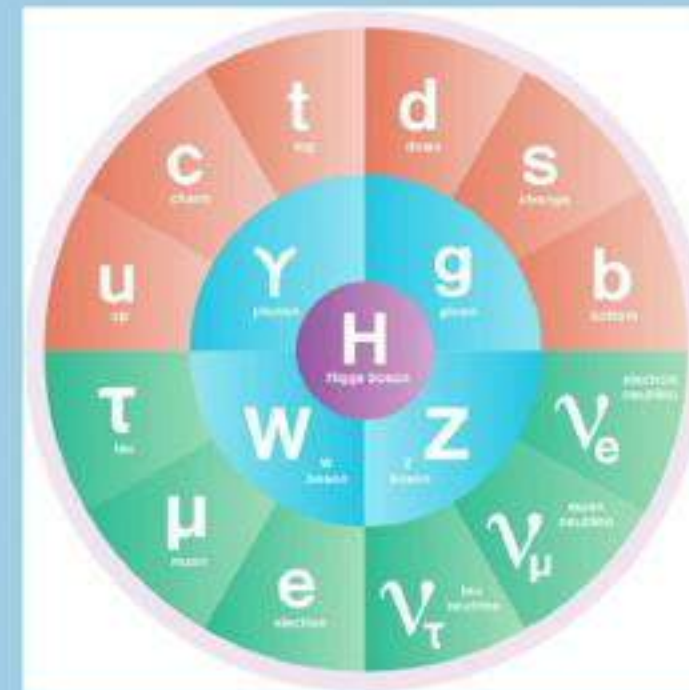
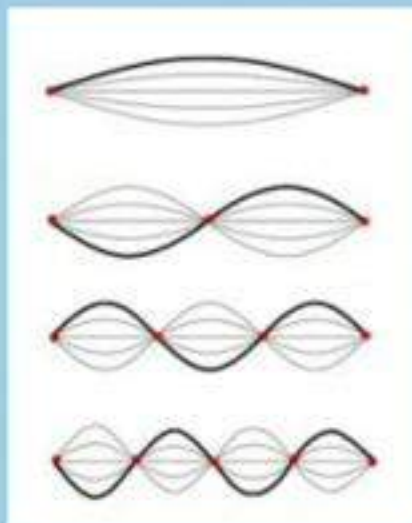
Why is quantum gravity interesting?

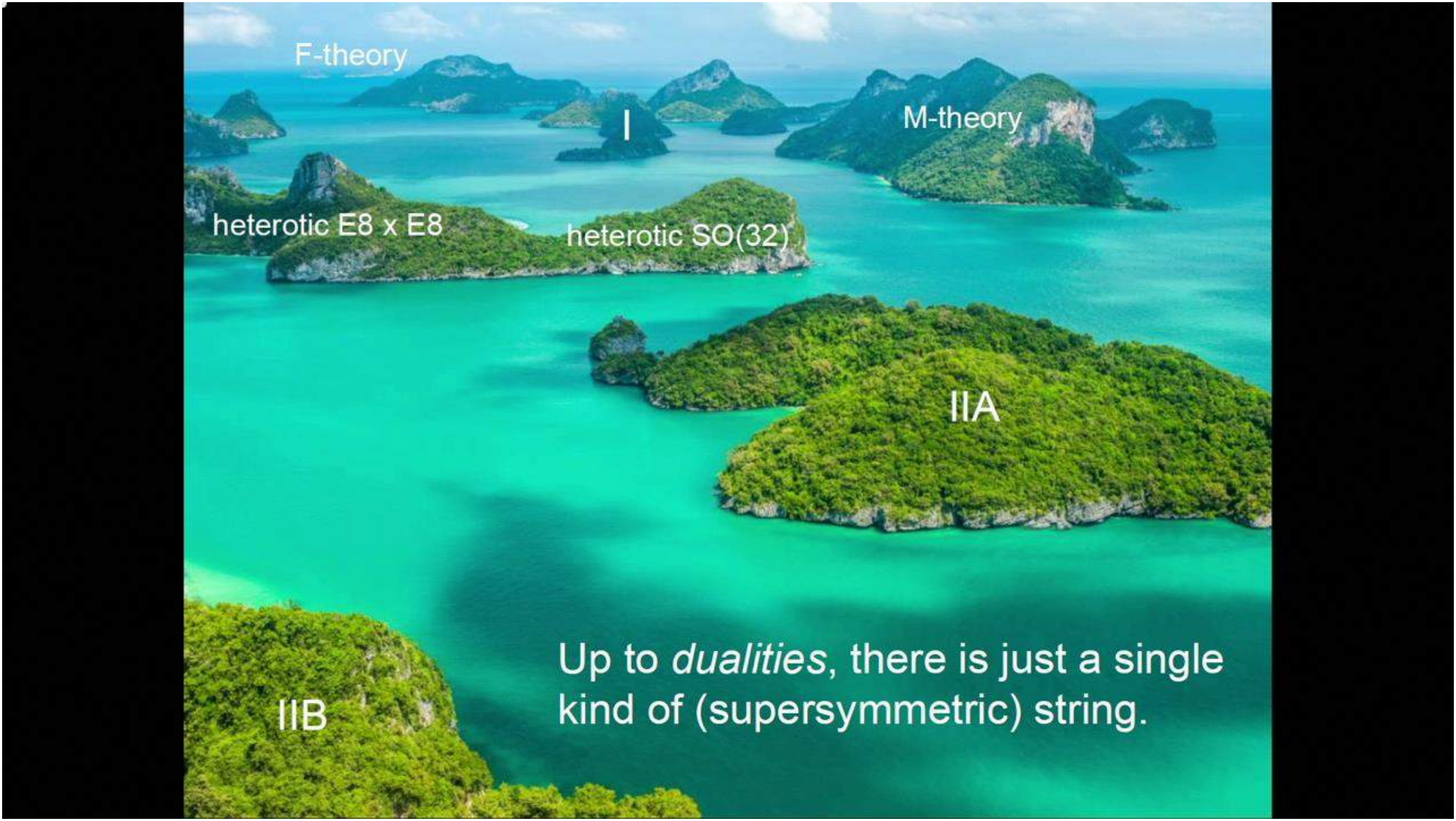
- By understanding more extreme conditions, we can learn more fundamental laws.
- Required to understand the origin of the universe, and to interpret certain observations in cosmology.
- Key to understanding the nature and origin of gravity.
- Best hope of understanding nature of physical law itself.

String theory

A finite theory of quantum gravity.

Fundamental constituents are strings, rather than point particles.



An aerial photograph of a tropical archipelago, likely in the Philippines, featuring numerous small, green, rocky islands surrounded by clear, turquoise water. The sky is blue with some white clouds. The image serves as a background for text labels representing different string theories.

F-theory

M-theory

heterotic $E_8 \times E_8$

heterotic $SO(32)$

IIA

IIB

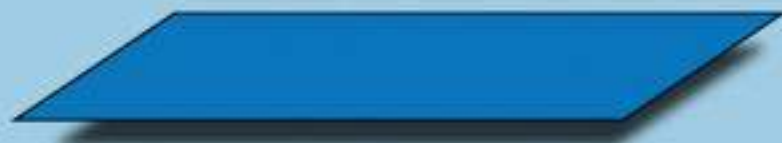
Up to *dualities*, there is just a single kind of (supersymmetric) string.

Compactification

String theory naturally exists in 10 dimensions.

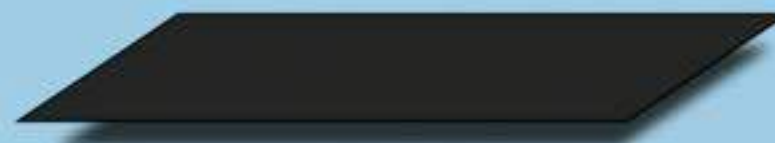
Its fundamental solution is 10d Minkowski space.

0123



4d spacetime

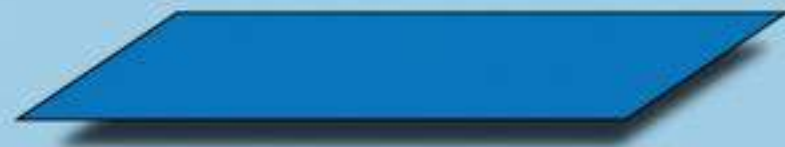
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6d Euclidean space

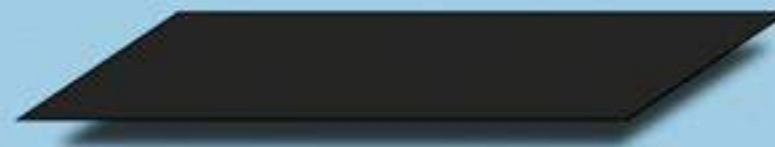
x

Compactification

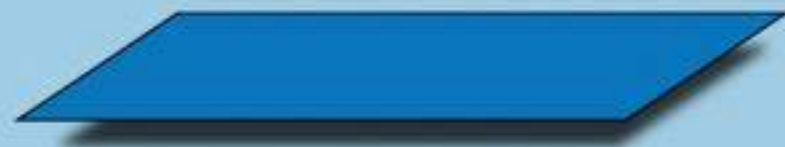
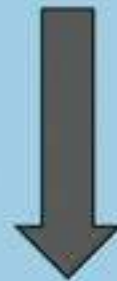


4d Minkowski space

x



6d Euclidean space



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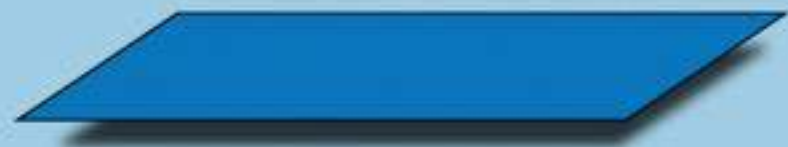


6d compact space

0123

456789

Einstein equations in vacuum: $\text{Ricci} = 0$



x



4d Minkowski space

6d compact space

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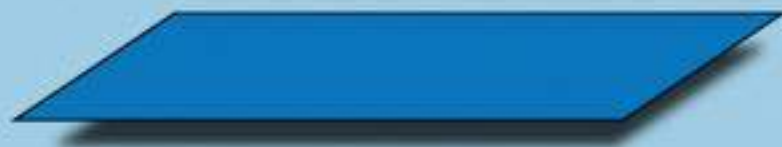
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Einstein equations in vacuum: **Ricci** = 0

$$R_{\mu\nu} = 0 \quad \text{and} \quad R_{mn} = 0$$

$$\mu, \nu \in \{0, 1, 2, 3\}$$

$$m, n \in \{4, 5, 6, 7, 8, 9\}$$



x



4d Minkowski space

6d compact space

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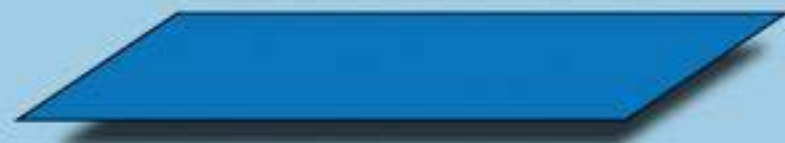
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Einstein equations in vacuum: $\text{Ricci} = 0$

