Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

The future of mathematics?

Kevin Buzzard

Imperial College London

MSR, 5th September 2019.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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- Possible: tools such as Lean will begin to do research semi-autonomously, perhaps uncover problems in the literature. Maybe these tools will replace research mathematicians.
- In April, Christian Szegedy from Google told me that he believes that computers will be beating humans at math within ten years.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

Who am I?

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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- The proof of Fermat's Last Theorem is long, and structurally extremely complex. The advent of the internet means that proofs are getting longer.
- Nervousness about the state of the mathematical literature was one reason I started to experiment with computer theorem provers.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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The future of mathematics?
Kevin Buzzard
Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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- So my personal main goal at this point is to bring other mathematicans into the area, so things begin to happen more quickly.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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- October 2019 – it's going to be interesting.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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- The theorem of quadratic reciprocity,
- Sylow's theorems,
- the fundamental theorem of algebra,
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Sian Carey, Anca Ciobanu, Clara List and Ramon Fernandez Mir have all formalised mathematics in Lean as part of projects.

The future of mathematics?
Kevin Buzzard
Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

See Computers and Mathematics, London Mathematical Society newsletter, September 2019 (pages 32 to 36) for more details of my work with undergraduates.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Genuine dumb problem: it's hard for a maths UG to install Lean and its maths library on Windows, because they need to use the command line. We can use CoCalc instead – it costs money, but I have run some successful webinars on it.

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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$$(\exists \varphi : X/\sim \to Y, \forall x \in X, \varphi(\pi(x)) = f(x)) \Leftrightarrow (\forall x, x' \in X,\ x \sim x' \Rightarrow f(x) = f(x')).$$

```
theorem passage_au_quotient (X Y : Type) (s : setoid X) (f : X → Y) :
(∃ φ : quotient s → Y, ∀ x : X, φ(⟦x⟧) = f x) ↔ (∀ x x', x ≈ x' → f x = f x') :=
```

Démonstration

Montrons les deux implications.

On commence par supposer la condition de gauche.

Fixons un φ vérifiant cette propriété.

Soit x et x' des éléments équivalents de X.

On veut montrer que $f(x) = f(x')$. Vu la propriété supposée pour φ, on peut réécrire le membre de gauche comme $\varphi(\pi(x))$ et celui de droite comme $\varphi(\pi(x))$.

```
rw ← Hφ  x,
rw ← Hφ x',
```

Le point clef est que, puisque $x \sim x'$, le théorème fondamentalde la théorie des quotients assure $\pi(x) = \pi(x')$

```
have clef : ⟦x⟧ = ⟦x'⟧,
{ exact quotient.sound Hxx' },
```

On conclue en reportant cette égalité dans notre objectif, qui devient une tautologie.

```
rw clef },
```

Réciproquement, supposons la condition de droite et construisons une fonction φ convenable.

```
intro hyp,
```

Le théorème fondamental assure que π est surjectif.

```
have surj : ∀ q, ∃ x : X, ⟦x⟧ = q,
{ apply quotient.exists_rep },
```

L'axiome du choix donne alors une fonction $\sigma : X/\sim \to X$ qui est un inverse à droite de π.

```
choose σ Hσ using surj,
```

Montrons que la fonction qui envoie q sur $f(\sigma(q))$ convient,

```
X Y : Type,
s : setoid X,
f : X → Y,
hyp : ∀ (x x' : X), x ≈ x' → f x = f x',
surj : ∀ (q : quotient s), ∃ (x : X), ⟦x⟧ = q
⊢ ∃ (φ : quotient s → Y), ∀ (x : X), φ ⟦x⟧ = f x
```

See Computers and Mathematics, London Mathematical Society newsletter, September 2019 (pages 32 to 36) for more details of my work with undergraduates.

The main frustrations for undergraduates were *pragmatic* rather than foundational. For example, lack of appropriate documentation (I will fix this).
Genuine dumb problem: it's hard for a maths UG to install Lean and its maths library on Windows, because they need to use the command line. We can use CoCalc instead – it costs money, but I have run some successful webinars on it.

Conclusions: it is possible to teach undergraduate mathematicians how to do some of their homework in Lean. Forthcoming paper by Iannone and Thoma will say something more formal about what I have achieved so far.