$$R_{\mu\nu} = 0 \quad \text{and} \quad R_{mn} = 0$$

Vacuum solution of string theory =

Compact Ricci-flat six-manifold



x



4d Minkowski space

6d compact space

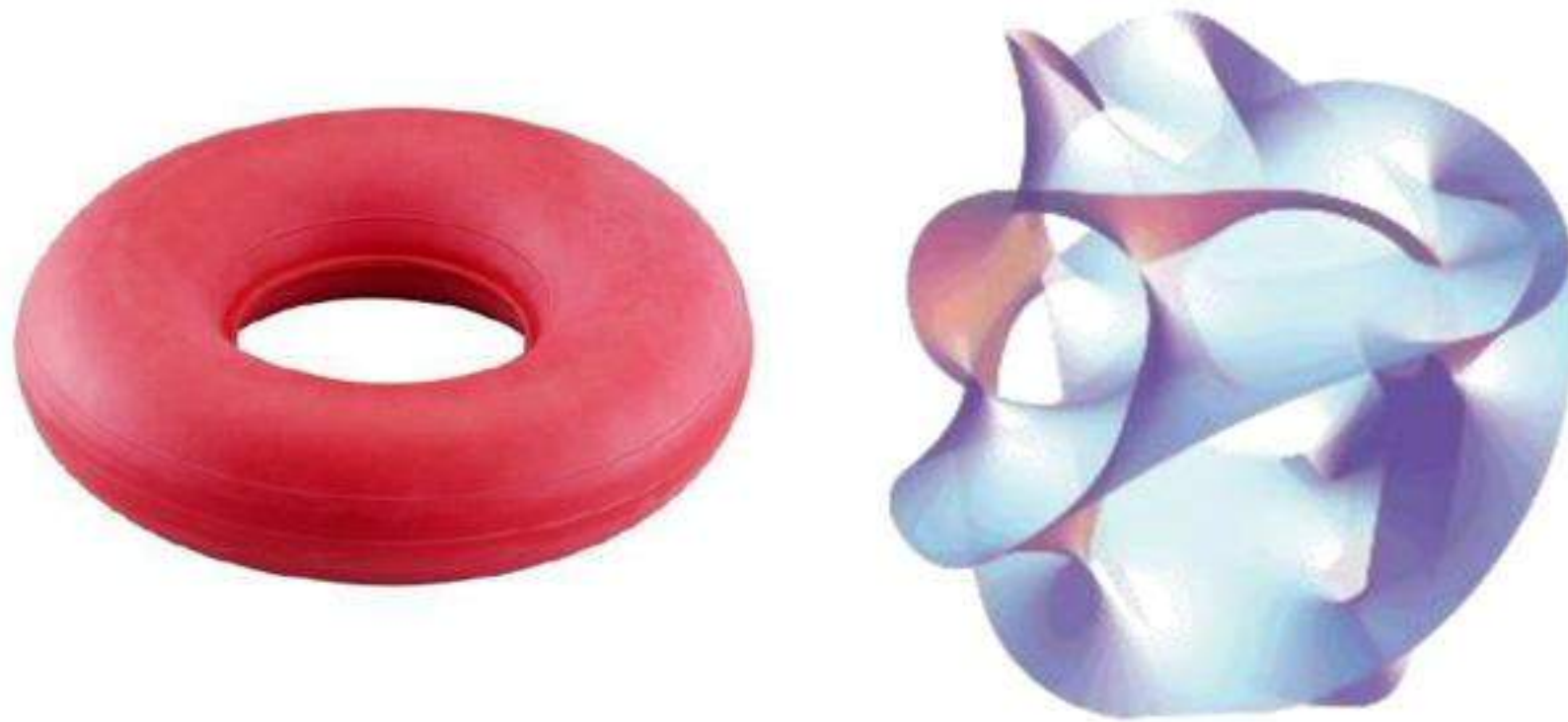
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String theory is unique

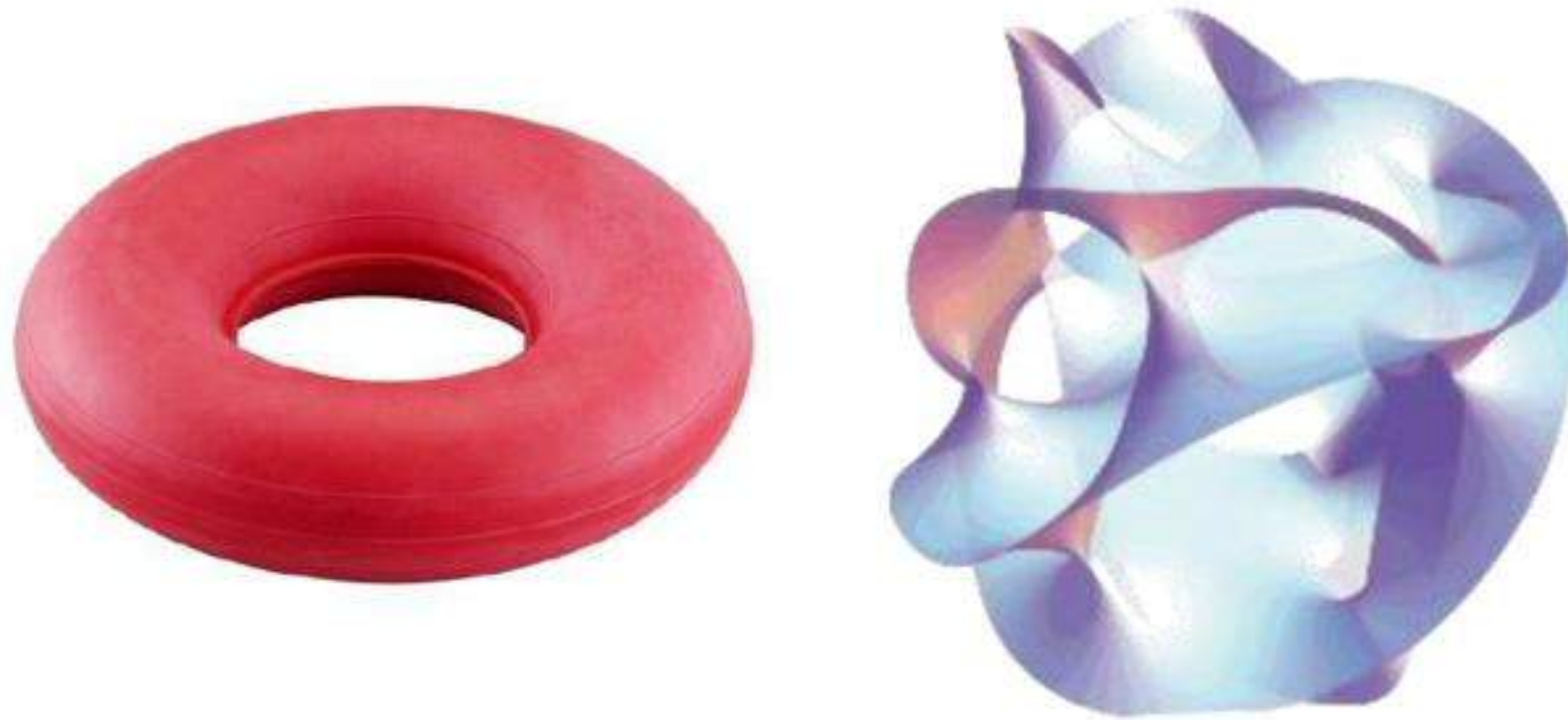
- The theory has **no fundamental dimensionless parameters**.
- The parameters we see at low energy are determined by the geometry of the internal space.
 - i.e. by the *solution* to Einstein's equations.

So all parameters in nature are the expectation values of fields, principally those that dictate the internal geometry (size and shape).



$$\phi(\vec{x}, t) = \text{Vol}_6(\vec{x}, t)$$

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$$\phi(\vec{x}, t) = \text{Vol}_6(\vec{x}, t)$$

- *Continuous* parameters correspond to massless scalar fields (moduli).
- These are catastrophic for cosmology.



Cosmological effects of light scalars:

Dissociation of light elements after nucleosynthesis

Overproduction of dark matter

Overclosure of the universe

Fifth forces

- The only cosmologically viable solutions of string theory are those without continuous deformations (moduli).
- We call these solutions ‘isolated vacua’, or just ‘vacua’.
- Their spin-zero fields are all massive,

$$m^2 \geq m_{\min}^2 > 0$$



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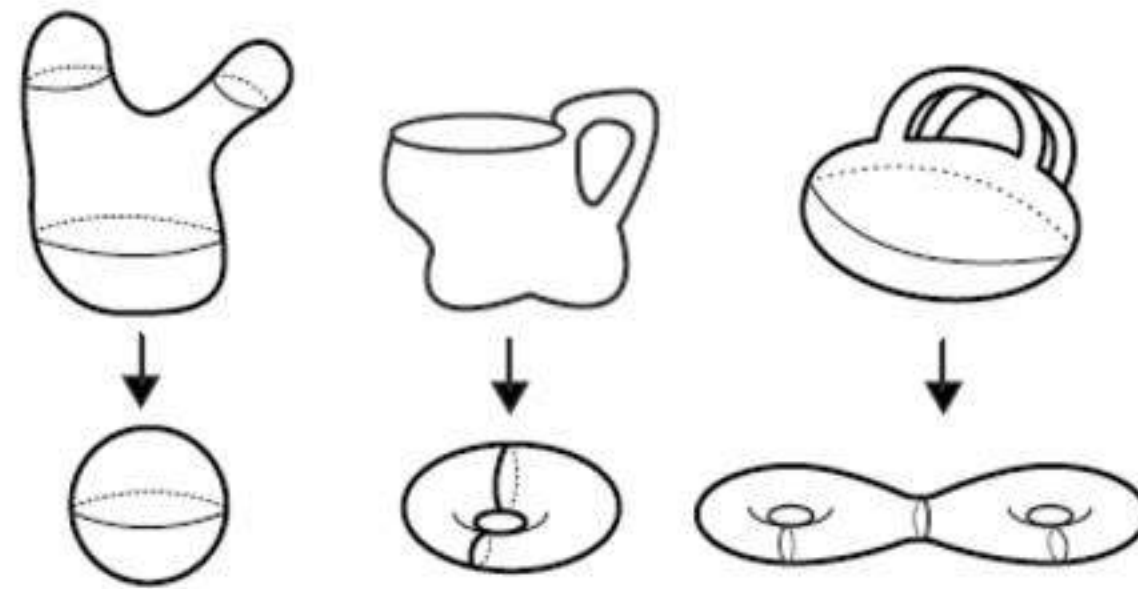
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- The only cosmologically viable solutions of string theory are those without continuous deformations (moduli).
- There are no fundamental parameters in the theory, and we demand there be no continuous parameters in the solution.
- All that remains are **discrete parameters in the solution**.
- When you have discarded the continuous data describing a geometry, what remains is **topology**.

A cosmologically viable solution of string theory is specified by the topological invariants of the internal six-manifold X .

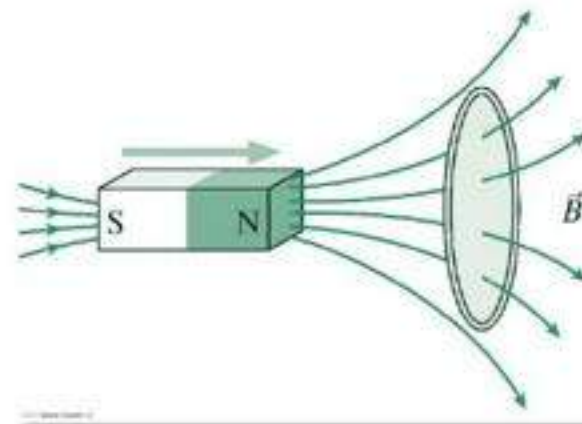
Heuristically, these invariants count holes, handles, etc., in X .



A cosmologically viable solution of string theory is specified by the topological invariants of the internal six-manifold X .

More precisely, they involve:

- Topology of X : Hodge numbers, intersection numbers
- Topology of **D-branes** on submanifolds S in X : wrapping numbers, bundles on S
- Number of units of **flux** threading cycles of X

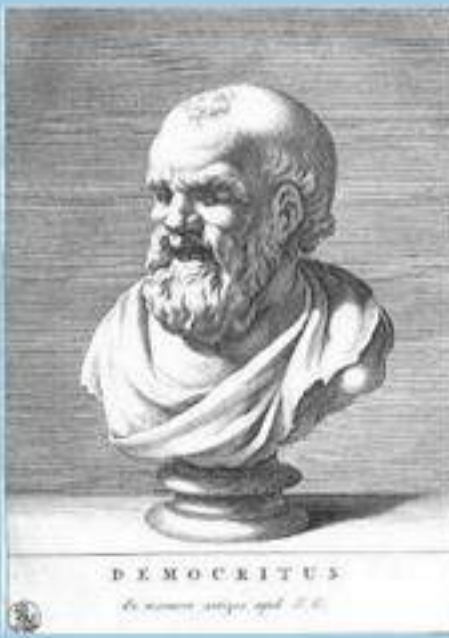


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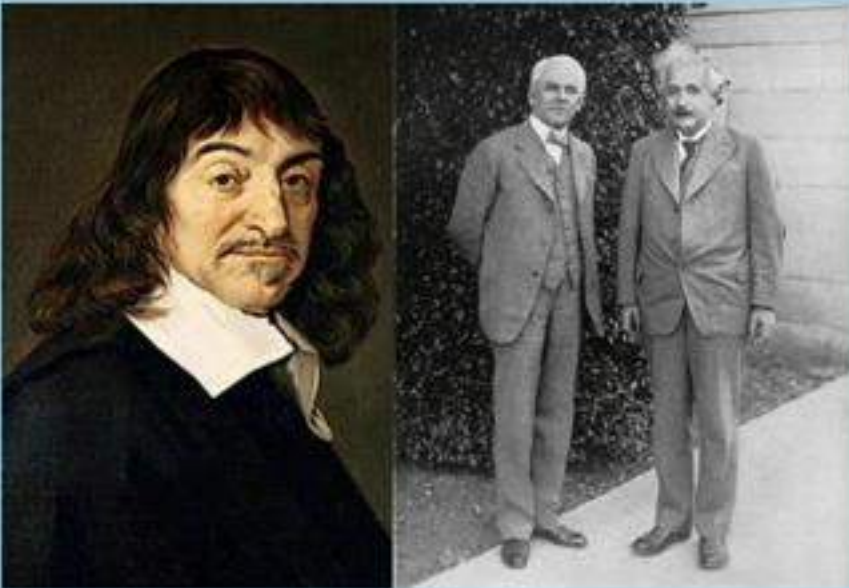
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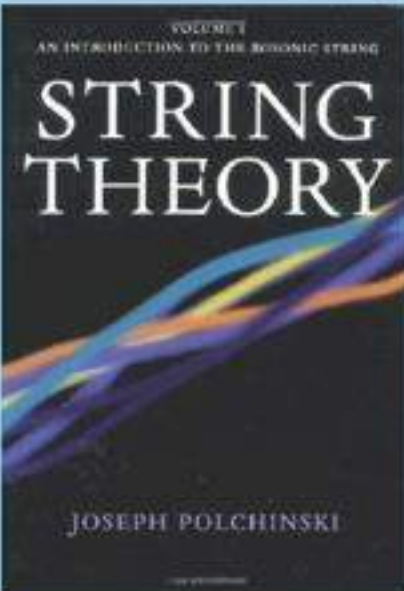
For each solution, the invariants amount to a **finite number N of integers.**



Matter is quantized



Force carriers are quantized



Parameters are quantized

$$\cancel{\alpha = \frac{1}{137}}$$

Cosmologically viable quantum gravity theories are specified by **finitely many integers**, the fundamental parameters in Nature.

$$\vec{p} \in \mathbb{Z}^N \quad N \sim \mathcal{O}(100 - 1000)$$

Not all $\vec{p} \in \mathbb{Z}^N$ are consistent/allowed.

There is no compelling evidence for any infinite family of vacua of string theory with $m_{\min}$ fixed. I assume the number of viable vacua is **finite**.

The set of such vacua is **the string landscape**.

Size of the landscape

- In known ‘islands’, number of cycles in internal space, hence N , is large:
 - Type IIB string theory on Calabi-Yau threefolds:
 $N \sim \mathcal{O}(100 - 500)$ e.g. $\dim H_4(CY_3, \mathbb{R}) \leq 491$
 $\mathcal{N}_{\text{vac}} \sim 10^N \sim 10^{500}$ Denef, Douglas
 - F-theory on Calabi-Yau fourfolds
 $N \lesssim \mathcal{O}(10^6)$ e.g. $\chi(CY_4) \leq 1,820,448$ Klemm, Lian, Roan, Yau
 $\mathcal{N}_{\text{vac}} \sim 10^{272,000}$ Taylor, Wang

String theory and reality

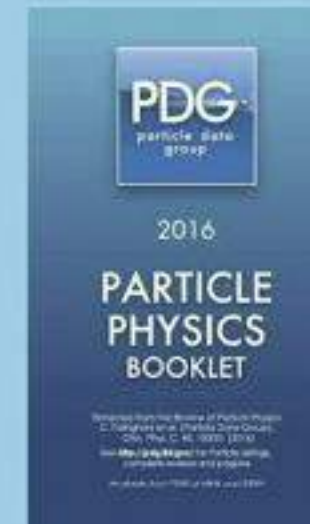
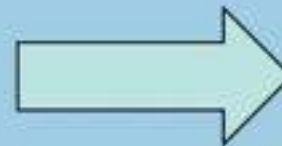
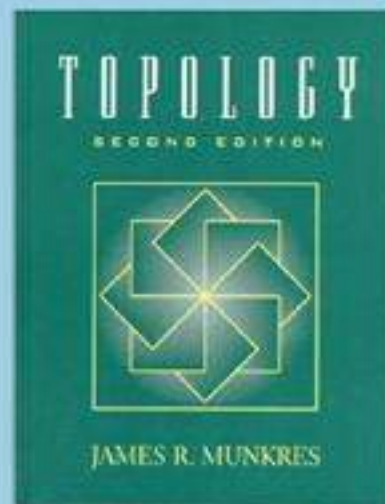
- Goal: connect solutions of string theory to the observed universe.
- To achieve this, must know:
 - What values can the quantized parameters take?
 - Given an allowed set of values, what would be observed in the resulting universe?
- Examples of observables:
 - Matter content, gauge group, vacuum energy, masses and couplings of visible and dark sector particles

Parameters to Observables

$$\mathcal{P} \rightarrow \mathcal{O}$$

$$\mathcal{P} = \left\{ \vec{p} \in \mathbb{Z}^N \mid \vec{p} \text{ allowed in string theory} \right\} \quad \text{N: number of discrete parameters}$$

$$\mathcal{O} = \left\{ \vec{o} \in \mathbb{Q}^M \mid \vec{o} \text{ possible values of } M \text{ measurable parameters} \right\}$$



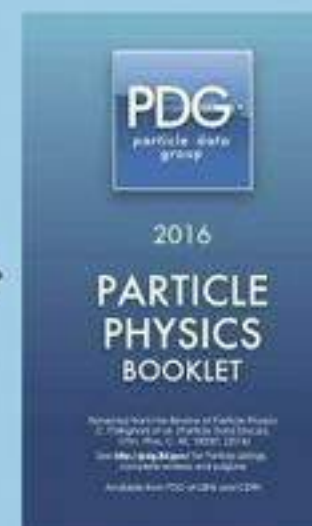
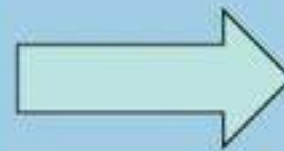
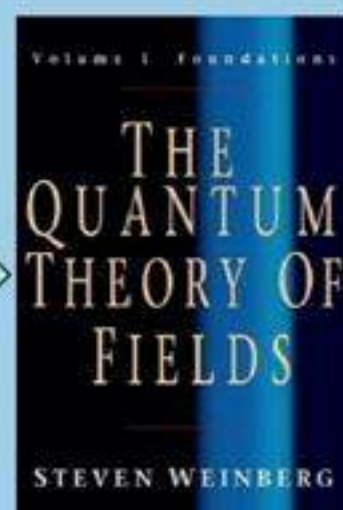
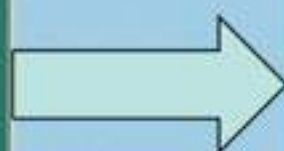
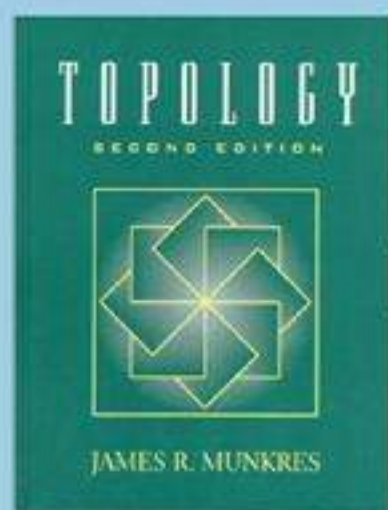
Parameters, Effective Theory, Observables

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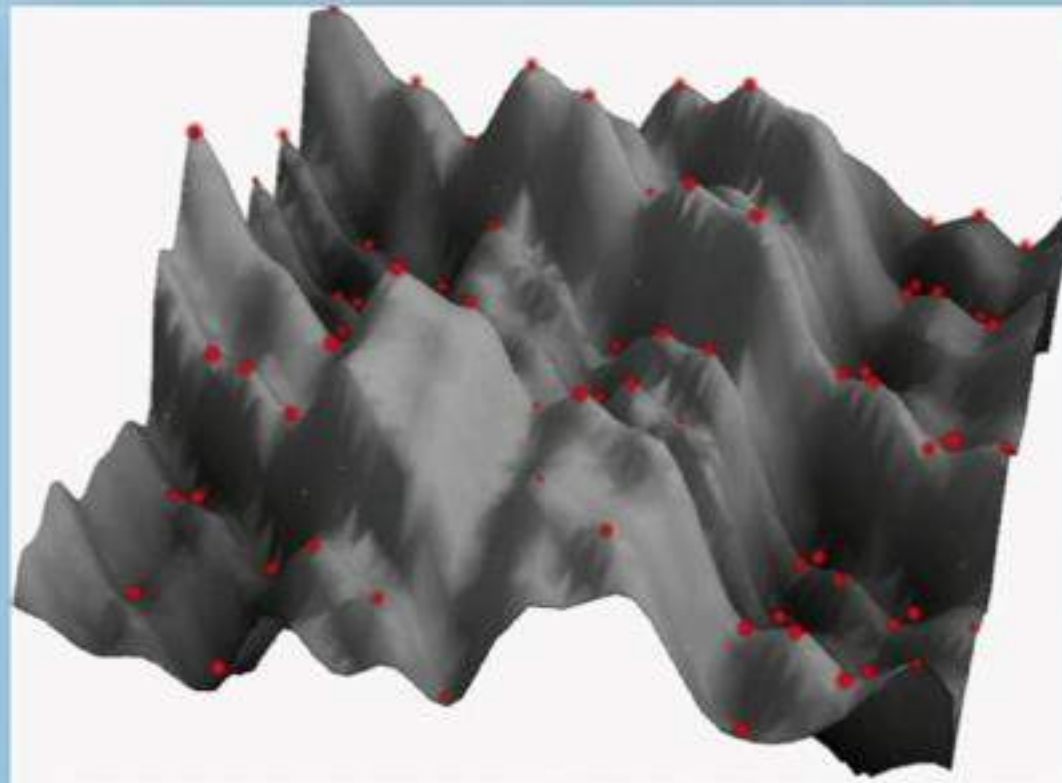
Parameters, Effective Theory, Observables

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⊃ Riemannian manifold $\mathcal{M}$ (“field space”), metric g ;

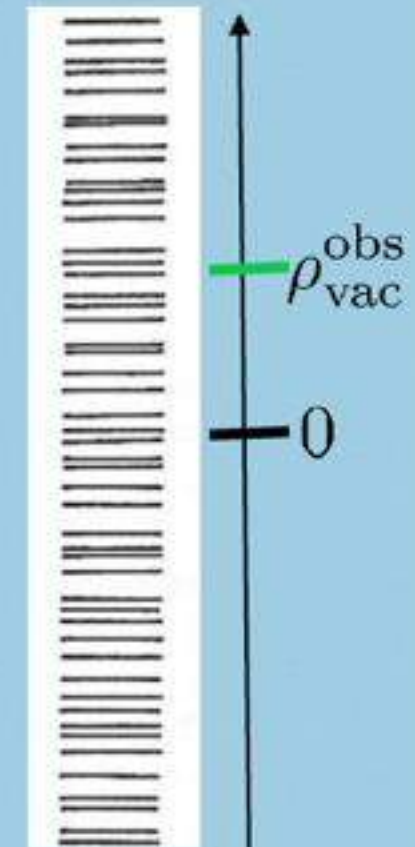
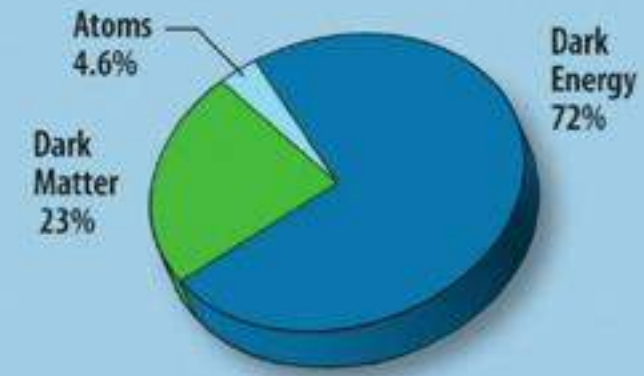
function $V : \mathcal{M} \rightarrow \mathbb{R}$ (“potential”)