In October 2019 all of the homework in my course will be in Lean format, and all of the course notes too. Like this. My job is to teach the undergraduates what a proof is. Lean is a wonderful tool for this.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

Example of what I have learnt myself from using Lean:
First part of first question on first problem sheet of my course:
“True or false – if x is a real number, and $x^2 - 3x + 2 = 0$, then $x = 1$.”

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Lean: "OK, so it now suffices to prove that (a) $2^2 - 3 \times 2 + 2 = 0$ and that (b) $2 \neq 1$.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Lean: "OK, so it now suffices to prove that (a) $2^2 - 3 \times 2 + 2 = 0$ and that (b) $2 \neq 1$.

Me in 2017: "…"

A few weeks later, this was fixed by computer scientists, who wrote a tactic which solved these goals.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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- I think there is a non-zero chance that some of our great castles are built on sand. But I think it's small.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

The statement of Fermat's Last Theorem can be explained to a high school kid. What does the proof of Fermat's Last Theorem look like?

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

The statement of Fermat's Last Theorem can be explained to a high school kid. What does the proof of Fermat's Last Theorem look like?

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

The statement of Fermat's Last Theorem can be explained to a high school kid. What does the proof of Fermat's Last Theorem look like?

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

The statement of Fermat's Last Theorem can be explained to a high school kid. What does the proof of Fermat's Last Theorem look like?

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- Then you prove some really profound theorems about some of these objects, using the rest of these objects.
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- The full proof takes thousands of pages.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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We accepted the proof of the odd order theorem in 1970 – that's why we gave John Thompson a Fields Medal.

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Introduction

What is this talk about?

Who am I?

Why Lean?

The Xena Project.

What can a mathematics undergraduate do?

Lean in Research

Can Lean handle modern maths?

Yes it can.

What next?

Summary

So if proper mathematicians aren't interested in a proof of the odd order theorem, what are they interested in?

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Example: Perfectoid spaces.

So if proper mathematicians aren't interested in a proof of the odd order theorem, what are they interested in?

Example: Perfectoid spaces.

	Proof of odd order theorem	Perfectoid spaces
Got author a Fields Medal?	Yes (1970)	Yes (2018)
High level mathematics?	No	Yes
Lots of PhD students and post-docs working in the area?	No	Yes
Talks happening about these things all over the world?	No	Yes
Mathematicans interested in 2019?	No	Yes

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Mathematical aside: why is formalising a definition hard work?

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Mathematical aside: why is formalising a definition hard work?

A real manifold is a topological space which locally looks like a ball. For this to typecheck we need to know that a ball is a topological space. This is not difficult.

A perfectoid space is a locally ringed space which locally looks like an affinoid perfectoid space. For this to typecheck we need to show that affinoid perfectoid spaces are locally ringed spaces (or actually something slightly weaker).

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

As I've said, my next step is to get more research mathematicians using the software.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Why? To make some powerful high-level tools which future mathematicians will use, we need to teach Lean hundreds, or maybe thousands, of high-level mathematical definitions.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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Why? To make some powerful high-level tools which future mathematicians will use, we need to teach Lean hundreds, or maybe thousands, of high-level mathematical definitions. Advances in comprehension of natural language will not do this for us.

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Why? To make some powerful high-level tools which future mathematicians will use, we need to teach Lean hundreds, or maybe thousands, of high-level mathematical definitions. Advances in comprehension of natural language will not do this for us. This has “synergy” written all over it.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

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We mathematicians don't see the modern complex mathematical objects which we use every day, in theorem provers. Yet. I just wrote some EU grant proposal to fund post-docs who will write a bunch of Lean code defining the objects which "make a mathematican tick". And then (following Tom Hales) we can start to make a database, or a network, mapping out the state of the beliefs of the elders.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

Conclusions:

- Lean's type theory seems to be perfect for modern pure mathematics.

Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
Yes it can.
What next?
Summary

Conclusions:

- Lean's type theory seems to be perfect for modern pure mathematics.
- Crucial next step: put some modern pure mathematics into it.
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Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
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What next?
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What is this talk about?
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The Xena Project.
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Lean in Research
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Introduction
What is this talk about?
Who am I?
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The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
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Introduction
What is this talk about?
Who am I?
Why Lean?
The Xena Project.
What can a mathematics undergraduate do?
Lean in Research
Can Lean handle modern maths?
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Lean in Research
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Thanks for coming!