vacua : local minima of V $\nabla V = 0, \quad \nabla \nabla V > 0$



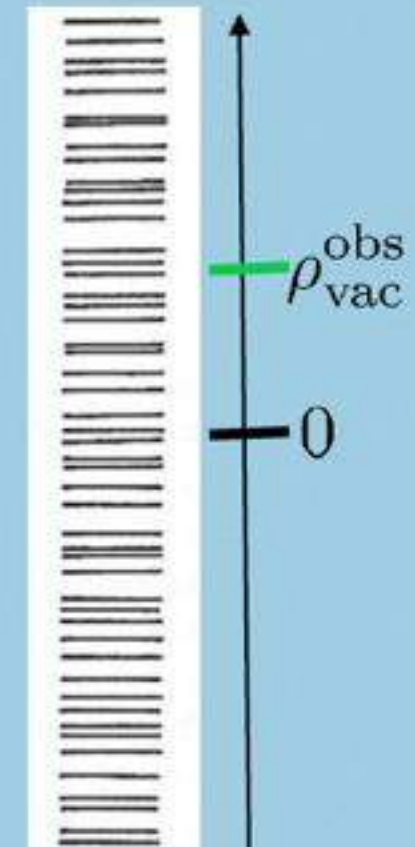
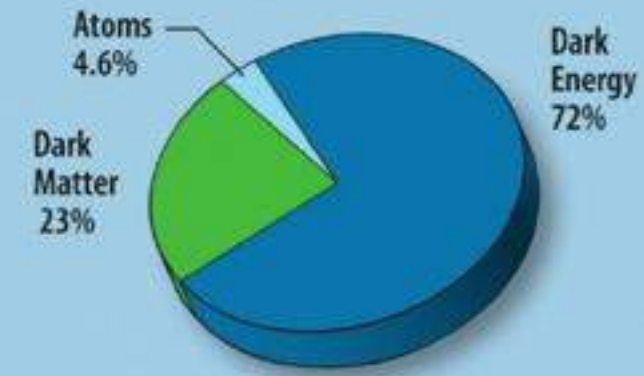
Observable: dark energy

- Dark energy density $\rho_{\text{vac}}^{\text{obs}} \approx 10^{-26} \frac{\text{kg}}{\text{m}^3} \approx 10^{-123} M_{\text{pl}}^4$
- No continuous parameter to adjust to match observations.
- If vacua of string theory have $\rho_{\text{vac}} \in (-M_{\text{pl}}^4, M_{\text{pl}}^4)$ sufficiently uniform,



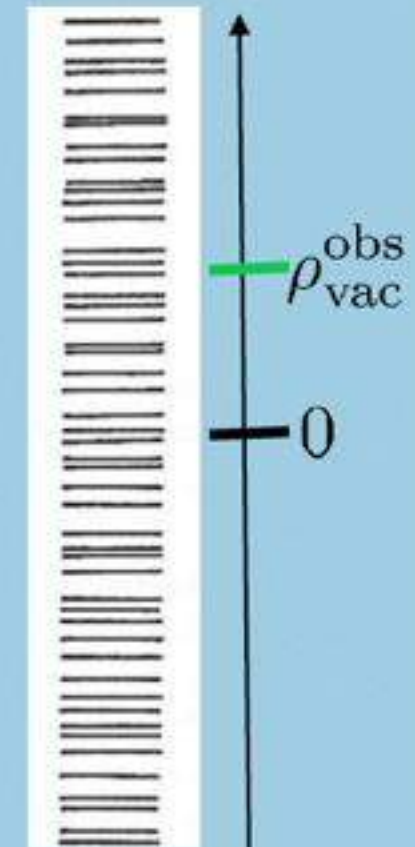
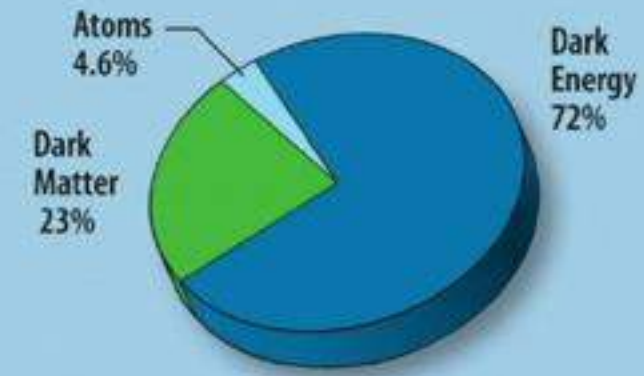
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- If vacua of string theory have $\rho_{\text{vac}} \in (-M_{\text{pl}}^4, M_{\text{pl}}^4)$ sufficiently uniform, need $\mathcal{N}_{\text{vac}} \gtrsim 10^{123}$ to accommodate observations.
- ‘Anthropic solution to cosmological constant problem’.



Plan

- I. Invitation to quantum gravity
- II. Quantized parameters and the string landscape
- III. Example: toric hypersurfaces
- IV. Targets for ML



Mehmet Demirtas



Cody Long



Mike Stillman



Andres Rios Tascon

Ricci-flat six-manifolds

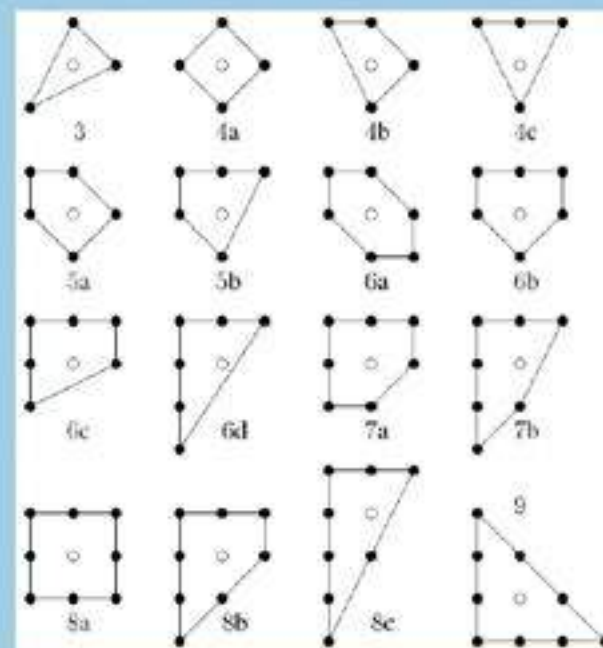


Finding Ricci-flat 6-manifolds directly is challenging. In the case of Calabi-Yau threefold (CY_3) hypersurfaces, the problem is combinatorial and can be automated.

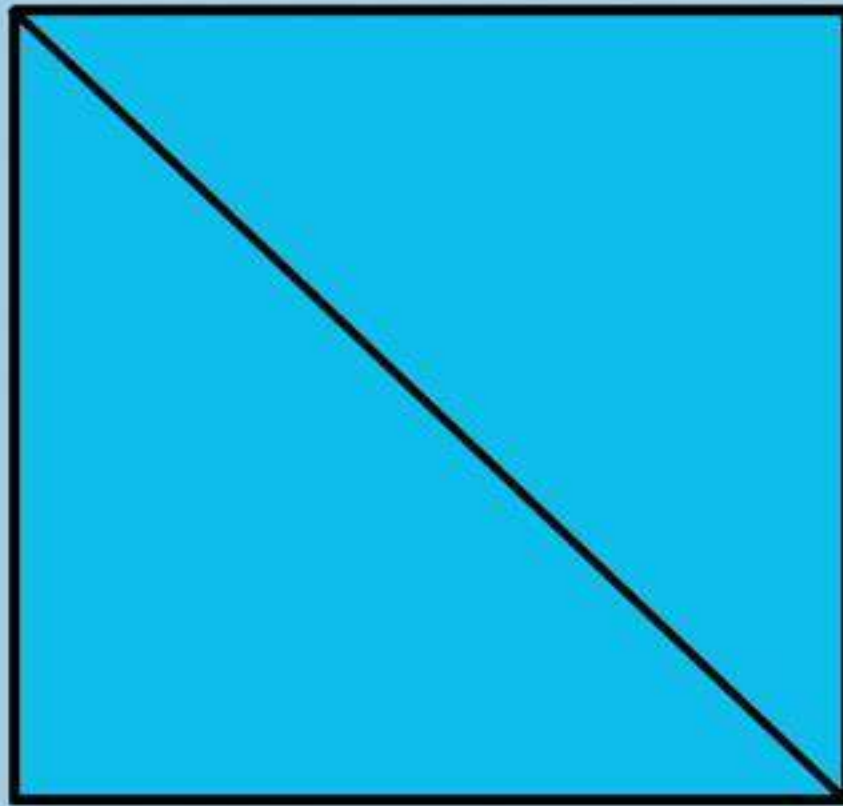
- Toric varieties are algebraic varieties that can be assembled by gluing copies of algebraic tori, $(\mathbb{C}_*)^n$, according to instructions encoded in a lattice polytope.
- Vast class of TV encoded in 4d reflexive polytopes $\Delta \subset \mathbb{Z}^4$.
- 4d reflexive polytopes have been classified.
- There are 473,800,776 of them.
- Each 'fine regular star triangulation' (FRST) of such a Δ yields a TV in which the generic anticanonical hypersurface is a smooth CY_3 .



Kreuzer

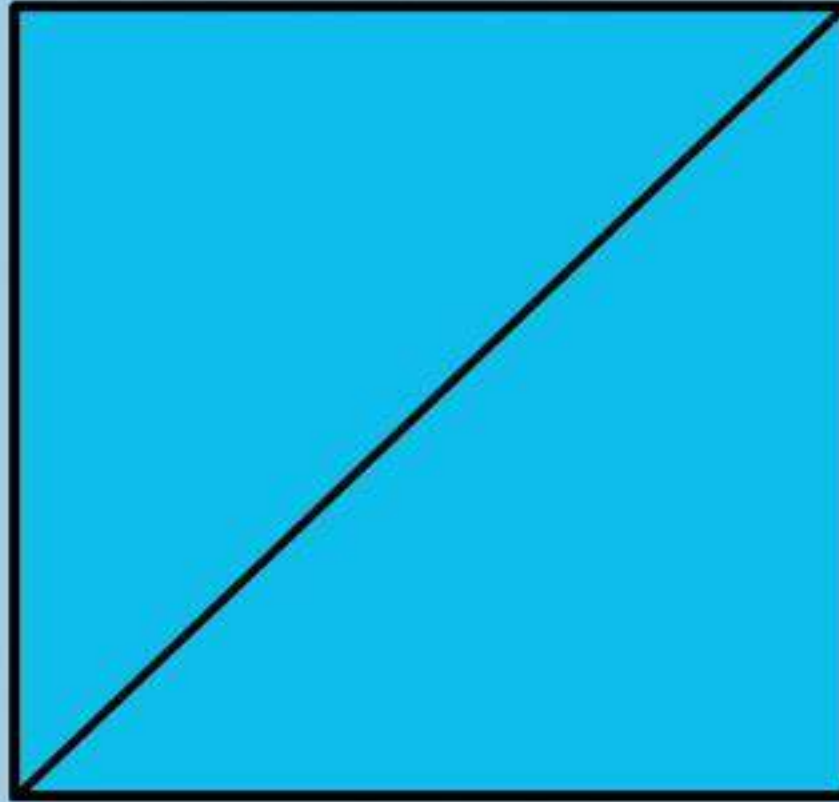


Triangulation



Triangulation of polytope = division into simplices.

Triangulation



Triangulation of polytope = division into simplices.

Fine: uses all the points.

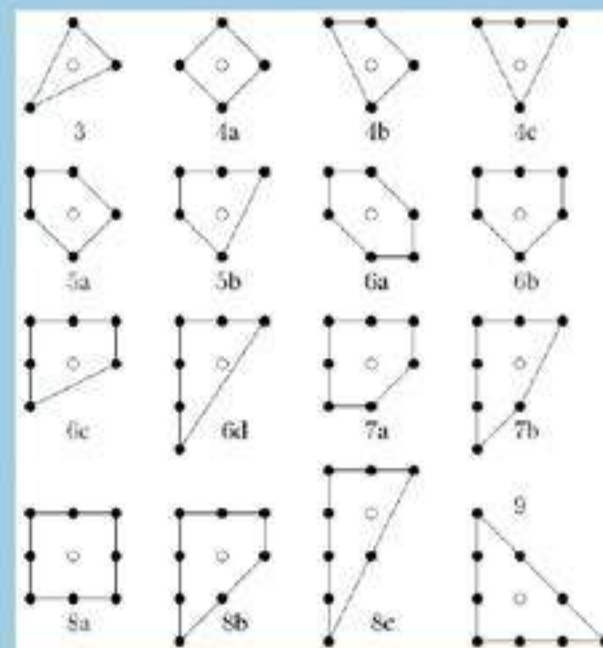
Star: all simplices contain origin.

Regular: descends from 1 higher dimension.

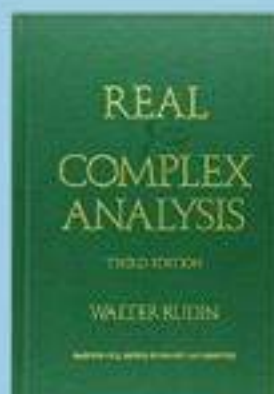
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Kreuzer



Task of triangulating polytopes is combinatorial, and amenable to automation.



analysis



algebraic geometry

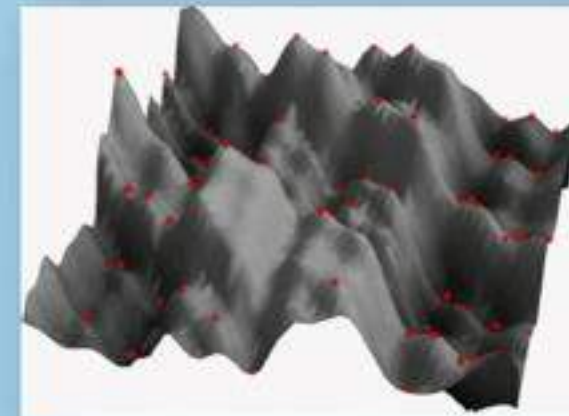


combinatorics

Example of discrete parameters

1. Choose a 4d reflexive polytope $\Delta \subset \mathbb{Z}^4$ 473,800,776
2. Choose a fine star regular triangulation of Δ , determining a toric variety V , and a CY_3 hypersurface $X \subset V$.
3. Choose quantized fluxes, $\vec{f}, \vec{h} \in H_3(X, \mathbb{Z}) \cong \mathbb{Z}^{2h^{2,1}(X)}$
4. Choose D7-branes $\{\vec{w}_A\} \in H_4(X, \mathbb{Z}) \cong \mathbb{Z}^{h^{1,1}(X)}$
5. For each D7-brane, choose bundle $\vec{\mathcal{F}}_A \in H^2(w_A, \mathbb{Z})$

One can then compute the potential, V .



Computational task

1. Triangulate 4d polytope
2. Examine fine regular star triangulation to obtain
 - a. Toric variety V
 - b. Intersection numbers of $X \subset V$

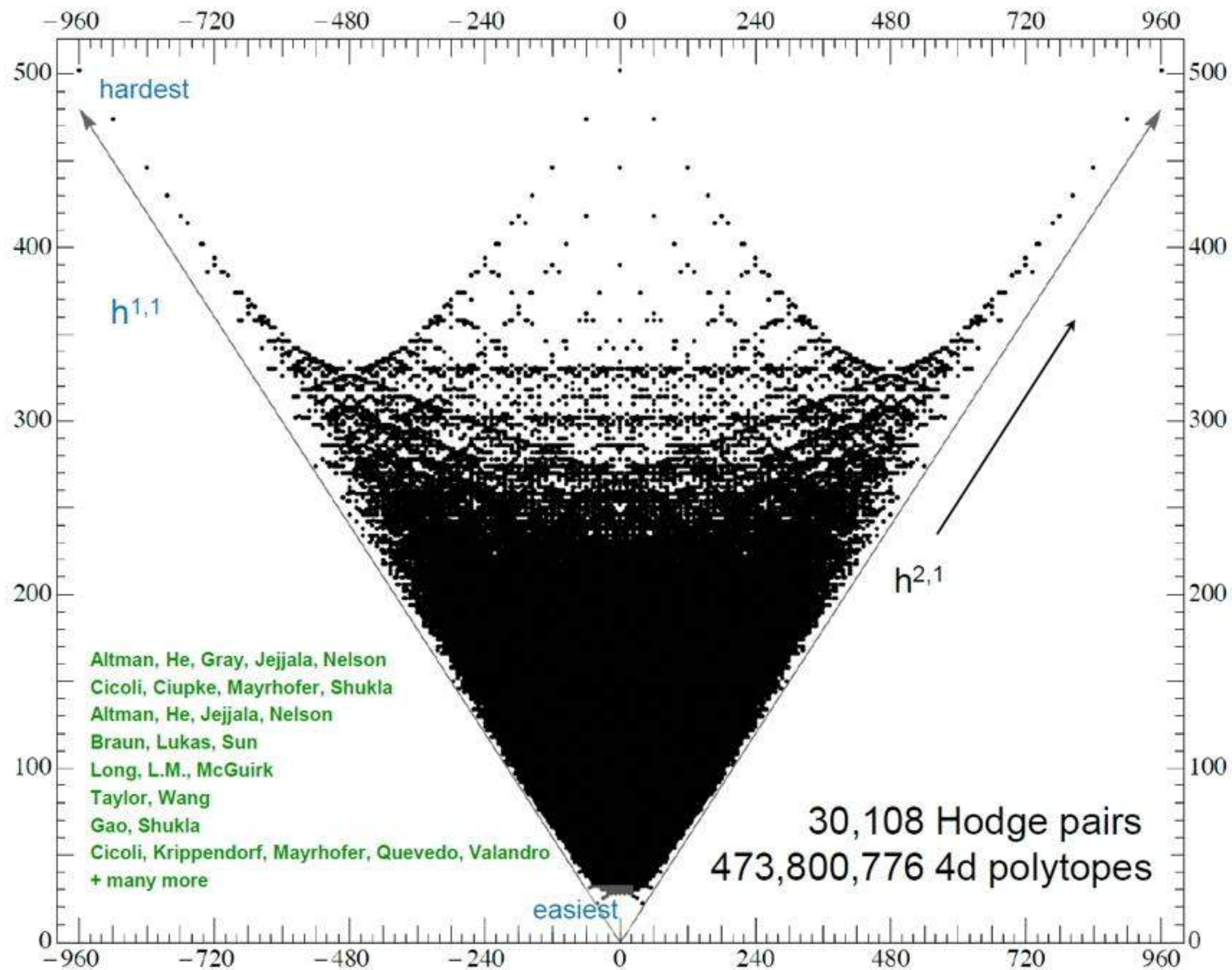
A. Braun

Oda, Park; Berglund, Katz, Klemm

Number of relevant points = $h^{1,1}(X) + 4$. ('favorable' case)

Number of intersection numbers $\sim (h^{1,1})^3$

$$1 \leq h^{1,1}(X) \leq 491$$



Number of FRSTs

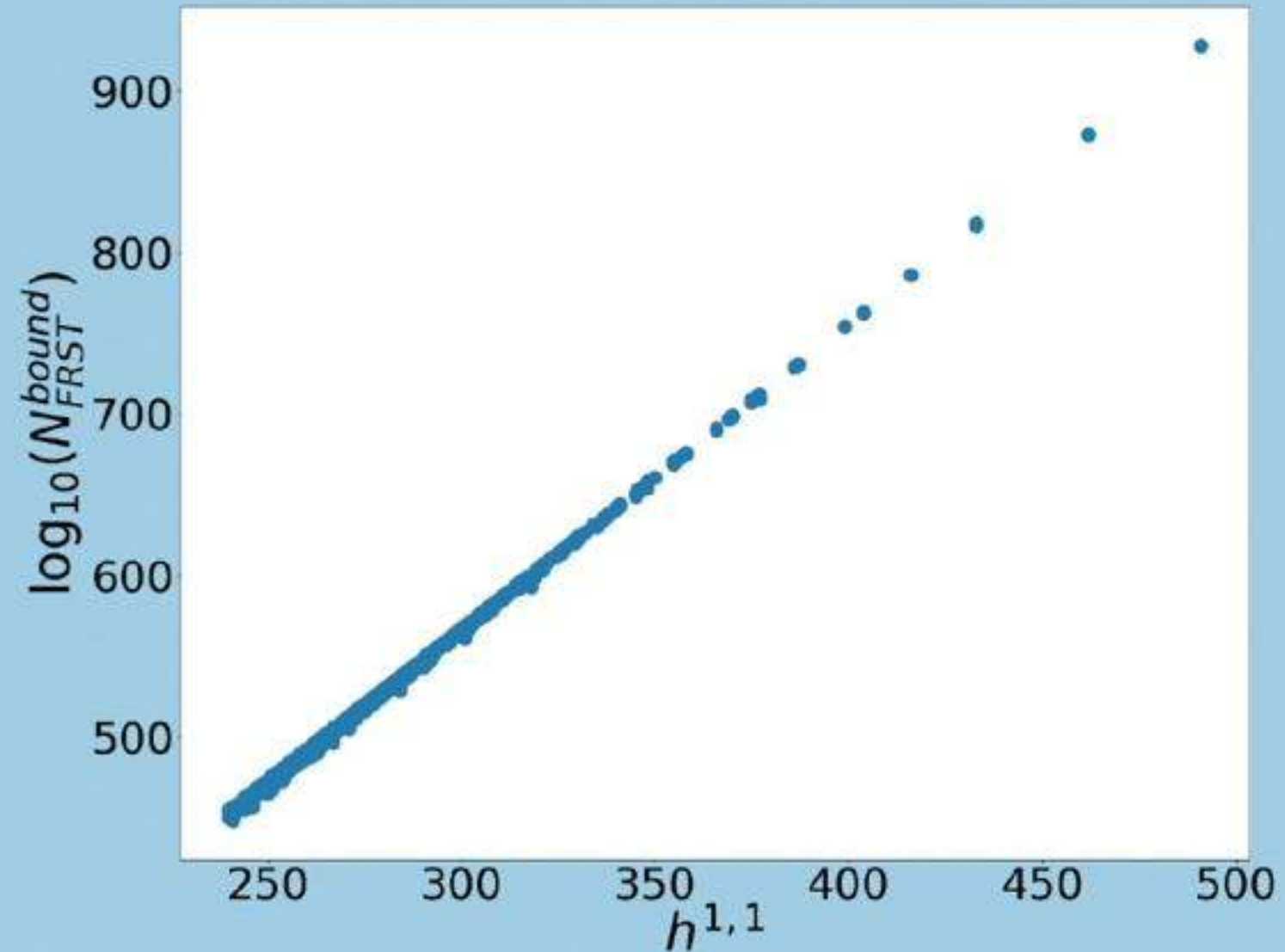
Number of triangulations/polytope grows combinatorially.

$$N_{\text{FRST}} \leq \binom{4V - 1}{h^{1,1} + 3}$$

V: volume of polytope

$$N_{\text{FRST}} \leq \binom{14,111}{494} \approx 1.5 \times 10^{928}$$

$$N_{\text{FRST}} \leq \binom{13,271}{465} \approx 2.7 \times 10^{863}$$



Why count solutions?

- Suppose string theory has a large but finite number $\mathcal{N}$ of isolated solutions that fulfill a set of criteria **C**.
 - e.g., three large dimensions, Standard Model gauge group, ...

Number of FRSTs

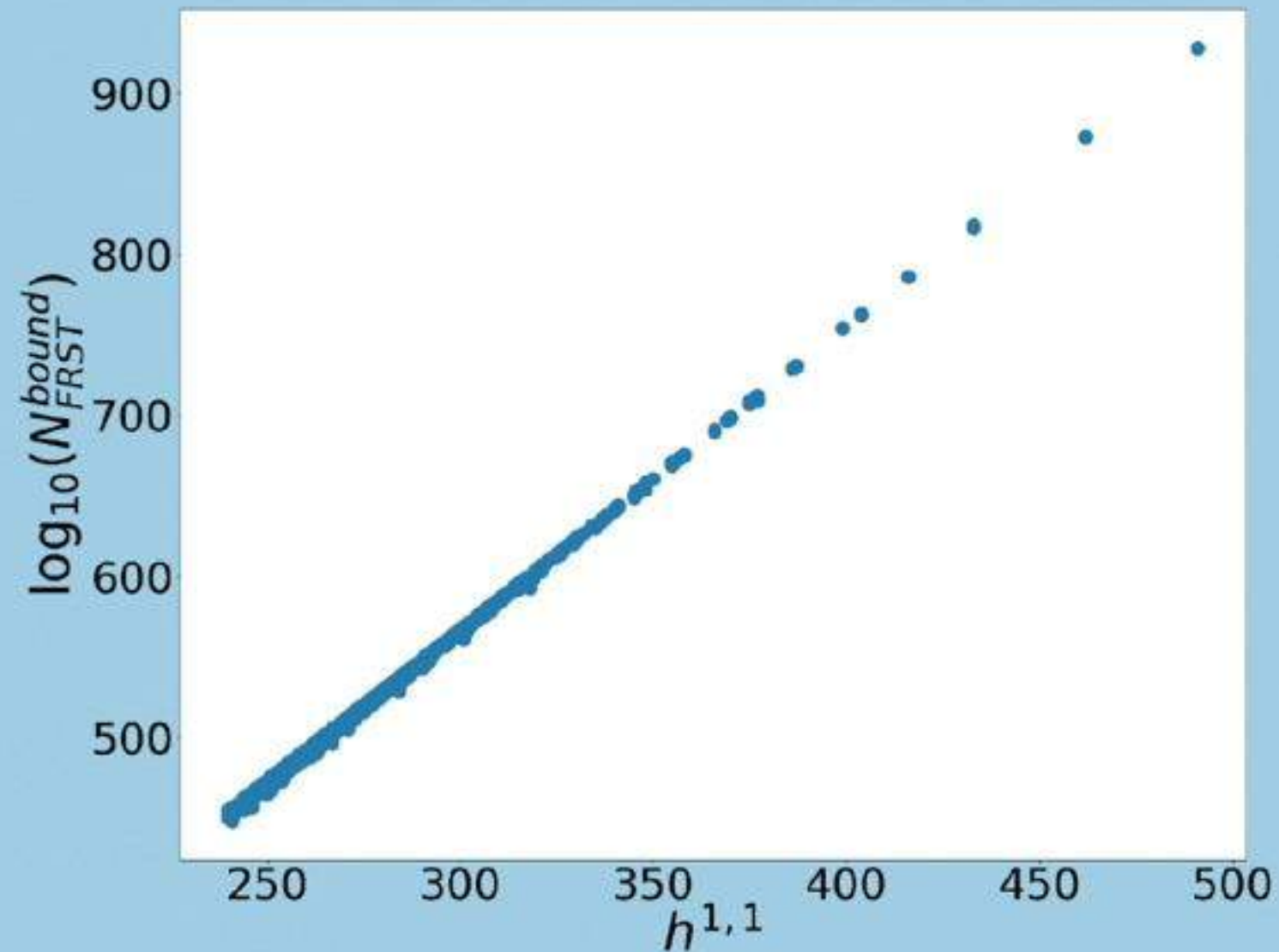
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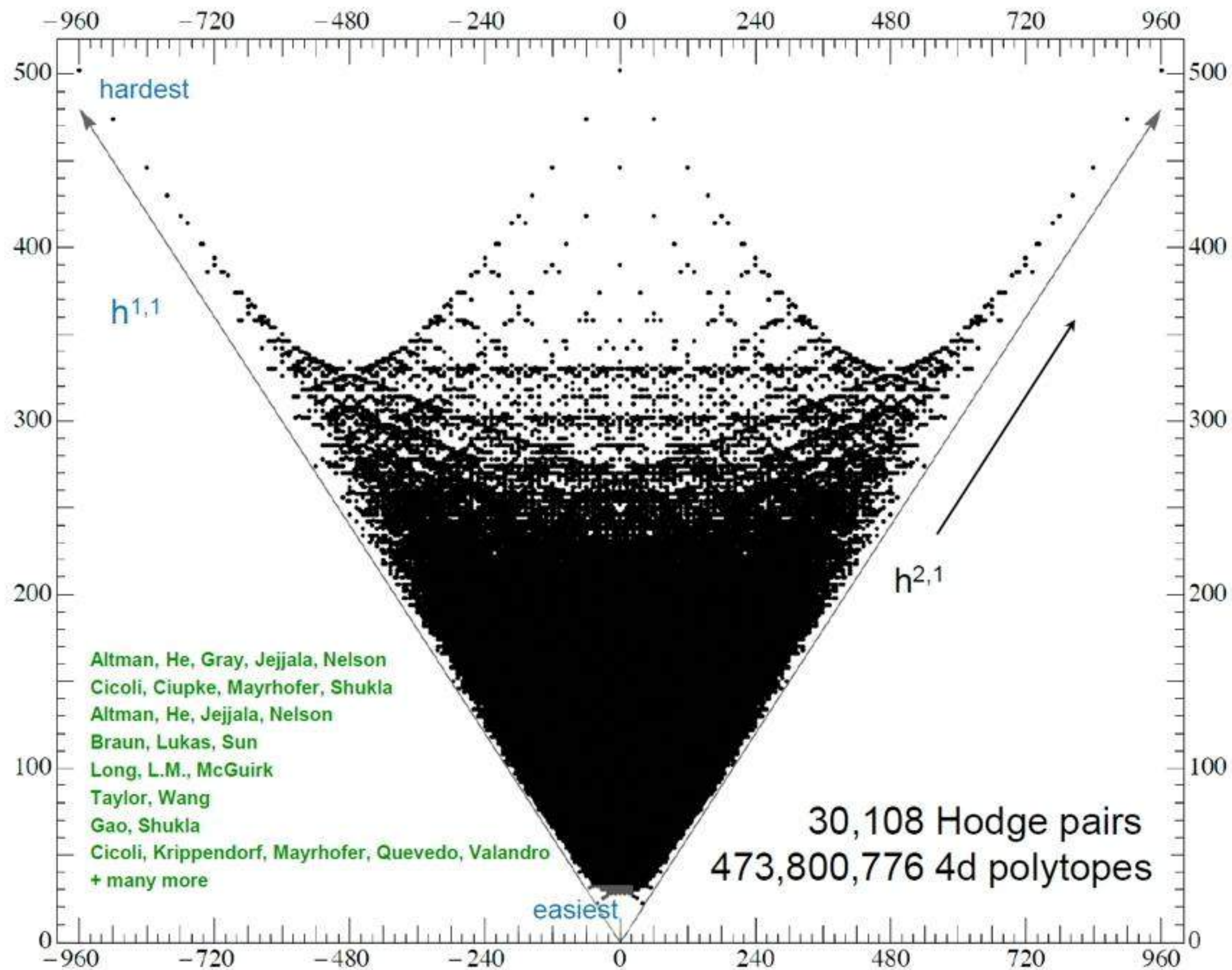
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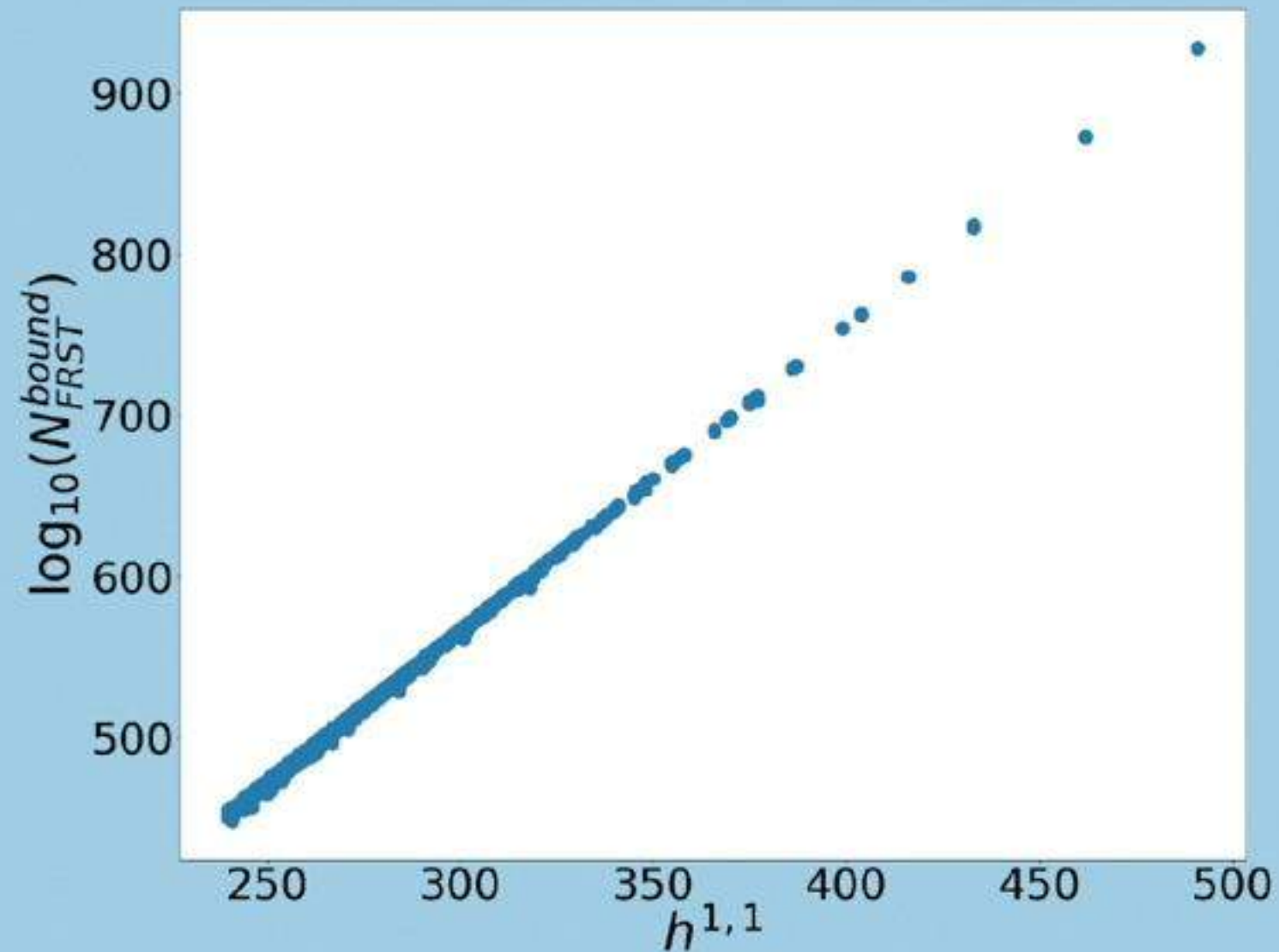
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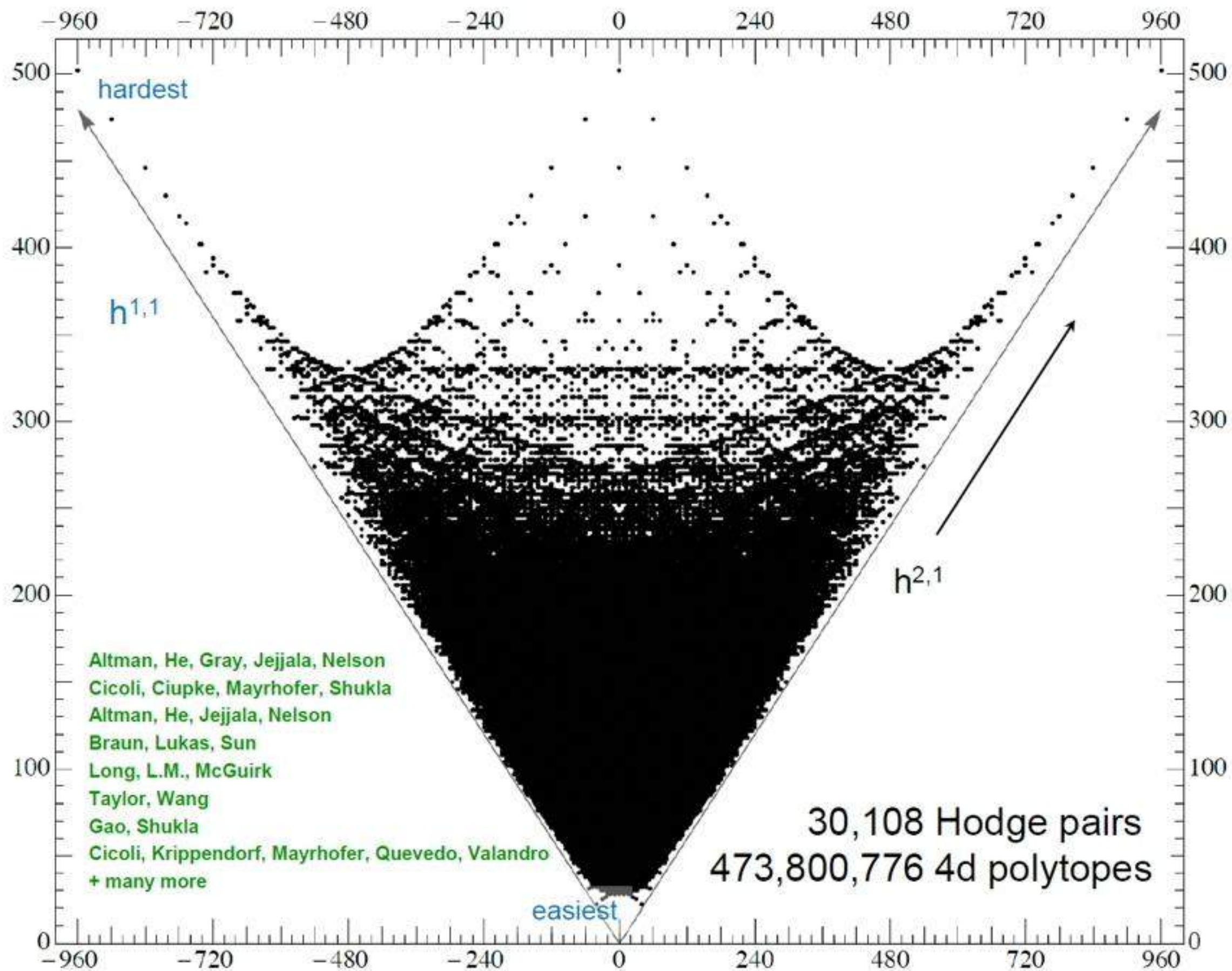
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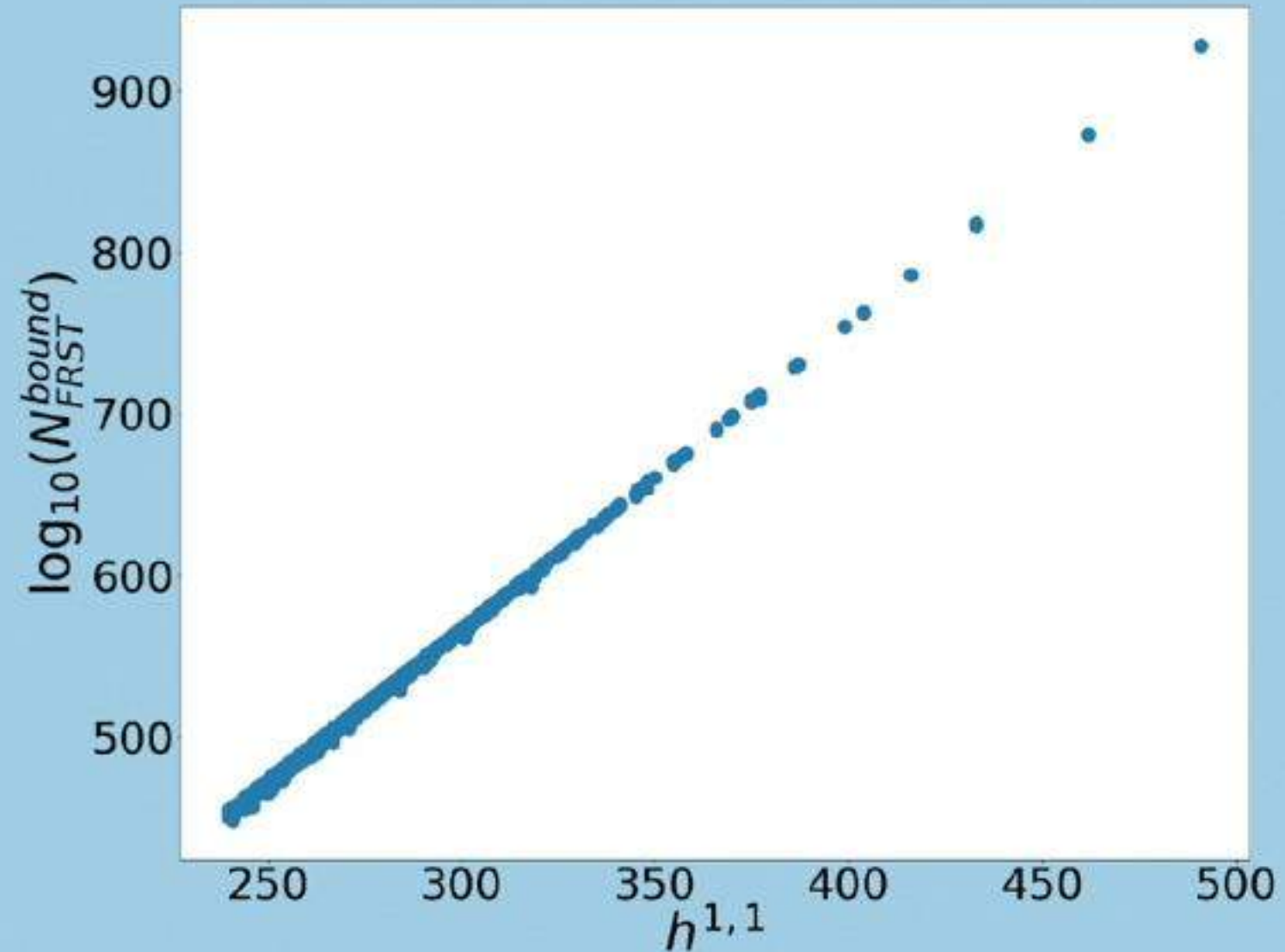
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$$N_{\text{FRST}} \leq \binom{4V - 1}{h^{1,1} + 3}$$

V : volume of polytope

$$N_{\text{FRST}} \leq \binom{14,111}{494} \approx 1.5 \times 10^{928}$$

$$N_{\text{FRST}} \leq \binom{13,271}{465} \approx 2.7 \times 10^{863}$$



Why count solutions?

- Suppose string theory has a large but finite number $\mathcal{N}$ of isolated solutions that fulfill a set of criteria **C**.
 - e.g., three large dimensions, Standard Model gauge group, ...
- If an **overwhelming fraction** of these solutions also have some property P,
 - e.g., more than one species of dark matter particlethen a reasonable **null hypothesis** is that P holds in our universe.
- This ‘counting measure’ prediction excludes **selection effects** during cosmic (pre)history.
- Observing P would be a successful prediction.
- Observing not-P would mean our universe involves **fine-tuning**, and/or **dynamical selection**, and/or is not described by solutions of string theory obeying **C**.

Number of FRSTs

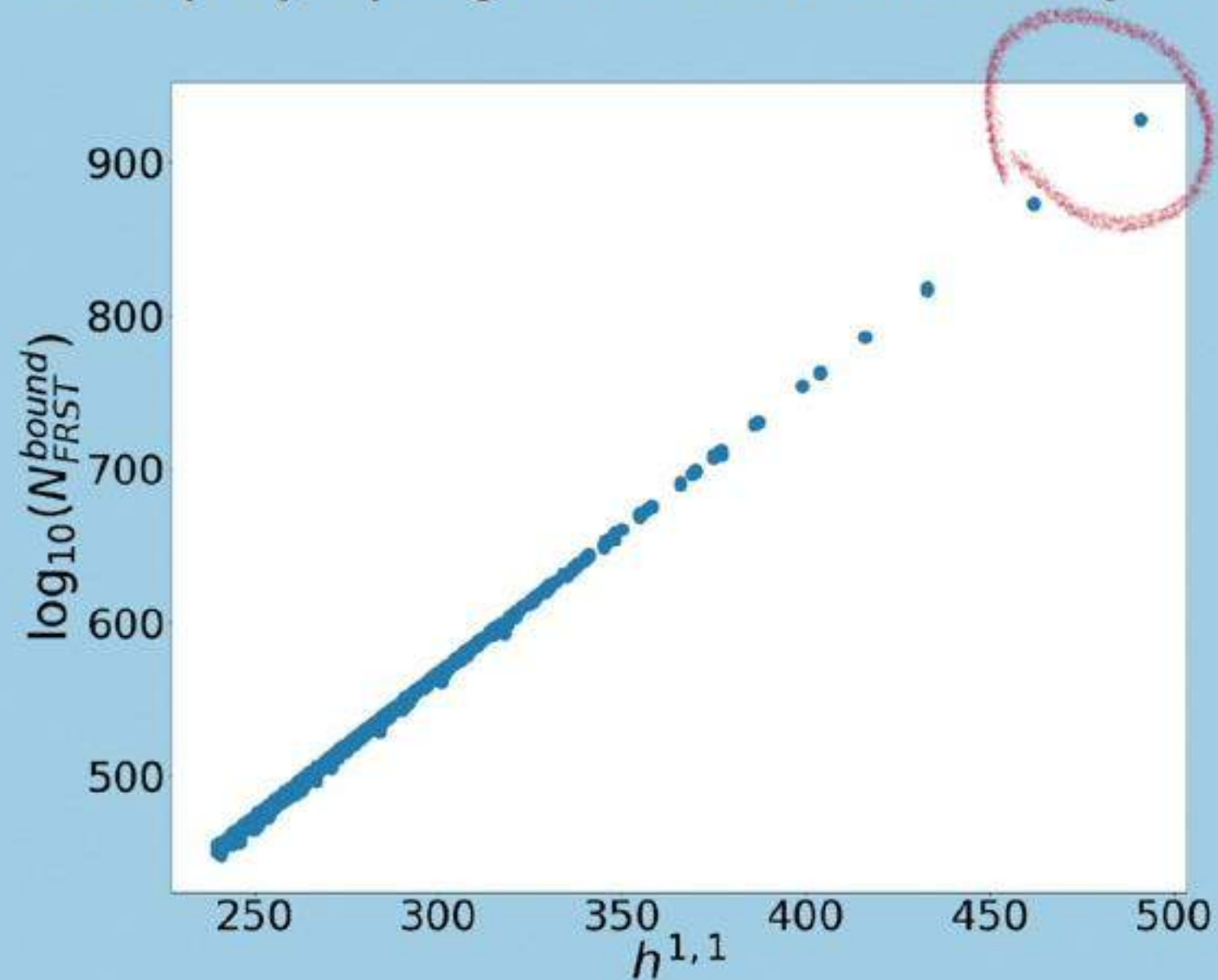
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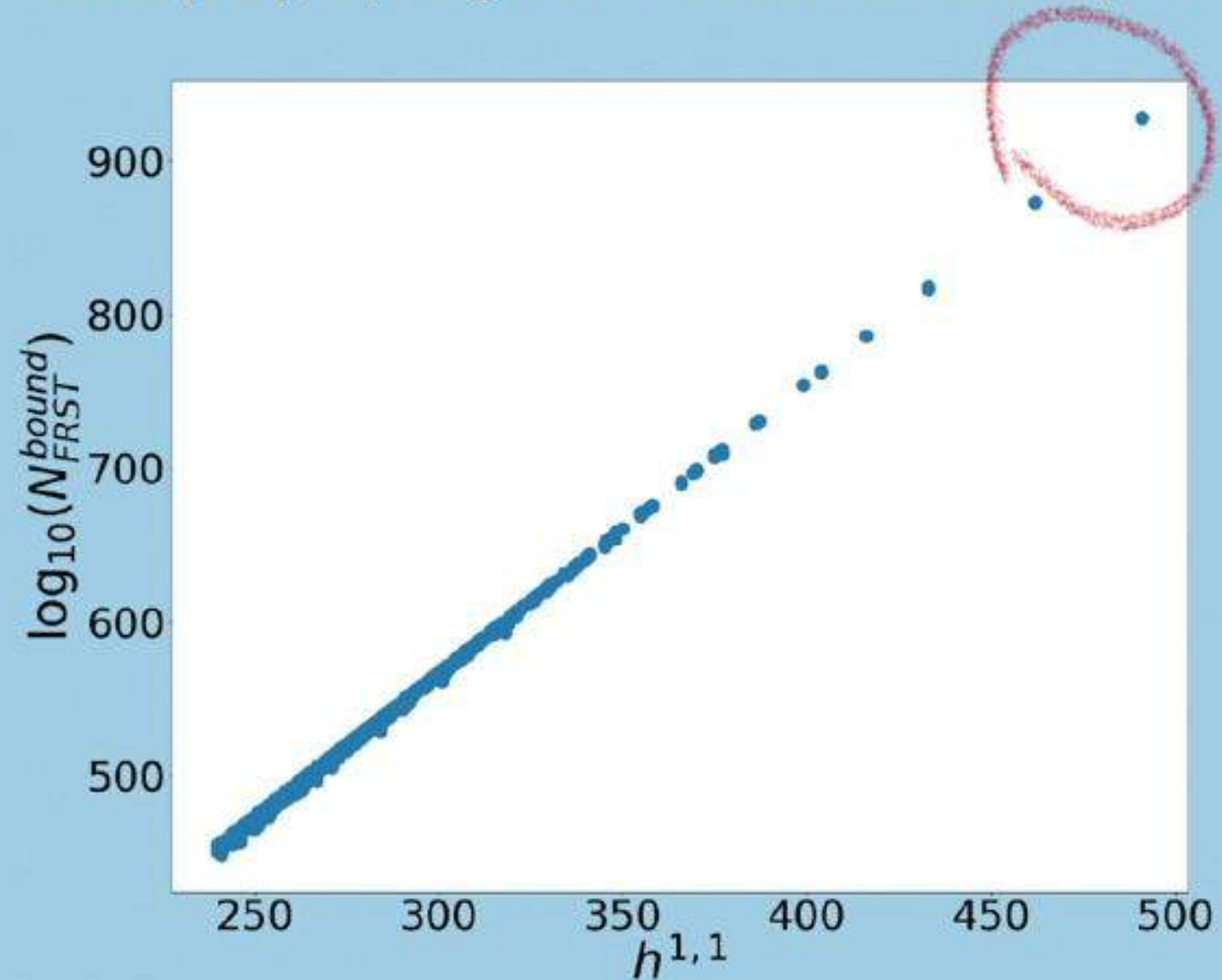
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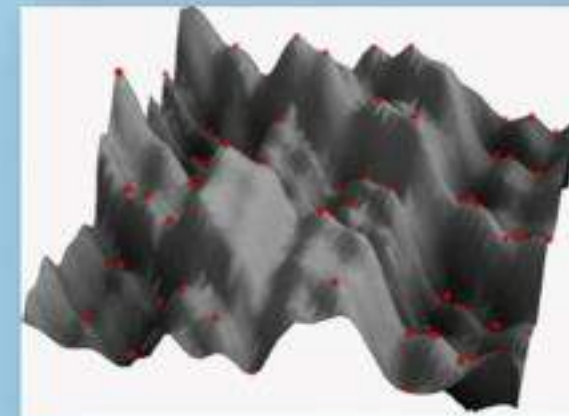
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Example of discrete parameters

1. Choose a 4d reflexive polytope $\Delta \subset \mathbb{Z}^4$ 473,800,776
2. Choose a fine star regular triangulation of Δ , determining a toric variety V , and a CY_3 hypersurface $X \subset V$.
3. Choose quantized fluxes, $\vec{f}, \vec{h} \in H_3(X, \mathbb{Z}) \cong \mathbb{Z}^{2h^{2,1}(X)}$
4. Choose D7-branes $\{\vec{w}_A\} \in H_4(X, \mathbb{Z}) \cong \mathbb{Z}^{h^{1,1}(X)}$
5. For each D7-brane, choose bundle $\vec{\mathcal{F}}_A \in H^2(w_A, \mathbb{Z})$

One can then compute the potential, V .

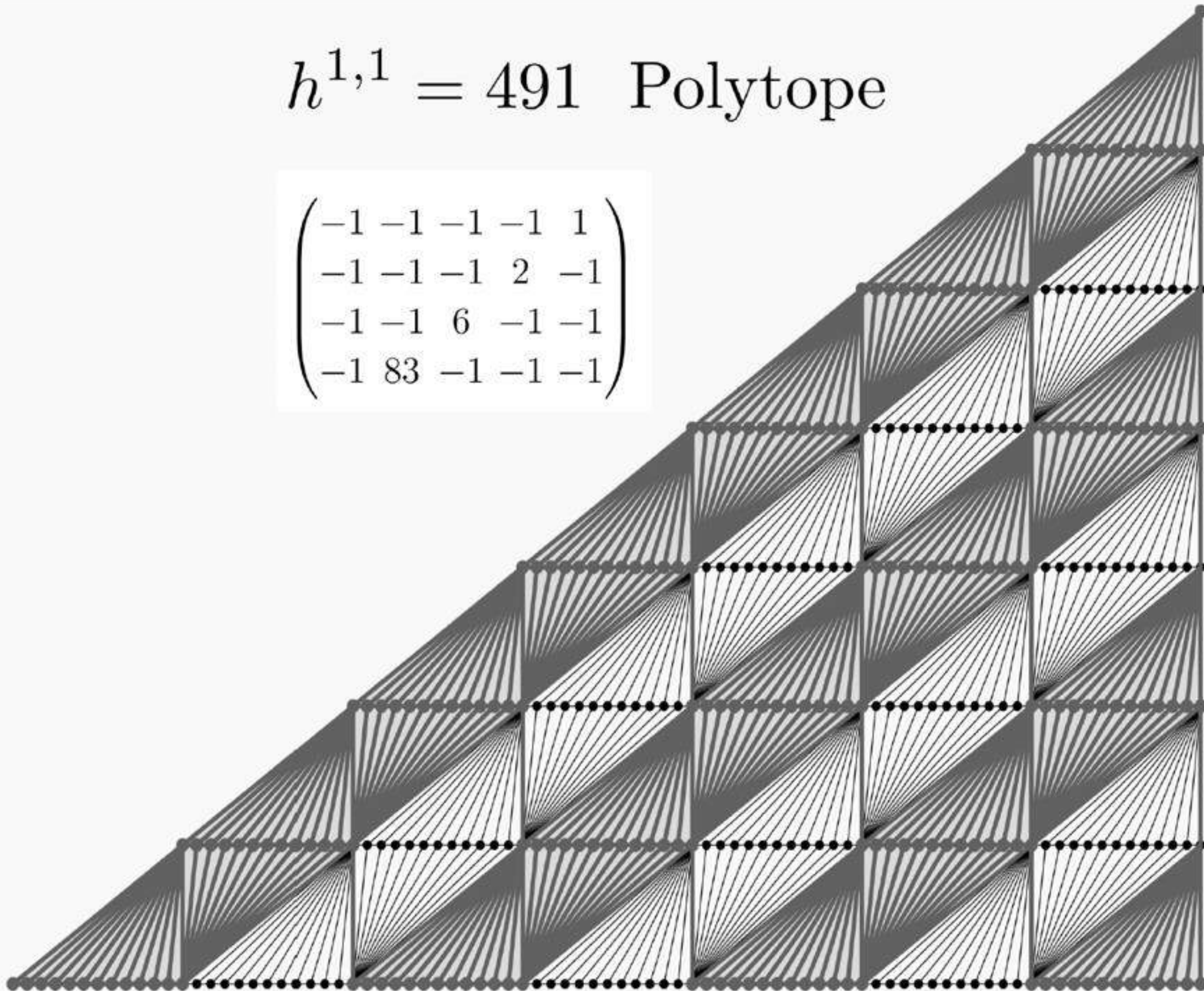


$$h^{1,1} = 491 \quad \text{Polytope}$$

$$\begin{pmatrix} -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 2 & -1 \\ -1 & -1 & 6 & -1 & -1 \\ -1 & 83 & -1 & -1 & -1 \end{pmatrix}$$

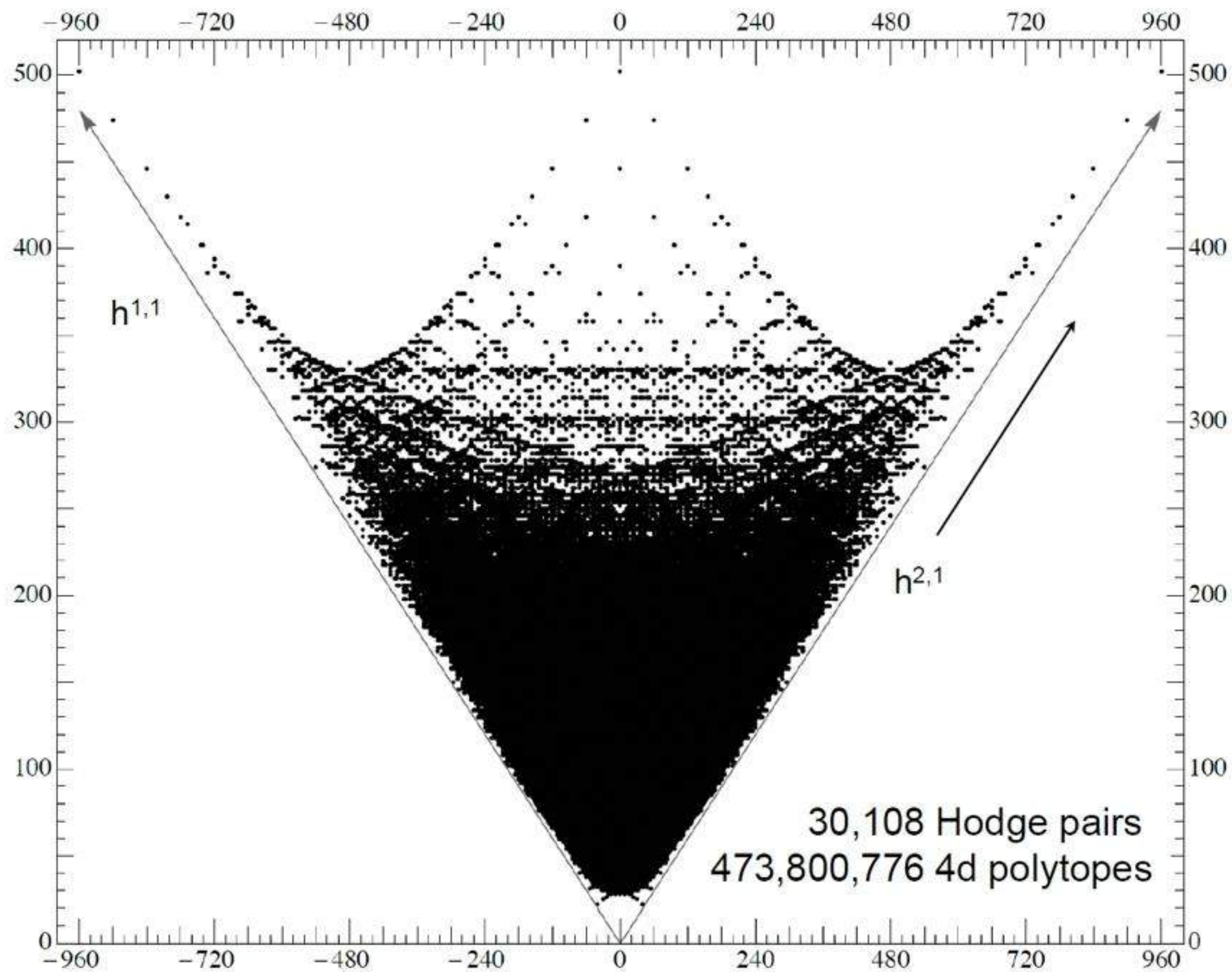
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Plan

- I. Invitation to quantum gravity
- II. Quantized parameters and the string landscape
- III. Example: toric hypersurfaces
- IV. Targets for ML



Target 1: Count Vacua

- Estimate or bound the number of triangulations of a polytope, when this is so large that direct enumeration is impossible.

Target 2: Find Desirable Vacua

- Given a polytope, find triangulations with a desired feature
 - e.g., top eigenvalue of metric $>$ threshold
 - e.g., given pattern of intersections of cycles
- ‘With’ a feature F : $P(F) \gg P_{\text{median}}(F)$
- Most features verifiable in ~seconds.

Target 3:

Predictions from Polytopes

- Given a polytope, predict features that hold for all triangulations.
- Or, for all triangulations in a new class that one identifies.

473,800,776 polytopes vs. $10^{928}(\text{??})$ triangulations

Summary of task

- We face a finite but extremely large set, the (CY_3 hypersurface subset of the) string landscape.
- We wish to evaluate various fitness functions on this set, and find local or global maxima.
- Milliseconds to seconds per evaluation.
- There are many correlations in this data, but there are not enough graduate students to extract them.
- Would be extremely helpful to find and exploit patterns via ML!
- Immense array of related problems, here and in other regions of the landscape.

Thanks!